Name: _____   **School:** _____

University of Houston
High School Mathematics Contest
Geometry Exam – Spring 2008

Directions:
You have 50 minutes to complete this exam. Calculators are not permitted. Choose the correct answer for each question, and write the <u>letter</u> (A, B, C, D or E) corresponding to that answer in the blank to the right of each question. There is no penalty for guessing. In the case of a tie, students' work will be used to determine the winner – so show all work clearly on either your exam or on scrap paper. Write your name and school on top of each page in the blanks provided.

Note: Geometric figures in the problems may not be drawn to scale.

1.   Given $\overline{AB}$ with endpoint $A(-2, 6)$ and midpoint $M(-5, 1)$, find the coordinates of endpoint B.

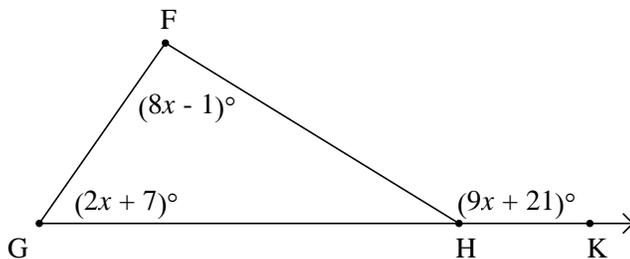(A) $\left(-\dfrac{3}{2}, -\dfrac{5}{2}\right)$

(B) $\dfrac{5}{3}$

(C) $(-8, -4)$

(D) $\left(-\dfrac{7}{2}, \dfrac{7}{2}\right)$

(E) $(-12, 8)$

1. _____

2. ΔFGH is shown below, with point K located on $\overrightarrow{GH}$. If $m\angle G = (2x + 7)^\circ$, $m\angle F = (8x - 1)^\circ$, and $m\angle FHK = (9x + 21)^\circ$, find the measure of $\angle FHG$.

2. _____

(A) 15° (B) 24° (C) 37° (D) $93\frac{9}{19}^\circ$ (E) 156°

3. Find the measure of an exterior angle of a regular convex decagon.

    (A) $36°$

    (B) $144°$

    (C) $18°$

    (D) $45°$

    (E) $135°$

3. _____

4. Find the area of an equilateral triangle with altitude 12cm.

    (A) $27\sqrt{3}$ cm^2

    (B) $48\sqrt{3}$ cm^2

    (C) $96\sqrt{3}$ cm^2

    (D) 72 cm^2

    (E) $72\sqrt{3}$ cm^2

4. _____

5. Given the following conditional statement:

    If a quadrilateral has four right angles, then it is a square.

Determine the truth value of the inverse, contrapositive, and converse, respectively.

    (A)   False, True, False
    (B)   False, False, False
    (C)   True, True, True
    (D)   True, False, True
    (E)   True, True, False

5. _____

6. Right triangle *DEF* is shown below with altitude $\overline{EG}$. If the length of $\overline{EG}$ is $2\sqrt{10}$ cm and the length of $\overline{GF}$ is 5 cm, find the length of $\overline{DF}$.
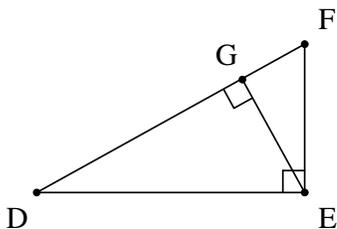
    (A) $\left(7\sqrt{10}\right)$ cm

    (B)   20 cm

    (C) $\sqrt{65}$ cm

    (D) $\left(\dfrac{5\sqrt{10}}{4}+5\right)$ cm

    (E)   13 cm

6. _____

7. Which of Objects 2-5 below could represent the image of Object 1 as the result of a single rotation (with no other transformations applied)?

