**Pre-Calculus Exam**
**2009 University of Houston Math Contest**

**Name:** _____

**School:** _____

Please read the questions carefully and give a clear indication of your answer on each question.

There is no penalty for guessing.

Judges will use written comments and/or calculations to settle ties.
Good luck.

# University of Houston High School Mathematics Contest
# Precalculus Exam – Spring 2009

Name: _____

School: _____

Grade: _____

Teacher: _____

**Directions:** You have 60 minutes to complete this exam. Calculators are not permitted. Your exam will be graded using the answer grid below. For each of the 30 questions, put the letter answer (A, B, etc.) in the corresponding box on the answer grid. You may change your answer, but illegible answers will be marked as incorrect.

**Answer grid:**

| | |
|---|---|
| 1 | 16 |
| 2 | 17 |
| 3 | 18 |
| 4 | 19 |
| 5 | 20 |
| 6 | 21 |
| 7 | 22 |
| 8 | 23 |
| 9 | 24 |
| 10 | 25 |
| 11 | 26 |
| 12 | 27 |
| 13 | 28 |
| 14 | 29 |
| 15 | 30 |

1. Is there a Real number t so that the points (2, 3) and (t, 1) are one unit apart?

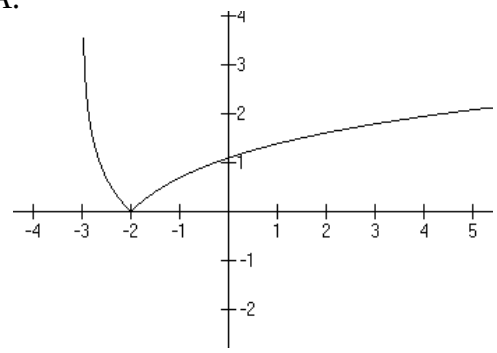
A. Yes
B. No

2. Given that a function and its inverse are defined over the Real numbers, and that f(3) = 6, f(6) = –1, and f(–1) = 0, what is $f^{-1}(-1)$?

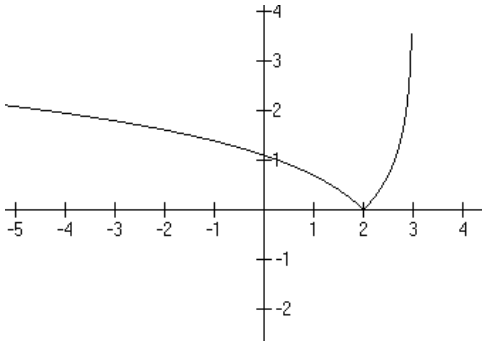
A. 6
B. 0
C. 3
D. Cannot tell from this information
E. $(-\infty, \infty)$

3. Which of the following is the graph of $f(x) = \left|\ln(3-x)\right|$ ?

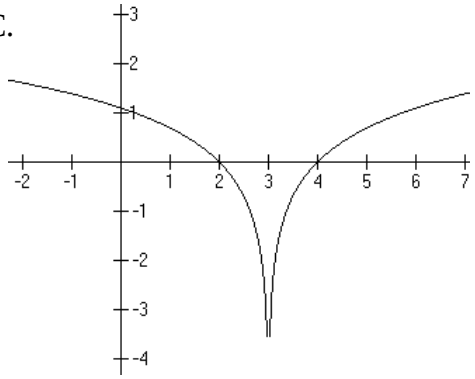
A.



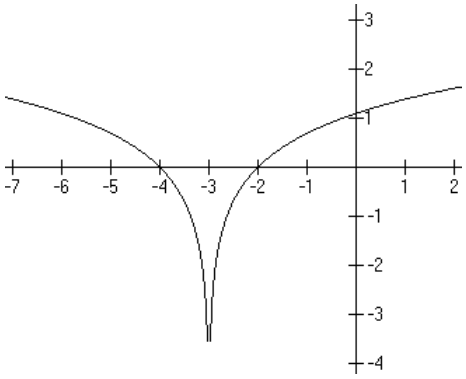
B.



C.



D.



4. What's the inverse function to $y = 0.25\ln(\sqrt{4-x})$?

A. $y = e^{4x^2} + 4$

B. $y = 4 - e^{4x^2}$

C. $y = 4 - e^{16x^2}$

D. $y = 4 - e^{8x}$

E. None of these

5. Given that
$x = \log_b(2),\quad y = \log_b(3),\quad z = \log_b(4),\quad w = \log_b(5),\quad \text{and}\quad p = \log_b(6)$.
Which of the following is equivalent to $\log(3)$?

