1. \( \lim_{x \to 0} \sec \left( \frac{\sin 2\pi x}{6x} \right) = \)
   
   (a) 1
   
   (b) \( \frac{2}{\sqrt{3}} \)
   
   (c) 2
   
   (d) \( \sqrt{2} \)
   
   (e) The limit does not exist

2. Suppose \( f(0) = 2, f'(0) = 3, f(2) = -1, f'(2) = -2, g(0) = 2, g'(0) = 4, g(2) = -1, g'(2) = 0 \). If \( h(x) = f(g(x)) \), then \( h'(0) = \)
   
   (a) -4
   
   (b) -8
   
   (c) 0
   
   (d) 12
   
   (e) 8

3. Let \( f \) be some function for which you know only that

   \[
   \text{if} \quad 0 < |x - 3| < 1, \quad \text{then} \quad |f(x) - 4| < 0.1.
   \]

   Which of the following statements are necessarily true?

   I. If \( |x - 3| < 0.1 \), then \( |f(x) - 4| < 0.01 \).
   II. If \( |x - 2.6| < 0.3 \), then \( |f(x) - 4| < 0.1 \).
   III. If \( 0 < |x - 3| < 0.5 \), then \( |f(x) - 4| < 0.1 \).
   IV. \( \lim_{x \to 3} f(x) = 4 \).

   (a) II and IV
   (b) IV only
   (c) I and II
   (d) II and III
   (e) II, III and IV
4. A function $f$ is defined on an interval $[a, b]$. Which of the following statements could be false?

I. If $f(a)$ and $f(b)$ have opposite sign, then there must be a point $c \in (a, b)$ such that $f(c) = 0$.

II. If $f$ is continuous on $[a, b]$, $f(a) < 0$ and $f(b) > 0$, then there must be a point $c \in (a, b)$ such that $f(c) = 0$.

III. If $f$ is continuous on $[a, b]$ and there is a point $c$ in $(a, b)$ such that $f(c) = 0$, then $f(a)$ and $f(b)$ have opposite sign.

IV. If $f$ has no zeros on $[a, b]$, then $f(a)$ and $f(b)$ have the same sign.

(a) II only
(b) II, III
(c) I, III, IV
(d) II, IV
(e) I, III

5. An equation for the normal line to the curve $2x^3 + 2y^2 = 5xy$ at the point $(1, 2)$ is

(a) $3x - 4y = 2$
(b) $3x + 4y = -5$
(c) $3x + 4y = 11$
(d) $4x - 3y = -2$
(e) $4x + 3y = 10$

6. Set

$$g(x) = \begin{cases} \sqrt{x} + 1 & 0 < x < 3, \\ bx + 2 & 3 \leq x < 5. \end{cases}$$

The values of $a$ and $b$ such that $g$ is differentiable on $(0, 5)$ are:

(a) $a = 4, b = 2$
(b) $a = -8/5, b = -2/5$
(c) $a = 2, b = 3/2$
(d) $a = -1, b = -3/4$
(e) $a = 8/5, b = 2/5$
7. Set \( f(x) = 4x^2 - x^3 \), and let \( L \) be the line \( y = 18 - 3x \), where \( L \) is tangent to the graph of \( f \). Let \( S \) be the region bounded by the graph of \( f \), the line \( L \) and the \( x \)-axis. The area of \( S \) is:

(a) \( \frac{103}{12} \)
(b) \( \frac{43}{6} \)
(c) \( \frac{101}{12} \)
(d) \( \frac{95}{12} \)
(e) \( \frac{53}{6} \)

8. A particle is moving along the \( x \)-axis so that its velocity at time \( t \), \( 0 \leq t \leq 10 \) is

\[ v(t) = \ln \left( t^2 - 3t + 3 \right). \]

During which time intervals is the particle moving to the left?

(a) \( 2 < t \leq 10 \)
(b) The particle never moves left
(c) \( 1 < t < 2 \)
(d) \( 0 \leq t < 1 \)
(e) \( 0 < t < 5 \)

9. Let \( R \) be the region bounded by the graph of \( f(x) = \sqrt{x - 1} \), the vertical line \( x = 10 \), and the \( x \)-axis. Find the volume of the solid generated when \( R \) is revolved about the line \( y = 3 \).

(a) \( \frac{135\pi}{2} \)
(b) \( \frac{99\pi}{2} \)
(c) \( \frac{189\pi}{2} \)
(d) \( \frac{119\pi}{2} \)
(e) \( \frac{137\pi}{2} \)
10. If \( \int_{1}^{4} f(x) \, dx = 5 \), \( \int_{3}^{4} f(x) \, dx = 7 \), and \( \int_{1}^{8} f(x) \, dx = 11 \), then \( \int_{3}^{8} f(x) \, dx = \).

(a) \(-9\)
(b) \(13\)
(c) \(-1\)
(d) \(9\)
(e) \(-13\)

11. If \( f \) is a continuous function and \( F(x) = \int_{0}^{x} \left[ (2t + 3) \int_{t}^{2} f(u) \, du \right] \, dt \), then \( F''(2) = \)

(a) \(-2f(2)\)
(b) \(-7f(2)\)
(c) \(7f'(2)\)
(d) \(3f'(2)\)
(e) \(7f(2)\)

12. A curve in the plane is defined by the parametric equations: \( x = e^{2t} + 2e^{-t} \), \( y = e^{2t} + e^{t} \). An equation for the line tangent to the curve at the point where \( t = \ln 2 \) is:

(a) \(7x + 10y = -8\)
(b) \(5x - 6y = -11\)
(c) \(5x - 3y = 7\)
(d) \(10x - 7y = 8\)
(e) \(3x - 2y = 3\)
13. Which of the following series converges to 2?