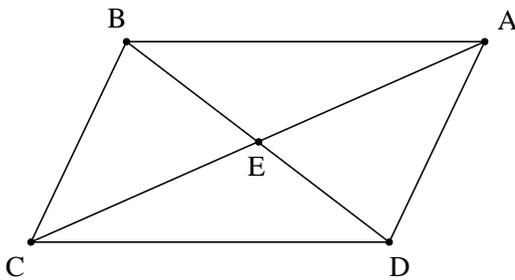P   q   b   �   ℛ

7. _____

Object 1   Object 2   Object 3   Object 4   Object 5

(A) Objects 2 and 3 only   (B) Objects 4 and 5 only   (C) Object 4 only
(D) Objects 2, 3, and 4 only   (E) Objects 2, 3, 4, and 5

8. In the quadrilateral below, $\overline{BA} \parallel \overline{CD}$, $\overline{BC} \parallel \overline{AD}$, $m\angle CBD = (6x-5)^\circ$, $m\angle ABD = (3x+4)^\circ$, and $m\angle BAD = (7x+5)^\circ$. Find the measure of $\angle ADB$.

8. _____

(A) 13° (B) 98° (C) 11° (D) 37° (E) 61°

9. In the circle below, chord $\overline{CD}$ bisects chord $\overline{AB}$ at point E. If the measure of $\overline{CE}$ is nine less than twice the measure of $\overline{BE}$, and the measure of $\overline{ED}$ is six more than the measure of $\overline{EA}$, find the measure of chord $\overline{CD}$. *Note: All units are in centimeters.*
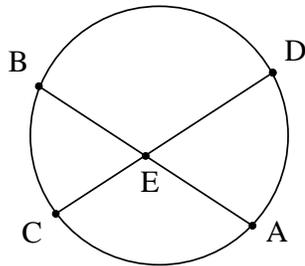
9. _____

(A) 24 cm   (B) 15 cm   (C) 12 cm   (D) 6 cm   (E) 3 cm

10. Which of the following polyhedra have 9 faces?

 10. _____

        I.   Nonagonal prism
        II.   Heptagonal prism
        III. Triangular prism
        IV. Octagonal pyramid
        V.   Nonagonal pyramid

(A) IV only   (B) II and IV only   (C) III only
(D) I and V only   (E) II, III, and IV only

11. The sum of the interior angles of a regular convex polygon is $1620°$. Find the measure of one of its interior angles.

 11. _____

(A) $162°$

(B) $147 \frac{3}{11}°$

(C) $135°$

(D) $11°$

(E) $9°$

12. Given $\triangle ABC$ and $\triangle DEF$ with $\overline{AC} \cong \overline{FD}$, $\angle A \cong \angle D$, and $\overline{BC} \cong \overline{FE}$, what theorem or postulate would follow directly to prove that $\triangle ABC \cong \triangle DEF$?

 12. _____

(A) Side-Side-Side
(B) Angle-Angle-Side
(C) Side-Angle-Side
(D) Angle-Side-Angle
(E) The triangles are not necessarily congruent

13. If 5 and 9 are each side lengths of an acute triangle, find a number that could be the third side length.

 13. _____

(A) 5   (B) 8   (C) 3   (D) $2\sqrt{14}$ (E) 14

14. In right triangle DEF, $\overline{DE}$ is the hypotenuse and G is the midpoint of $\overline{DE}$. If
$GF = 6x - 2$ and $GE = 4x + 8$, find the length of $\overline{DE}$.

(A) 5

(B) 14

(C) $\dfrac{104}{5}$

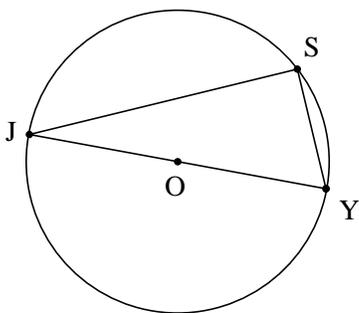(D) 56

(E) Cannot be determined

14. _____

15. Draw ΔACD with B located between A and C. Given that $\angle C \cong \angle ADB$, $AB = 4$ and
$AD = 6$, find BC.

(A) 5

(B) $2\sqrt{6}$

(C) 8

(D) 9

(E) 10

15. _____

16. ΔSJY is inscribed in circle O having diameter $\overline{JY}$. If $OY = 5$ and $SY = 3$, find the
cosine of $\angle SJY$.