A. y

B. $\dfrac{p-x}{x+w}$

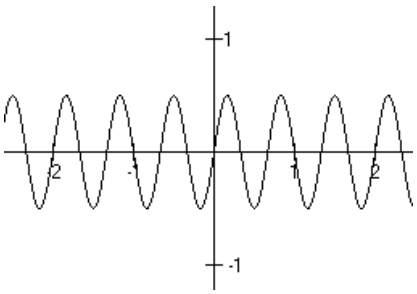
C. $x - (z - p)$

D. y -x

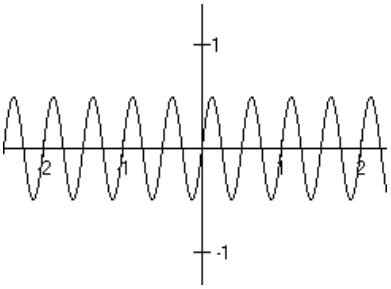
E. $\dfrac{1}{3} + \dfrac{p}{z}$

6. Which of the following graphs is expressed by the function y = ½ sin (5πx)?

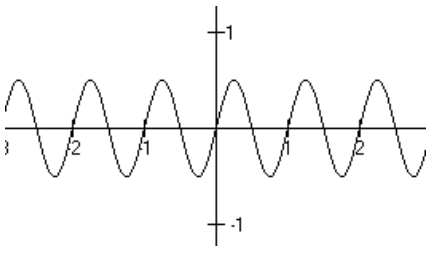
A.



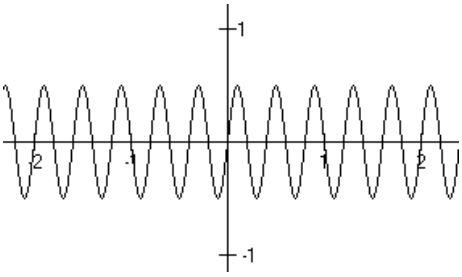

B.




C.




D.




E. None of these

7. Let f(x) be an odd function that has the property: $3f(x) - f(-x) = \sin(x)$.

What is the value of $f\left(\frac{\pi}{2}\right)$?

A. $-\frac{1}{2}$ B. $-1$ C. $\frac{1}{4}$ D. 1 E. None of these

8. Given $i = \sqrt{-1}$, evaluate $\sum_{k=0}^{2009} i^k$.

A. i B. $-i$ C. 0 D. $1 + i$ E. $-1$

9. Simplify $\dfrac{\frac{\cos\theta + 1}{\cos\theta} + 1}{\frac{\cos\theta - 1}{\cos\theta} - 1}$

A. $-2\cos\theta - 1$ D. $-1$

B. $\csc\theta$ E. $\cot\theta - 1$

C. $2\sin\theta + 1$

10. Use the fact that $-9$ is a root of the given polynomial to solve the equation: $x^3 + 7x^2 - 19x - 9 = 0$. Which of the following is a solution?

A. 1 D. $-1$

B. $1 + \sqrt{3}$ E. None of these

C. $1 - \sqrt{2}$

11. Which of the following are in the solution space of this system of equations?

$$\begin{cases} x^2 + y^2 < 1 \\ y - x^2 \geq 0 \end{cases}$$

I. $(0, \sqrt{0.3})$
II. $(1, 0)$
III. $(1, 1)$
IV. $(0.5, 0)$
V. $(\sqrt{0.3}, \sqrt{0.4})$

A. None of them
B. All of them
C. I and V only
D. IV and V only
E. IV only

12. If $f(x+1) = \dfrac{1}{2x}$, what is f(2x + 3)?

A. $\dfrac{1}{2x-2}$ B. $\dfrac{1}{4x+4}$ C. $\dfrac{1}{4x-4}$ D. $\dfrac{1}{4x^2}$ E. None of these

13. Which of the following does not simplify to cos x for every value for which the expression is defined?

A. $\sqrt{\dfrac{\tan x - \dfrac{\sin^3 x}{\cos x}}{\tan x}}$
B. $\dfrac{\cot x}{\csc x}$
C. $\dfrac{\cos x}{\csc^2 x - \cot^2 x}$
D. All of these simplify to cos x
E. None of these simplify to cos x

