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Calculator Exam – 2011

Directions: Select the answer to each question which is closest to the actual

answer. For example, suppose a question requests the value of $\sin(2)$. Your calculator will tell you that $\sin(2)$ is approximately 0.9092974268. Now suppose the answer choices are

- a. .3
- b. .55
- c. .75
- d. .95
- e. .85

Then the correct response is d.

1. $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \dots + \frac{1}{15} =$

- a. .7
 - b. .71
 - c. .72
 - d. .73
 - e. .74
2. Solve $\sin(x) + x = 3$.
- a. 2.14
 - b. 2.16
 - c. 2.18
 - d. 2.2
 - e. 2.22
3. Give the area of a circular sector of radius $3\sqrt{\pi}$, having a central angle of 2 radians.
- a. 27.75
 - b. 28
 - c. 28.25
 - d. 28.5
 - e. 28.75
4. Give the value of x where the graph of $f(x) = x^2 + 3\sin(x) + 4$ is closest to the x axis.
- a. -1
 - b. -0.98
 - c. -0.96
 - d. -0.94
 - e. -0.92

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5. Give the x coordinate of the solution to the system $\begin{cases} 2.37x + .42y = 6.01 \\ -1.31x + 31y = -3.15 \end{cases}$.
- 2.535
 - 2.536
 - 2.537
 - 2.538
 - 2.539
6. Give the positive value of x where the circle of radius 4 centered at the origin intersects the graph of the equation $\frac{x^2}{25} + y^2 = 1$.
- 4
 - 3.99
 - 3.98
 - 3.97
 - 3.96
7. Let $f(x) = 3x^2 + 2x + 7$. $\frac{f(-1)}{f(12 - f(3))} - 2f(1) + f(10 - f(2)) =$
- 461
 - 462
 - 463
 - 464
 - 465
8. The graph of the quadratic function $f(x) = ax^2 + bx + c$ passes through the points (1.2,2.3), (2.1,-1.5) and (3.6,4.2). Give the value of a .
- 3.31
 - 3.32
 - 3.33
 - 3.34
 - 3.35
9. A circle is drawn so that it is centered at (0,0) with radius 1. A second circle is drawn so that it is centered at (2,0) with radius 1/2. A third circle is drawn so that it is centered at (4,0) with radius 1/3. A fourth circle is drawn so that it is centered at (6,0) with radius 1/4. This process is continued until the last circle is drawn so that it is centered at (200,0) with a radius that follows the pattern of the earlier circles. What is the total area of all of the circles.
- 5.125
 - 5.130
 - 5.135
 - 5.140
 - 5.145

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10. Give the sum of the reciprocals of the first 25 prime numbers.
- 1.80
 - 1.81
 - 1.82
 - 1.83
 - 1.84
11. Give the sum of the reciprocals of the two largest prime factors of 17516.
- 0.03
 - 0.04
 - 0.05
 - 0.06
 - 0.07
12. A line is tangent to the circle of radius 1 centered at the origin, and it crosses the y axis at $(0,3)$. Give the x coordinate of the point of intersection of the circle with the line.
- .93
 - .94
 - .95
 - .96
 - .97
13. An amount of money is invested in an account that compounds interest annually. The interest rate for the first year is 1.07%, and the interest rate increases each year by 0.01%. If there is \$415,000 in the account 40 years after the amount of money is invested, then what is the size of the original investment?
- \$250,000
 - \$251,000
 - \$252,000
 - \$253,000
 - \$254,000
14. Let $x_0 = 1$ and $x_n = \cos(x_{n-1})$ for $n = 1, 2, 3, \dots$. Give the value of x_{236} .
- 0.55
 - 0.65
 - .075
 - 0.85
 - 0.95
15. Give the largest integer x for which $x + 200 \cos\left(\frac{x}{2}\right) < 0$.
- 20
 - 80
 - 140
 - 200
 - 260

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16. A number is written in base 4 as 3,120,331,212. Another number is written in base 3 as 21,111,210,213,222. Write the absolute value of the difference of the numbers in base 10.

- a. 3,000,000
- b. 3,500,000
- c. 4,000,000
- d. 4,500,000
- e. 5,000,000

17. Six positive integers are chosen at random from the interval $[0,1000]$ and named

$x_1, x_2, x_3, x_4, x_5, x_6$. Then the value $\frac{x_1}{1000} - 2\cos(2x_2) + 3\sin(3x_3) - \frac{x_4}{x_5 + x_6 + 1000}$

is computed. What is the probability that the resulting value is negative?

- a. 0.4
- b. 0.45
- c. 0.5
- d. 0.55
- e. 0.6

18. The determinant of $\begin{pmatrix} x_1 & x_2 \\ x_3 & x_4 \end{pmatrix}$ calculated for every possible matrix where the numbers

x_1, x_2, x_3, x_4 are chosen as integers in the interval $[-5,5]$. How many distinct values are calculated?

- a. 40
- b. 50
- c. 60
- d. 70
- e. 80

19. How many integers are there in the interval $[1,10000]$ for which the sum of the digits is an integer multiple of 5?

- a. 2000
- b. 2100
- c. 2200
- d. 2300
- e. 2400

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20. A prime point has the form (a,b) where a and b are integers and either $|a|$ or $|b|$ is a prime number. Give the number of prime points contained in the interior of the

ellipse $\frac{x^2}{2500} + \frac{y^2}{10000} = 1$.

- a. 4725
 - b. 5250
 - c. 6025
 - d. 7150
 - e. 7825
21. **Tie Breaker – This problem will only be checked to break ties associated with first through sixth place.** A fair coin is tossed 100 times and the results are recorded in the order they are obtained, with H representing “heads” and T representing “tails.” A streak occurs when multiple heads or tails are tossed in a row, and the length of the streak is the number of heads or tails in the streak. For example, the table below shows an example of 100 tosses, where the results are recorded from left to right and top to bottom. In this example, the longest streak is of length 6, and it is shaded in the table. In general, if a fair coin is tossed 100 times, what is the probability that a streak of length at least 6 will occur?

H	H	T	H	T	T	T	H	T	H
H	H	T	T	T	T	T	H	T	H
H	T	T	T	H	T	H	H	H	T
H	H	T	T	T	T	H	T	H	H
T	T	T	T	T	T	H	H	T	H
T	H	H	T	T	T	H	T	H	H
H	T	H	H	T	T	T	T	H	H
H	T	H	T	H	H	T	H	H	H
T	T	H	T	T	T	H	T	H	T
T	T	T	H	T	H	H	T	H	H

Tie Breaker Answer: _____