16. _____

(A) $\dfrac{4}{5}$

(B) $\dfrac{3}{10}$

(C) $\dfrac{3\sqrt{91}}{91}$

(D) $\dfrac{10\sqrt{109}}{109}$

(E) $\dfrac{\sqrt{91}}{10}$

17. If the surface area of a sphere is $\dfrac{9\pi}{4}$ cm^2, find its volume.

17. _____

(A) $\dfrac{9\pi}{16}$ cm^3

(B) 36π cm^3

(C) $\dfrac{9\pi}{2}$ cm^3

(D) $\dfrac{32\pi}{81}$ cm^3

(E) $\dfrac{27\pi}{16}$ cm^3

18. Circles c_1 and c_2 are externally tangent as shown below. If $m\widehat{GF} = 125°$ and $m\widehat{NO} = 84°$, find $m\widehat{UE}$.

18. _____
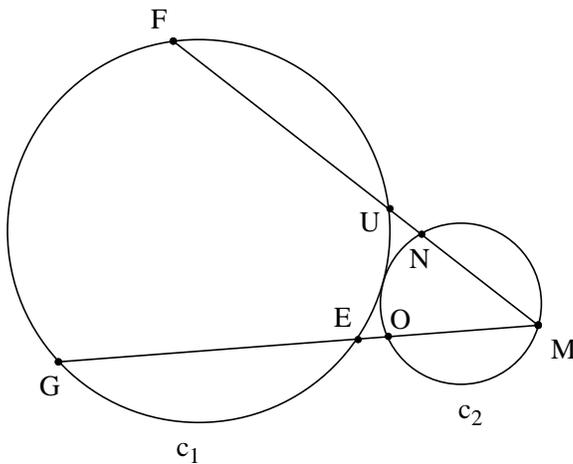
(A) $62.5°$
(B) $41°$
(C) $125°$
(D) $42°$
(E) $84°$