14. What is the domain for $f(x) = \dfrac{1}{\sqrt{5^{2x-1} - 125}}$?

A. $(-\infty, \infty)$

B. $(-\infty, 2) \cup (2, \infty)$

C. $\{3, 4\}$

D. $(2, \infty)$

E. None of these

15. Which of these functions is/are even?

I. $f(x) = \sqrt[3]{x^3 + 4x}$

II. $f(x) = \sqrt{|\cos x|}$

III. $f(x) = \sqrt[3]{x^2 - 64}$

IV. $f(x) = \left|\sqrt[3]{x^7}\right|$

A. All of them are even.

B. II, III, and IV only

C. None of them are even.

D. IV only

E. II and IV only

16. If $\dfrac{x^2 + y^2}{x + y} = 4$, find all possible values for $(x - y)$.

A. $(-\infty, \infty)$

B. $[-4, 4]$

C. $\{-2, 2\}$

D. $\{-4, 4\}$

E. $\{0\}$

F. No solution

17. Let $f_1(x) = \dfrac{x-2}{x-1}$. Let $f_2(x) = f_1 \circ f_1(x)$. Let $f_3(x) = f_1 \circ f_1 \circ f_1(x)$ and so on in this pattern of compositions. What is $f_{101}(x)$?

A. $\dfrac{x^{101}-202}{x^{101}-101}$

B. $\dfrac{x-2}{x-1}$

C. $\dfrac{101x-202}{101x-101}$

D. $x$

E. None of these

18. Below is a graph of two functions. The parabola is f(x) and the line is g(x). Assuming that all of the expressions below are integers, which of them has a value of 1?

I. $5f(0) - g(0)$

II. $\left(f \circ g^{-1}\right)(2)$

III. $\left|\left(f \circ g^{-1}\right)(8)\right|$

IV. $(f + g)(3) - (f - g)(3) - 9$

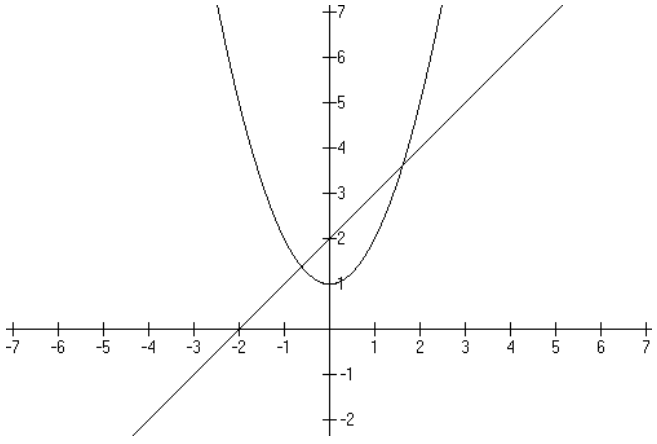
A. All of them.

B. I and II only

C. III and IV only

D. II and IV only

E. None of the statements yields 1 as an answer.



19. Use trigonometric substitution for $x = 2\tan\theta$, $0 < \theta < \frac{\pi}{2}$ to rewrite the expression $\sqrt{4 + x^2}$ as a trigonometric function of $\theta$.

A. 2

B. $\tan\theta$

C. $2\sec\theta$

D. $2(\sin\theta + \cos\theta)$

E. $\csc\theta + 1$

20. The displacement from equilibrium of an oscillating weight suspended by a spring is given by $y(t) = \frac{1}{4}\cos(4t)$. Find the displacement when $t = \frac{29\pi}{12}$.

A. $-\frac{1}{8}$

B. 1

C. $-\frac{\sqrt{3}}{8}$

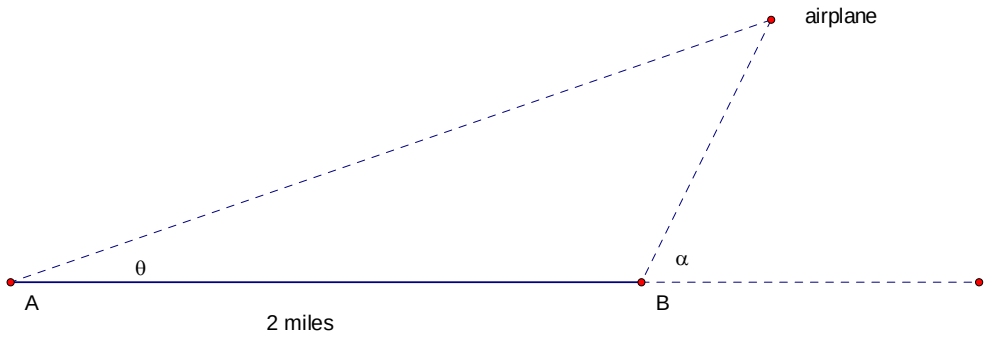
D. $\frac{1}{8}$

E. $\frac{\sqrt{3}}{8}$

21. Given:
$f(\theta) = \sin\theta$ is an odd function and $f(\theta) = \cos\theta$ is an even function.
What can be said about $p(\theta) = \sin(\theta)\cdot\cos(\theta)$ ?

