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University of Houston
High School Math Contest – Spring 2011
Algebra 2 Test

1. If $i^2 = -1$, then $i - i^2 + i^3 - i^4 + i^5 - i^6 + ... + i^{2011} =$

A. 0 B. 1 C. i D. $1005 - 1006i$ E. -1

2. Which of the following is a solution to the equation $3x^2 - 4x + 2 = 0$?

A. $\frac{2+\sqrt{10}}{3}$ B. $\frac{2-i\sqrt{2}}{3}$ C. $2 + i\sqrt{2}$ D. $\frac{-4-i\sqrt{2}}{6}$ E. $2 + 4i$

3. The graph of the equation $9x^2 + 18x - 4y^2 + 16y - 43 = 0$ is:

A. An ellipse with center $(1,-2)$
B. A circle with center $(-1,2)$
C. A hyperbola with center $(1,-2)$
D. A hyperbola with center $(-1,2)$
E. An ellipse with center $(-1,2)$

4. A point (x, y) in the plane is called a lattice point if *x* and *y* are both integers. How many lattice points are contained in the solution set of the following system of inequalities?

$x^2 + y^2 < 25$
$x \geq 0$
$y \geq 0$

A. 25 B. 22 C. 26 D. 13 E. 4

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5. Solve for x: $8^x + 8^x + 8^x + 8^x = \frac{1}{128}$.

A. -3 B. $-\frac{3}{8}$ C. $-\frac{7}{12}$ D. $-\frac{7}{5}$ E. $-\frac{8}{3}$

6. One of Clark's tasks at work is to scan the company's old records into the computer. Since Clark does not want to work too hard each day, he counts the number of documents in his in-box each workday morning and scans 20% of them each day. (He rounds the 20% to the nearest whole number each day.) Unfortunately for Clark, his boss adds 10 documents to his in-box each evening. If Clark starts with 30 documents in his in-box on Monday morning and follows this procedure, how many documents will be in his in-box on the next Monday morning at the beginning of the day. Assume Clark and his boss work Monday-Friday.

A. 10 B. 30 C. 45 D. 43 E. 44

7. Simplify: $\frac{\sqrt{x+3}}{\sqrt{x+3}+2} + \frac{2}{\sqrt{x+3}-2}$

A. $\frac{x+7}{x-1}$ B. $\frac{x+5}{x+1}$ C. $\frac{x+7}{x+3}$ D. $\frac{\sqrt{x+3}}{x-1}$ E. 1

8. Let *n* be a natural number greater than 1. Let $P = n(n+1)(n+2)$. Which of the following is NOT a true statement?

A. P is even
B. There exists a natural number *d* such that EXACTLY one of the following is true:
   I. $P = 5d - 1$
   II. $P = 5d$
   III. $P = 5d + 1$
C. P is divisible by 3
D. P is divisible by 4
E. P is divisible by 6

9. Find the product of the solutions to the following equation:

$8|x+1|^2 - 2|x+1| = 15$

A. $-\frac{15}{8}$
B. $-\frac{5}{4}$
C. $\frac{3}{2}$
D. $\frac{45}{16}$
E. $-\frac{9}{4}$

10. Tom has pennies, nickels and dimes in his change jar. The number of dimes in the jar is one less than three times the number of pennies. The number of nickels in the jar is twice the number of pennies and dimes combined. The total value of the coins in the jar is $6.90. Find the total number of coins in the jar.

A. 79 B. 69 C. 117 D. 107 E. 138

11. Suppose $f(x) - f(x-1) = 4$ and $f(0) = 3$. Find $f(-1)$.

A. 1
B. -1
C. 7
D. 0
E. It cannot be determined from the information given.

12. Define a new operation "$\oplus$" on the set of all integers by the formula

$x \oplus y = x + xy$. Which of the following statements are true for all integers *a* and *b*?

I. $a \oplus b = b \oplus a$ II. $a \oplus 0 = a$ III. $0 \oplus a = a$ IV. $b \oplus 1 = b$

A. II and III
B. I and IV
C. II only
D. II and IV
E. All are true

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13. Find the equation of the line that passes through the point $(-1,3)$ and is perpendicular to the line $3x+4y=6$.

A. $4x+3y=5$ B. $12x+16y=36$ C. $3x-4y=15$

D. $-20x+15y=65$ E. $-4x+3y=-13$

14. If $\sqrt{\frac{x}{y}\sqrt[3]{\frac{y}{x}\sqrt[4]{\frac{x}{y}}}}=\left(\frac{y}{x}\right)^P$ find P.

A. $\frac{17}{24}$ B. $\frac{3}{8}$ C. $-\frac{3}{8}$ D. $\frac{-1}{12}$ E. $\frac{1}{24}$

15. Suppose $f(x)=x^2+4x+c$ and $g(x)=\sqrt{x+s}+d$. If $f(x)$ and $g(x)$ are inverses of each other, find the value of $s+c+d$.

