# Math Contest – Precalculus Test, 2008

Name: 

School: 

Grade: 

Teacher: 

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<th>Question</th>
<th>Answer Letter</th>
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1. Where is the graph of \( f(x) = -3\sin(\pi x) \) decreasing in the interval \((0, \pi)\)?
Use the convention that the turn-around point is NOT included in the answer interval(s).

A. \((0, \frac{\pi}{2})\)

B. \((\frac{\pi}{2}, \pi)\)

C. \((0, \frac{1}{2})\) and \((\frac{3}{2}, \frac{5}{2})\)

D. It is decreasing on the whole interval.

E. It is increasing on the whole interval.

2. A wire of length \( (\pi \sqrt{2})x \) is bent into a circle. What is the area of the circle?

A. \(2\pi x^2\)

B. \(\frac{\pi x^2}{2}\)

C. \(2(\pi x)^2\)

D. \(\frac{x^2}{2\pi}\)

E. none of the above
3. Evaluate: \[
\left( \frac{\sin \frac{19\pi}{3}}{\cos \left(-\frac{\pi}{6}\right)} \right) \cdot \tan \left(-\frac{101\pi}{4}\right)
\]

A. 0  
B. 1  
C. \(\frac{\sqrt{3}}{2}\)  
D. \(\frac{\sqrt{3}}{4}\)  
E. \(-1\)

4. What is the area of the polygon between these two graphs?
\[
f(x) = |x - 3| + 1 \\
g(x) = -|x - 3| + 3
\]

A. 1 sq. unit  
B. 2 sq. units  
C. \(4\sqrt{2}\) sq. units  
D. 3 sq. units  
E. 4 sq. units

5. What is the range for \(f(x) = \frac{|x|}{x}\)?

A. \{-1, 1\}  
B. all Real numbers  
C. \([-1, 1]\)  
D. \((\infty, 0) \cup (0, \infty)\)  
E. \([0, \infty)\)
6. What is the domain for $f(x) = \sin^{-1}(x + 1)$?

A. $[-1, 1]$  
B. $[0, 2]$  
C. $(0, \infty)$  
D. $[-2, 0]$  
E. $(-\infty, \infty)$

7. Where is the asymptote for $f(x) = 2\log_3(1 - 3x) + 10$?

A. $y = 3$  
B. $x = 0$  
C. $y = 1$  
D. $x = 3$  
E. $x = \frac{1}{3}$

8. The following table was found by evaluating a function of $x$:

<table>
<thead>
<tr>
<th>$x$</th>
<th>1/2</th>
<th>1</th>
<th>2</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>3</td>
</tr>
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</table>

Which of the following statements are true?

1. $y$ is a logarithmic function of $x$
2. $x$ is an exponential function of $y$
3. $y$ is an exponential function of $x$
4. $y$ is an increasing function
5. $y$ is a decreasing function

A. 1 and 4 only  
B. 2 and 4 only  
C. 3 and 5 only  
D. 1 and 5 only  
E. 3 and 4 only
9. Given this fact:

The maximum of two real numbers a and b is
max (a, b) = \frac{1}{2} (a + b + |a - b|)

Which of the following is a formula for the minimum of the same two numbers a and b?

A. min (a, b) = |a| + |b|
B. min (a, b) = \frac{1}{2} \sqrt{a^2 + b^2}
C. min (a, b) = \frac{1}{2} (a - b + |a - b|)
D. min (a, b) = \frac{1}{2} (|a + b| - |a - b|)
E. none of the above

10. A utility company has a fleet of vans. The annual operating cost of each van is estimated to be C = 0.11 m + 2300 where C is cost measured in dollars and m is measured in miles. The company wants the annual operating cost of each van to be less than $10,000. What is the preferred domain for the formula?

A. m = 70,000    D. \( (-\infty, 700) \)
B. [0, 70,000)    E. [0, 10,000)
C. [7,000, \( \infty \))

11. Find a, b, and c so that the function: y = a sin (bt + c) matches the following graph. You are given the following 3 points on the graph: (0, \( \frac{3}{4}, 0 \)), and (\( \frac{5}{4}, 3 \)).

A. \(-3 \sin (x + \frac{1}{4})\)
B. \(3 \sin (2x + \frac{1}{2})\)
C. \(-3 \sin (\pi x + \pi/4)\)
D. \(3 \sin (2\pi x + \pi/8)\)
E. \(-3 \sin(x + \pi/4)\)
12. Which of the following is the interval described by \( \frac{1}{|x|} < 1 \)?