I. \( \sum_{n=0}^{\infty} \frac{1}{2^n} \)

II. \( \sum_{n=1}^{\infty} \frac{n}{2n^2 + 1} \)

III. \( \sum_{n=1}^{\infty} \frac{4}{3^n} \)

(a) I only  
(b) II only  
(c) I and III  
(d) III only  
(e) II and III

14. The function \( f(x) = \int_{3}^{x} \sqrt{16 + t^2} \, dt \) has an inverse. Find \( (f^{-1})'(0) \).

(a) 5  
(b) \( \frac{1}{4} \)  
(c) 4  
(d) 3  
(e) \( \frac{1}{5} \)

15. Suppose that the power series \( \sum_{k=0}^{\infty} a_k (x - 1)^k \) converges at \( x = 3 \). Which of the following series must be convergent?

I. \( \sum_{k=0}^{\infty} a_k \)

II. \( \sum_{k=0}^{\infty} a_k 3^k \)

III. \( \sum_{k=0}^{\infty} (-1)^k a_k \)

IV. \( \sum_{k=0}^{\infty} (-1)^k a_k 2^k \)

(a) I only  
(b) II and III  
(c) I and III  
(d) II, III, and IV  
(e) III and IV
16. A point \((x, y)\) is moving along a curve \(y = f(x)\). At the instant when the slope of the curve is \(-3/5\), the \(x\)-coordinate of the point is decreasing at the rate of 4 units per second. The rate of change, in units per second, of the \(y\)-coordinate is

(a) \(12/5\)
(b) \(3/20\)
(c) \(-12/5\)
(d) \(5/12\)
(e) \(-20/3\)

17. The region in the first quadrant bounded by the graph of \(y = e^{2x}\), the vertical line \(x = \ln 3\), and the \(x\)-axis is revolved about the \(y\)-axis. Find the volume of the solid that is generated.

(a) \(2\pi [3 \ln 3 - 2]\)
(b) \(\pi [9 \ln 3 + 4]\)
(c) \(9\pi/2\)
(d) \(\pi [9 \ln 3 - 4]\)
(e) \(\pi \left[\frac{5}{2} \ln 3 - \frac{1}{2}\right]\)

18. The length of the graph of \(f(x) = \ln \sec x\), \(0 \leq x \leq \pi/3\) is:

(a) \(3 + \sqrt{2}\)
(b) \(\ln \left(2 + \sqrt{3}\right)\)
(c) \(\ln \left(\sqrt{3}\right)\)
(d) \(2 + \sqrt{3}\)
(e) \(\ln \left(\frac{1 + \sqrt{3}}{2}\right)\)
19. \( \{a_n\} \) is a sequence of real numbers. Which of the following statements are necessarily true?

I. If \( a_n > 0 \) for all \( n \) and \( a_n \to L \), then \( L > 0 \).

II. If \( \{a_n\} \) is not bounded below, then it diverges.

III. If \( a_n \geq 0 \) for all \( n \) and \( a_n \to L \), then \( L \geq 0 \).

IV. If \( \{a_n\} \) is increasing and bounded above, then it converges.

(a) III only
(b) I, III
(c) II, IV
(d) II, III, IV
(e) II, III

20. Let \( f \) be a function such that \( |f^{(n)}(x)| \leq 1 \) for all \( x \) and \( n \). Find the least integer \( n \) such that the Taylor polynomial of degree \( n \) at \( x = 0 \) approximates \( f(1/2) \) to within 0.0005.

(a) 2
(b) 3
(c) 4
(d) 5
(e) 6

21. Evaluate the improper integral \( \int_{-\infty}^{\infty} \frac{1}{e^x + e^{-x}} \, dx \).

(a) \( \pi/2 \)
(b) 2
(c) 1
(d) \( \pi/4 \)
(e) divergent
22. The curves $r = \sqrt{2}$ and $r = 2 \cos \theta$ are shown in the figure. Find the area of the shaded region.

(a) $\pi + 1$
(b) $\frac{\pi + 1}{2}$
(c) $\pi - \frac{1}{4}$
(d) $\frac{\pi - 2}{4}$
(e) $\frac{\pi - 1}{2}$

23. The function $f$ is infinitely differentiable, $f(2) = 4$, and

$$f^{(n)}(2) = \frac{(n-1)!}{3^n} \text{ for all } n \geq 1.$$  

The interval of convergence of the Taylor series for $f$ in powers of $x - 2$ is:

(a) $-3 < x < 3$
(b) $-1 \leq x < 3$
(c) $-3 \leq x \leq 3$
(d) $-1 < x < 5$
(e) $-1 < x \leq 5$

24. \[ \lim_{x \to \infty} e^{-x^2} \int_0^x 2te^{t^2} \, dt = \]

(a) 0
(b) $\frac{1}{2}$
(c) 1
(d) $e$
(e) The limit does not exist.
25. An advertising company has designed a campaign to introduce a new product to city of 2 million people. Let \( P = P(t) \) denote the number of people who are aware of the product at time \( t \) and assume that \( P \) increases at a rate proportional to the number of people still unaware of the product. If no one knew about the product at the beginning of the campaign \( [P(0) = 0] \) and 40% of the people are aware of the product after 3 months of advertising, how long will it take for 90% of the population to be aware of the product?

(a) 11.65 mos  
(b) 10.23 mos  
(c) 14.72 mos  
(d) 12.39 mos  
(e) 13.52 mos