Algebra II Exam
2009 University of Houston Math Contest

Name: _____

School: _____

Please read the questions carefully and give a clear indication of your answer on each question.

There is no penalty for guessing.

Judges will use written comments and/or calculations to settle ties.
Good luck.

University of Houston
2009 High School Math Contest
Algebra II Exam

Name_____

School_____

Exam Time : 1 hour
No calculator allowed.
Write the letter (a, b, c, d or e) corresponding to your answer in the table below.
There is no penalty for guessing.
In the event of a tie, students' work will be used to determine the winner – so show your work clearly on your test or scratch paper.

Problem	Answer	Problem	Answer
1		14	
2		15	
3		16	
4		17	
5		18	
6		19	
7		20	
8		21	
9		22	
10		23	
11		24	
12		25	
13		26	

Algebra II Exam

1. Which of the following are solutions to the equation $x^{2010}+1=0$?

I. 1 II. -1 III. i IV. $-i$

(a) I and II
(b) II
(c) III
(d) III and IV
(e) II and III

2. John is the scheduler for his town's softball league. There are 8 teams in the league. In the regular season, each team plays all the other teams twice, and there is only one game per evening. John also reserves the softball field for 3 extra evenings, in case there are games cancelled due to weather. How many evenings does John need to reserve on the field's calendar for the regular season of the softball league?

(a) 28
(b) 56
(c) 31
(d) 59
(e) 53

3. $\log_4 x = -\frac{3}{2}$ Solve for x.

(a) -6
(b) $\frac{1}{8}$
(c) 8
(d) $\frac{1}{6}$
(e) $\frac{1}{64}$

4. For which of the following bases, b, is the number 121_b a perfect square?

(a) 4
(b) 5
(c) 3
(d) all of the above
(e) none of the above

5. Simplify

$$\frac{2x^2-5x+3}{3x^2+x-4}\times\left(\frac{2x^2+3x-9}{3x^2+13x+12}\right)^{-\frac{1}{2}}$$

(a) $\sqrt{\frac{2x-3}{3x+4}}$

(b) $\sqrt{\frac{3x+4}{2x-3}}$

(c) $\sqrt{\frac{2x+3}{3x-4}}$

(d) $\sqrt{\frac{3x-4}{2x+3}}$

(e) $\sqrt{\frac{x-1}{x+3}}$

6. The Puzzleberry Citrus Orchards runs an unusual end of the season sale every year. The bags they give out hold just 7 oranges each. The orchard then charges \$0.25 for each bag of seven oranges, and \$0.75 for each additional single orange. According to this system, which of the following would cost the most?

(a) 11 oranges
(b) 32 oranges
(c) 50 oranges
(d) 57 oranges
(e) 70 oranges

7. To keep their profits high without raising the price of a can of beans, the Keen Bean Company is planning on reducing the size of their cans. They start with a can of height *h* inches and radius *r* inches, which has volume $V = \pi r^2 h$ cubic inches. Which of the following will result in a can with the smallest volume?

(a) Keep h the same and reduce r by 20%
(b) Keep r the same and reduce h by 20%
(c) Reduce h by 10% and reduce r by 10%
(d) Increase h by 10% and reduce r by 30%
(e) All of the above will have the same volume.

8. Consider the function f(x) such that $f(mn) = f(m-n)$ for all real numbers *m* and *n*. If $f(4) = 3$, find $f(-2) + f(6)$.

(a) 0
(b) 6
(c) 3
(d) 2
(e) It cannot be determined with the information given.

9. Let *x* and *y* be real numbers such that $2^x = 24$ and $24^y = 32$. Find xy.

(a) 16
(b) 8
(c) 5
(d) 4
(e) 9

10. Let a, b and c be real numbers, and let $P(x) = ax^9 + bx^5 + cx + 3$. If $P(-5) = 17$, find $P(5)$.

(a) -17
(b) -11
(c) 14
(d) 17
(e) It cannot be determined from the information given.

11. Bob noticed that right now his dad, Steve, is five times older than his age. Last year, Bob's age was half that of his brother Bill. Nine years from now, Steve will be twice as old as Bill. What is the sum of the current ages of Bob, Bill and Steve?

(a) 55
(b) 33
(c) 35
(d) 64
(e) 65

12. Find the vertex of the parabola given by the equation $f(x) = 3x^2 - 18x - 29$.

(a) $(9, -110)$
(b) $(-3, -56)$
(c) $(3, -38)$
(d) $(-3, -2)$
(e) $(3, -56)$

13. Find the equation of the line which passes through the center of both of the following circles:

$$(x+3)^2+(y-3)^2=9$$
$$(x-1)^2+(y+2)^2=16$$

(a) $5x+4y=-3$
(b) $4x+5y=3$
(c) $5x+4y=3$
(d) $x+2y=5$
(e) $x-2y=5$

14. Let h and k be the roots of the equation $2x^2-9x+c=0$. If $4hk=11$, find $h+k+c$.

(a) $\frac{47}{8}$
(b) 20
(c) 1
(d) 10
(e) 11

15. A wire which is x inches long is bent into the shape of a rectangle. If the length of the rectangle is 2 inches more than three times its width, what is the area of the rectangle?