A. \((-\infty, -1) \cup (1, \infty)\)  
B. \((0, \infty)\)  
C. \((-\infty, 0) \cup (0, \infty)\)  
D. \((-1, 1)\)  
E. \((\infty, \infty)\)

13. Which of the following is the formula for the area of an equilateral triangle expressed as a function of a side length \( x \)?

A. \(A = \frac{x^2}{2}\)  
B. \(A = \frac{x^2}{4}\)  
C. \(A = \frac{x^2\sqrt{3}}{4}\)  
D. \(A = \frac{x\sqrt{2}}{3}\)  
E. none of the above

14. What is the behavior of the graph as \( x \) approaches 2 through values slightly less than 2 for the graph: \( f(x) = -\frac{x + 3}{x - 2} \)?

A. \(y \) approaches 0  
B. \(y \) approaches 3/2  
C. \(y \) approaches -3  
D. \(y \) approaches \(\infty\)  
E. \(y \) approaches \(-\infty\)
15. Which of the following is the graph of \( f(x) = -\sqrt{3-x} \)?
16. Given: 
\[ f(x) = \begin{cases} 
\sqrt{x} & x > \frac{3\pi}{2} \\
\sin x & -\frac{\pi}{2} < x \leq \frac{3\pi}{2} \\
x^2 & x \leq -\frac{\pi}{2} 
\end{cases} \]

Calculate: 
\[ \frac{f(12) + f\left(\frac{\pi}{3}\right)}{f(-2)} \]

A. \[ \frac{\sin(12) + \sqrt{3}}{8} \]  
B. \[ \frac{5\sqrt{3}}{8} \]  
C. \[ -1 \]  
D. \[ \frac{2\sqrt{3} + 1}{-2\sin(2)} \]  
E. none of the above

17. What is the y-intercept for the inverse function to \( f(x) = \frac{x + 3}{x - 1} \)?

A. \( (0, -3) \)  
B. \( (0, 3) \)  
C. \( (-3, 0) \)  
D. \( (3, 0) \)  
E. none of the above

18. If the point \( (a, b) \) is reflected about the line \( y = 3x \), what are the point’s new coordinates?

A. \( (b + 3, a + 3) \)  
B. \( \left(\frac{3b - 4a}{5}, \frac{3a + 4b}{5}\right) \)  
C. \( (a, b) \)  
D. \( \left(\frac{b}{3}, \frac{a}{3}\right) \)  
E. \( \left(\frac{a + b}{3}, \frac{a - b}{3}\right) \)
19. Which of the following are true statements?

1. Given two natural numbers a and b: \( \sqrt{ab} \leq \frac{a + b}{2} \leq \frac{\sqrt{a^2 + b^2}}{2} \)

2. The following is symmetric with respect to the x-axis: 
   \[ y = 2^x - 2^{-x} \]

3. Let \( S(x) = 0.5(3^x - 3^{-x}) \), \( S(3x) = S(x) + 4S(x) \).

4. \( \log_3(8\sqrt{2}) = \frac{3}{2} \).

A. 2 and 3 only  
B. 1, 2, and 3 only  
C. none of them  
D. 1 and 3 only  
E. all of them

20. Give the equation of the circle that is tangent to the x-axis, tangent to the y-axis and tangent to the line \( 3x + 4y = 12 \).

A. \( (x - 1)^2 + (y - 1)^2 = 4 \)  
B. \( (x - 1)^2 + (y - 1)^2 = 1 \)  
C. \( (x + 1)^2 + (y + 1)^2 = 1 \)  
D. \( (x + 1)^2 + (y + 1)^2 = 4 \)
21. Given \( f(x) = \sin(x) \) and \( g(x) = \frac{x-2}{2} \), \( h(x) = \pi x + \frac{\pi}{6} \), and \( k(x) = \cos x \)

Calculate: \( (k \circ h \circ g^{-1} \circ f)(x) \) for \( x = \frac{\pi}{6} \).

\[
\begin{align*}
A. & \quad \frac{1}{2} \\
B. & \quad 1 \\
C. & \quad -\frac{\sqrt{3}}{2}
\end{align*}
\]

D. \(-\frac{1}{2}\)  
E. \(-1\)

22. Find the length of one side of the shaded triangle given that the largest triangle is an equilateral triangle with side length 3 inches. Each side of the largest triangle is divided into two pieces; one of length 2 inches (a) and the other of length 1 inch (b). Use the Laws of Sines and Cosines. Hint: the shaded triangle is equilateral.

\[
\begin{align*}
A. & \quad \frac{3\sqrt{7}}{7} \\
B. & \quad \frac{2\sqrt{5}}{6} \\
C. & \quad \frac{5\sqrt{3}}{3}
\end{align*}
\]

D. \(\frac{\sqrt{3}}{5}\)  
E. \(\frac{\sqrt{2}}{3}\)