A. 6 B. 4 C. 2 D. 0 E. -2

16. Given $h(x)=-2^{x+1}+1$, find $h(0)+h(-2)$.

A. 1 B. $-\frac{1}{2}$ C. $\frac{1}{2}$ D. 2 E. $\frac{1}{4}$

17. Suppose $f(x)=x-3$ and $g(x)=ax^2+bx+c$. If $(g\circ f)(3x+1)=3x^2-2x+3$, find b.

A. -2 B. $\frac{2}{3}$ C. $-\frac{4}{3}$ D. $-\frac{2}{3}$ E. 1

18. Which of the following is the set of all real numbers which satisfy the inequality $\frac{2}{x-3}\geq\frac{-2}{x+5}$?

A. $\{x:-5<x\leq -1\}\cup\{x:x>3\}$

B. $\{x:x>3\}$

C. $\{x:x\geq -1\}$

D. All real numbers

E. No solution

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19. If $\log_b 3 = P$ and $\log_b 4 = Q$, find $\log_b 18$.

A. $\frac{P^2 Q}{2}$ B. $2P + \frac{Q}{2}$ C. $\frac{P}{2} + 2Q$ D. $P + Q^2$ E. $P^2 + \sqrt{Q}$

20. In Calculand, the unit of length is one tetradigit. If a square with sides of length $\sqrt{7}$ tetradigits has an area of $3\sqrt{2}\ cm^2$, find the length, in tetradigits, of the side of a square with area $\sqrt{8}\ cm^2$.

A. $\frac{14}{3}$ tetradigits B. $\frac{12}{7}$ tetradigits C. $\frac{\sqrt{42}}{3}$ tetradigits

D. $\frac{4\sqrt{7}}{21}$ tetradigits E. 8 tetradigits

21. Lee made a 120 mile trip at an average speed of 50 miles per hour. The first part of the trip was made by bus at an average speed of 30 miles per hour. The second part of the trip was by train at an average speed of 60 miles per hour. How many miles did Mike travel by bus?

A. 40 B. 96 C. 45 D. 30 E. 24

22. If $x^2 - 5x - 6 = 0$, then find the value of $x^4 - 10x^3 + 26x^2 - 5x - 6 =$

A. 36 B. 42 C. 30 D. 6 E. 0

23. Which of the following is a root of the equation $x^2 + \sqrt{8}x + 2 = 0$?

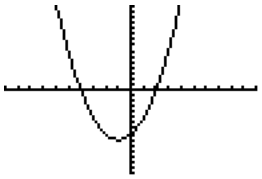
A. $2 - \sqrt{2}$ B. $-\sqrt{2}$ C. $\sqrt{2}$ D. $\frac{\sqrt{2}}{2}$ E. $\frac{-i\sqrt{2}}{2}$

24. Find the equation of the line that passes through the points of intersection on the graphs of the following functions:

$f(x) = 2x^2 + 4x - 16$
$g(x) = x^2 + 5x - 10$

A. $y = -2x + 20$ B. $y = 6x - 4$ C. $y = 3x - 2$ D. $y = -6x + 32$ E. $y = 0$

25. Michael is learning how to use his graphing calculator. He is very pleased that he figured out how to produce the graph below. Unfortunately, Michael did not label his axes or write down which viewing window he used to produce this graph. Which of the following functions could be graphed in this picture?



I. $f(x) = 2(x-2)^2 - 24$ II. $f(x) = 2(x+1)^2 - 18$
III. $f(x) = x^2 - 6x - 72$ IV. $f(x) = -x^2 - 2x + 8$
V. $f(x) = x^2 + x - 2$ VI. $f(x) = x^2 + 2x - 8$

A. I and III
B. VI only
C. II, III, V and VI
D. IV and VI
E. II, V and VI

26. Given the point $A(2,4)$ and the point $B(1,-2)$, which of the following equations is satisfied by all points $P(x,y)$ such that P is equidistant from A and B.

A. $x = -2(y-4)^2$
B. $x = \frac{3}{2}$
C. $2x + 12y - 15 = 0$
D. $12x + 2y + 15 = 0$
E. $2x + 4y - 15 = 0$

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27. How many real solutions exist for the following equation?

$\sqrt{x+1+\sqrt{x}} = \sqrt{x+\sqrt{x+7}}$

A. 4 B. 3 C. 2 D. 1 E. 0

28. An algebra teacher illustrates the concepts involved in solving linear equations by giving each student a card with a different expression written on it. The idea is for a student to set his expression equal to his neighbor's expression and solve the resulting equation. The list of expressions is printed below.

The distinct possible equations are found by setting one of these expressions equal to a different expression. What is the probability that a given equation has no solution?

Expressions:

$x+3$	$x-7$	$x-1$	$3x-18$
$2x-4$	$3x$	$5x-21$	$-1-x$
$4x+3$	$1-2x$	$7-x$	$4+8x$
$2x+8$	$2-x$	$4+2x$	$7x+7$

A. $\frac{1}{24}$ B. $\frac{1}{6}$ C. $\frac{1}{12}$ D. $\frac{1}{16}$ E. $\frac{1}{10}$

29. Find the difference between the largest and smallest real root of the polynomial $p(x) = x^4 - 4x^3 - 2x^2 + 4x + 1$

A. $3+\sqrt{5}$ B. $2+\sqrt{5}$ C. 4 D. 2 E. $2\sqrt{5}$

30. Solve for x: $\log_2(x+3) + \log_2(x+9) = 4$

A. $x = -1,\ x = -11$
B. $x = 1,\ x = 11$
C. $x = 11$
D. $x = -1$
E. No solution