19. If the ratio of the lateral area of two similar hexagonal prisms is 4:121, find the ratio of their volumes.

19. _____

(A) $4:121$
(B) $8:1331$
(C) $2:11$
(D) $16:14641$
(E) $64:1771561$

20. If quadrilateral *NICE* is reflected across the line $y = x + 2$, give the coordinates of each of the vertices of the image $N'I'C'E'$.
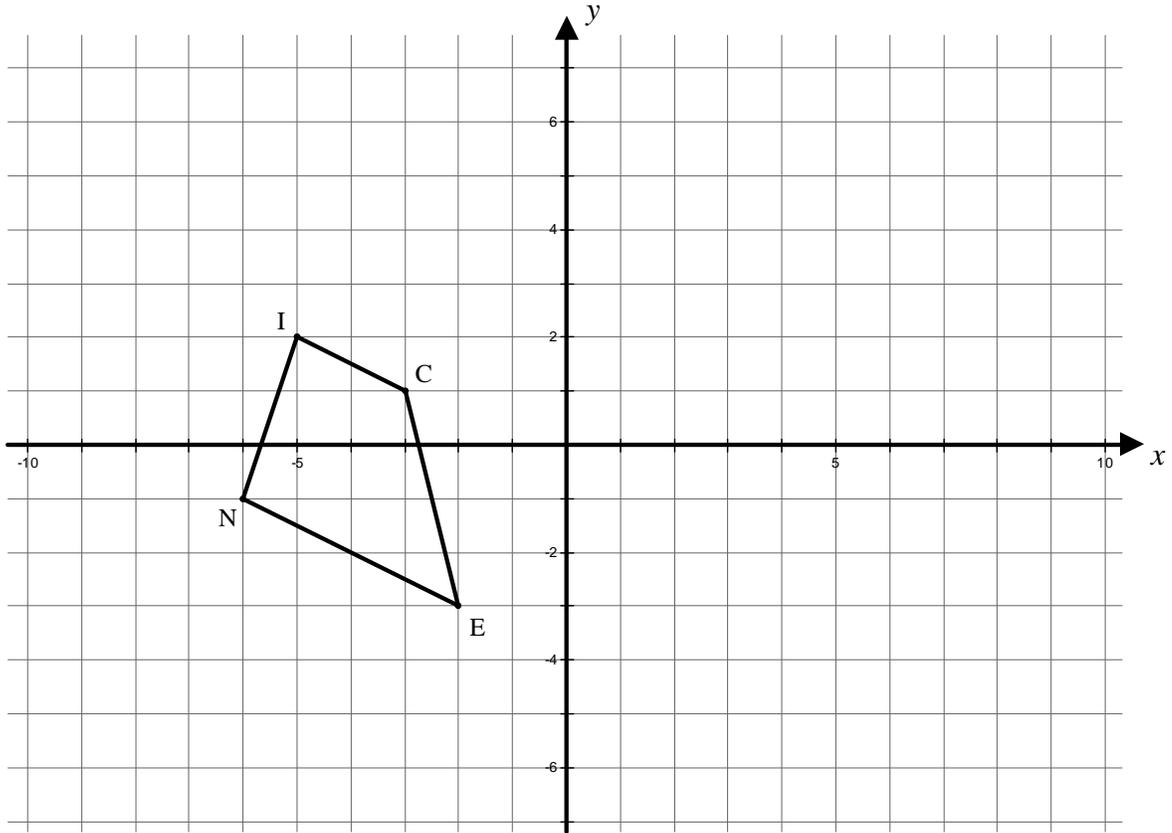

(A) $N'(-3, -4)$, $I'(0, -3)$, $C'(-1, -1)$, $E'(-5, 0)$

(B) $N'(-6, 5)$, $I'(-5, 2)$, $C'(-3, 3)$, $E'(-2, 7)$

(C) $N'(10, -1)$, $I'(9, 2)$, $C'(7, 1)$, $E'(6, -3)$

(D) $N'(6, -1)$, $I'(5, 2)$, $C'(3, 1)$, $E'(2, -3)$

(E) $N'(-1, -6)$, $I'(2, -5)$, $C'(1, -3)$, $E'(-3, -2)$

20. _____

Exam is <u>continued</u> on the next page…

21.   In Taxicab Geometry, the shortest distance between two points is defined as the minimum number of city blocks a taxi would need to travel to get from one point to the other, assuming square blocks of equal size and all streets oriented only horizontally and vertically.

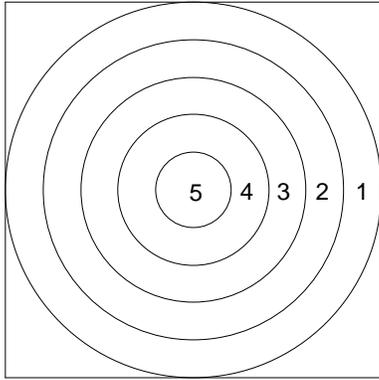In the diagram at the right, for example, the distance between the points $A(2,1)$ and $B(6,4)$ is 7.
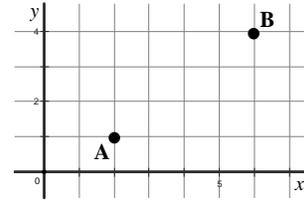
Using this Taxicab definition of distance, draw the set of points that are equidistant from points $C(2,5)$ and $D(8,3)$.