A. p is even
B. p is odd
C. p is neither even nor odd
D. p is symmetric about the x-axis

22. The angles of elevation $\theta$ and $\alpha$ to an airplane are being continuously monitored by two observation points that are 2 miles apart. See figure. Which of the following is the equation that gives the distance d between the plane and point B in terms of $\theta$ and $\alpha$?




A. Distance = $\sqrt{4-2\cos(\theta-\alpha)}$

B. Distance = $\sin^2\alpha+\sin^2\theta$

C. Distance = $\dfrac{2\sin\theta}{\sin(\alpha-\theta)}$

D. Distance = $\dfrac{\sin\alpha}{\cos\theta}$

E. It cannot be determined from the given information.

23. Find a positive value k such that $x^2+2kx=-3k-3$ has exactly one real number solution.

A. $k=\dfrac{3+\sqrt{15}}{2}$

B. $k=\dfrac{3+\sqrt{21}}{2}$

C. $k=\dfrac{2+\sqrt{11}}{3}$

D. $k=\dfrac{5-\sqrt{11}}{2}$

E. $k=\dfrac{2+\sqrt{15}}{3}$

24. Given a $30^o$–$60^o$– $90^o$ right triangle with perimeter $3+3\sqrt{3}$ cm and area $1.5\sqrt{3}\,\text{cm}^2$. What is the length of the shortest side?

A. $2\sqrt{2}$ cm B. $\sqrt{5}$ cm C. 2 cm D. $\sqrt{3}$ cm E. $3\sqrt{2}$ cm

25. Find consecutive integers n and n + 1 such that this expression lies between them: $\log(.003)$.

A. 0 and 1
B. – 2 and – 1
C. 2 and 3
D. – 3 and – 2
E. None of these

26. Let f be the function defined below. Is this function one-to-one?

$$f(x)=\begin{cases} -2^{-x} & x \le 0 \\ \ln x & 0 < x < 3 \\ x+1 & x \ge 3 \end{cases}$$

A. Yes B. No

27. Which of the following statements are true?

I. $e^{\ln x} = x$ for all Real numbers.

II. The range of the function $y = \dfrac{e^x+1}{e^x-1}$ is all Real numbers.

III. $\sec(\arcsin(x-1)) = (2x-x^2)^{-\frac{1}{2}}$

IV. $\dfrac{(2n+1)!}{(2n)!} = 2n+1$ for n a natural number

A. None of them are true.
B. I and II are true.
C. III and IV are true.
D. I and IV are true.
E. None of them are true.

28. Solve the following system:

$$\begin{cases} 2^x \cdot 3^y = 4 \\ x + y = 5 \end{cases}$$

A. $x > 0$ and $y \le -5$

B. $\left( \frac{2\ln 2 - 5\ln 3}{\ln 2 - \ln 3}, \frac{3\ln 2}{\ln 2 - \ln 3} \right)$

C. $\left\{ (\ln 2,\ \ln 5),\ \left( \frac{\ln 2 - \ln 3}{\ln 5},\ \frac{\ln 5}{\ln 2 + \ln 3} \right) \right\}$

D. $\left( \frac{\ln 2 - \ln 3}{\ln 5}, \frac{5\ln 2}{\ln 2 - \ln 3} \right)$

E. $(-\infty, \infty)$

29. Solve for x: $e^x = e^{-x} + 1$

A. $\ln\left( \frac{2 - \sqrt{3}}{4} \right)$

B. $\ln\left( \frac{1 + \sqrt{5}}{2} \right)$

C. $\ln\left( \frac{3 - \sqrt{5}}{2} \right)$

D. $\ln\left( \frac{3 + \sqrt{2}}{5} \right)$

E. No solution

30. Let A denote the area of the right triangle in the first quadrant that is formed by the y-axis and the lines $y = mx$ and $y = m$ (note that $m > 0$). Express the area of the triangle as a function of m.

A. $A = \frac{1}{2} m^2$

B. $A = m^2 + 1$

C. $A = \frac{1}{2} m$

D. $A = m$

E. It cannot be done.