(a) $3x^2+2x$
(b) $\frac{3}{64}x^2-\frac{1}{8}x-\frac{1}{4}$
(c) x^2
(d) $\frac{3}{8}x^2-x-2$
(e) $3x^2-8x+16$

16. Let $f(x)=\frac{3x-4}{2x+1}$. Find its inverse $f^{-1}(x)$.

(a) $\frac{2x+4}{3x-1}$
(b) $\frac{-x-4}{2x}$
(c) $\frac{x+4}{3-2x}$
(d) $\frac{x+4}{2x-3}$
(e) $\frac{2x+1}{3x-4}$

17. Solve $\sqrt{7+\sqrt{x+\sqrt{x+2}}}=3$.

(a) 2
(b) 4
(c) 14
(d) -2
(e) 7

18. Let *a*, *b* and *c* be nonzero digits (1-9). Let abc, bca and cab be the three digit numbers obtained by placing a, b and c next to each other. Which of the following is NOT always true about the sum $S = abc + bca + cab$

(a) S is divisible by 3
(b) S is divisible by 2
(c) S > 330
(d) S is divisible by 37
(e) $\dfrac{S}{(a+b+c)}$ is an integer

19. Solve $|x+3|+|x-5|\le 10$.

(a) $[2,6]$
(b) $(-\infty,-4)\cup(6,\infty)$
(c) $[-4,6]$
(d) $[-13,15]$
(e) $(-\infty,-3)\cup(5,\infty)$

20. Find the area of the region in the first quadrant which is the solution set to the following system of linear inequalities:

$$3x + y \le 15$$
$$y \ge 2x$$
$$x \ge 0$$
$$y \ge 0$$

(a) 45
(b) 15
(c) 30
(d) $\dfrac{45}{2}$
(e) $\dfrac{75}{2}$

21. Hal P. Miself's two hour self-help seminar was so bad that 40% of his audience left after the first 30 minutes. By the end of the one hour coffee break, 50% of those remaining after the first half hour left. When Hal stopped to look for something in his briefcase after 1 ½ hours of the seminar, 60% of the people remaining after the coffee break left, and the rest stayed until the end of the seminar. There were only 30 people in the room to hear Hal's one good piece of advice at the end of his talk. How many people were in the audience when Hal started speaking?

(a) 100
(b) 360
(c) 250
(d) 375
(e) 500

22. One jelly bean is drawn at random from a jar containing red and black jelly beans. If there were 2 more black jelly beans in the jar, the probability of drawing a black jelly bean would be 1/3. If there were 2 less black jelly beans in the jar, the probability of drawing a black one would be ¼. How many red jelly beans are actually in the jar?

(a) 34
(b) 24
(c) 14
(d) 48
(e) 10

23. Which of the following is a solution to the equation $x^2 + \sqrt{12}\ x - 28 = 0$?

(a) $-\sqrt{3} - \sqrt{31}$
(b) -14
(c) $-\sqrt{3} + 5i$
(d) $-6 + \sqrt{31}$
(e) $-6 + 5i$

24. Which of the following is a true statement for all nonzero real numbers x and y satisfying the inequality $\frac{1}{x} - \frac{1}{y} > 0$?

(a) $x > y$
(b) $y > x$
(c) $\frac{1}{x} + \frac{1}{y} > 0$
(d) $\frac{x+1}{x} - \frac{y+1}{y} > 1$
(e) $\frac{x}{xy} < \frac{y}{xy}$

25. Emma has a collection of wooden blocks. She has three sizes of blocks: large blocks are cubes with 4 inch sides, medium blocks are cubes with 3 inch sides, and small blocks are cubes with 2 inch sides. Emma has three times as many small blocks as large blocks. She also has two more small blocks than medium blocks. Her block collection fits perfectly into a $8in.\times 10in.\times 12in.$ box when she picks them all up. If Emma were to stack all of her blocks into a vertical tower one block wide, how tall would this tower be?

(a) 9 feet
(b) 40 feet
(c) 108 feet
(d) 8 feet
(e) 7 feet and 5 inches

26. Given that $\left(2^a\right)^b = 2^a \times 2^b$, find the value of a if $b = 3$.

(a) $\frac{2}{3}$
(b) $\frac{3}{2}$
(c) 0
(d) 1
(e) a can be any real number