(A)

(B)

(C)

(D)

(E)
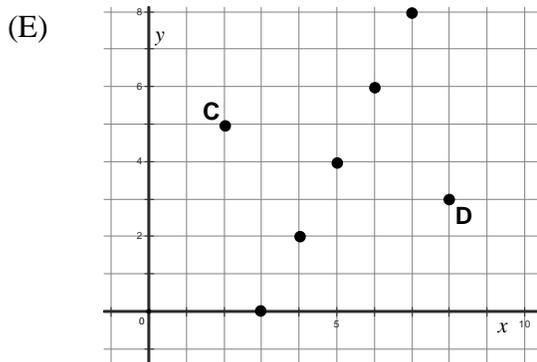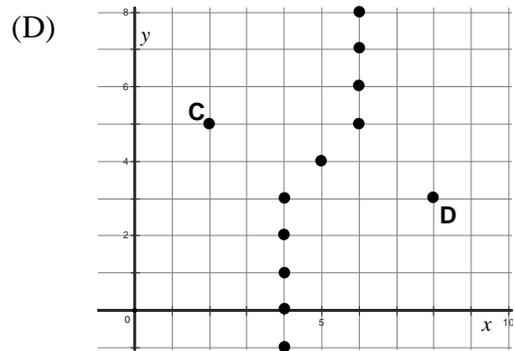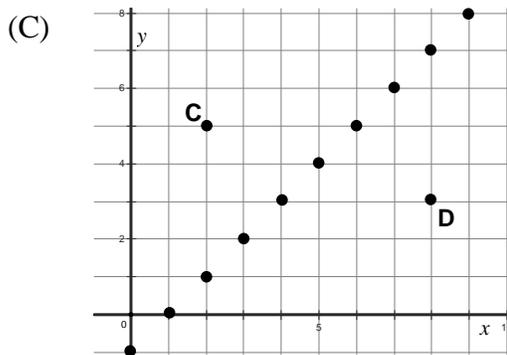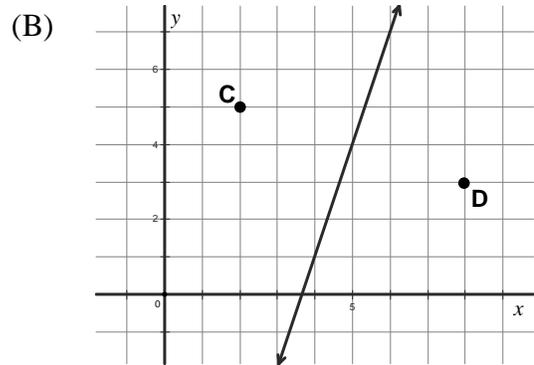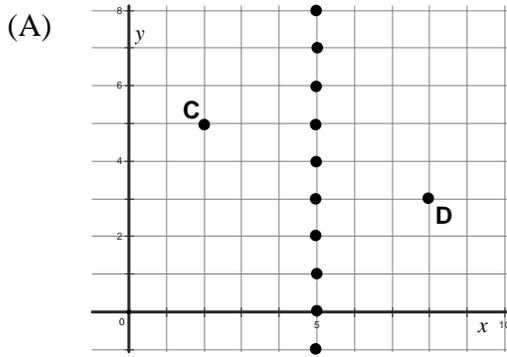
21. _____

22.

The square dartboard above is circumscribed about the largest of five concentric circles, and the ratio of the radii of the circles is 1:2:3:4:5. The numbers on the dartboard above represent the point values obtained for hitting that particular ring on the board. If a dart is thrown at the dartboard, what is the probability of hitting a ring worth an odd number of points? (Assume that all darts thrown are randomly distributed within the outer square.)

(A) $\dfrac{3}{5}$

(B) $\dfrac{3\pi}{20}$

(C) $\dfrac{\pi}{10}$

(D) $\dfrac{\pi}{4}$

(E) $\dfrac{3\pi}{5}$

22. _____

23.   A construction has been performed on triangle ABC below, with all relevant lines and arcs of circles shown. Point D represents the _____ of the triangle.
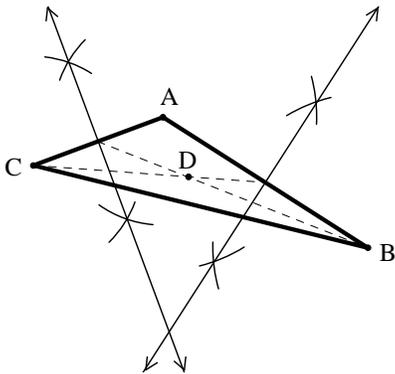
(A) Incenter
(B) Circumcenter
(C) Orthocenter
(D) Centroid
(E) Center of dilation

23. _____

24. Diameters $\overline{BD}$ and $\overline{CE}$ of circle P are shown below, along with chord $\overline{ED}$. If $\angle PED$ measures $70°$ and the radius of the circle measures 12 cm, find the area of the shaded sector.

24. _____

(A) $\dfrac{8\pi}{3}$ cm^2

(B) 16π cm^2

(C) 22π cm^2

(D) 32π cm^2

(E) 56π cm^2
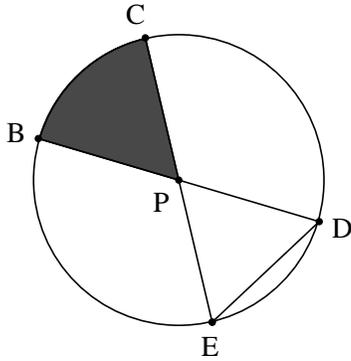
25. Find the number of diagonals in a convex 16-gon. (Hint: Look for a pattern.)

25. _____

(A) 104

(B) 112

(C) 120

(D) 208

(E) 2520

26. Triangle ABC is drawn on a coordinate plane with vertices $A(-5, -3)$, $B(-3, 7)$, and $C(4, 3)$. Find the length of the altitude drawn from vertex B.

26. _____

(A) $\sqrt{65}$

(B) $\dfrac{3}{2}$

(C) $\dfrac{\sqrt{221}}{2}$

(D) $\sqrt{117}$

(E) $2\sqrt{13}$

27. A cylindrical can of radius 9 cm and height 15 cm is shown below, and the gray shaded area represents a sticker that is placed on the lateral surface of the can. A central angle measuring $100°$ is shown on the upper base of the cylinder, and the arc subtended by the central angle coincides with the edge of the sticker. Find the area of the sticker.
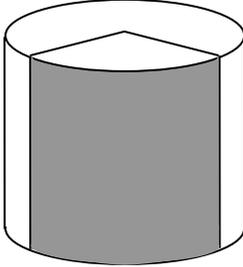
(A) $\dfrac{45\pi}{2}$ cm^2

(B) $\dfrac{675\pi}{2}$ cm^2

(C) 135 cm^2

(D) 270π cm^2

(E) 75π cm^2

27. _____

28. A spherical scoop of ice cream with radius 4 cm fits entirely inside a right circular ice cream cone so that the scoop of ice cream is tangent to the plane containing the circular base of the cone, and the point of tangency P is the center of the base. (In other words, the ice cream reaches the opening of the cone – but does not show above the cone itself.) The height of the cone is 16 cm. Find the lateral area of the cone.

(A) $16\sqrt{17}\,\pi$ cm^2

(B) $128\sqrt{2}\,\pi$ cm^2

(C) 96π cm^2

(D) 192π cm^2

(E) $\dfrac{512\pi}{3}$ cm^2
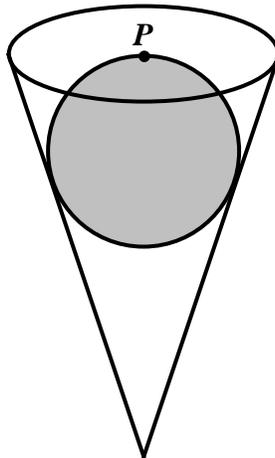
28. _____

29.   A pencil-like shape is formed by joining a regular right hexagonal prism and a regular right hexagonal pyramid, as shown in Figure 1 below. The regular hexagon (representing the base of both the prism and the pyramid) has a side length of 10 cm. The prism has height 40 cm, and the pyramid has slant height $\sqrt{219}$ cm.

The solid is shown again in Figure 2, with a bold segment on the base of the prism which joins the midpoints of opposite sides of the hexagon. Find the area of the cross-section formed if the object is sliced through this bold segment, perpendicular to the base and passing through the apex of the pyramid.

10 cm
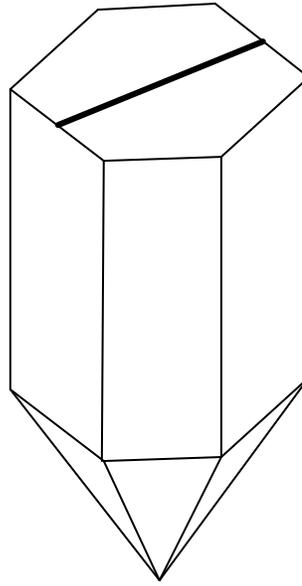
40 cm
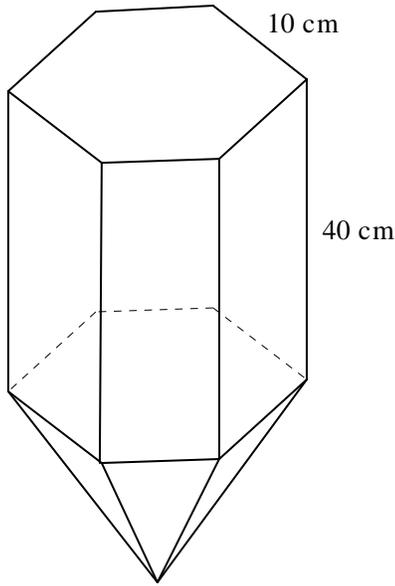
Slant height of pyramid $= \sqrt{219}$ cm

Figure 1   Figure 2

(A) 920 cm^2

(B) $\left(400\sqrt{3}+15\sqrt{73}\right)\text{cm}^2$

(C) $520\sqrt{3}\ \text{cm}^2$

(D) $\left(800+10\sqrt{119}\right)\text{cm}^2$

(E) $460\sqrt{3}\ \text{cm}^2$

29. _____

Exam is <u>continued</u> on the next page…

30. The net below is comprised of a square of side length 12 cm, surrounded by four congruent isosceles trapezoids with bases measuring 12 and 6 cm, and legs measuring 5 cm. One of the trapezoids shares a side with a square of side length 6 cm. Find the volume of the solid formed by this net.

30. _____

6 cm

5 cm

12 cm
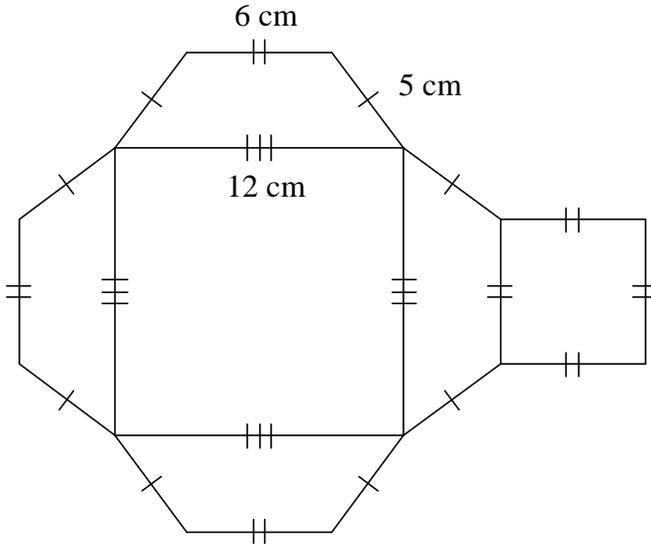
(A) 324 cm^3

(B) 192 cm^3

(C) 336 cm^3

(D) $84\sqrt{7} \text{ cm}^3$

(E) 420 cm^3

END OF EXAM ☺
(Make sure all answer blanks are filled in with the letter of the correct answer.)