1. \( \lim_{x \to 0} \sec \left( \frac{\sin 2\pi x}{3x} \right) = \)
   
   (a) 2
   (b) \(-2/\sqrt{3}\)
   (c) \(-2\)
   (d) \(\sqrt{3}\)
   (e) The limit does not exist

2. Find the numbers \( a \) such that \( \lim_{x \to 0} \frac{e^{ax^2} - \cos 2x}{x^2} = 8 \).

   (a) \( a = 3 \)
   (b) \( a = 6 \)
   (c) \( a = 4 \)
   (d) \( a = 2 \)
   (e) \( a = 1 \)

3. A function \( f \) is defined on an interval \( [a, b] \). Which of the following statements are always true?

   I. If \( f \) is differentiable on \( (a, b) \), and \( f(a) \) and \( f(b) \) have opposite sign, then there must be a point \( c \in (a, b) \) such that \( f(c) = 0 \).

   II. If \( f \) is continuous on \( [a, b] \), and \( f(a) \) and \( f(b) \) have opposite sign, then there must be a point \( c \in (a, b) \) such that \( f(c) = 0 \).

   III. If \( f \) is continuous on \( [a, b] \) and there is a point \( c \) in \( (a, b) \) such that \( f(c) = 0 \), then \( f(a) \) and \( f(b) \) have opposite sign.

   IV. If \( f \) is differentiable on \( (a, b) \) and has no zeros on \( [a, b] \), then \( f(a) \) and \( f(b) \) have the same sign.

   (a) I, III, IV
   (b) I, II
   (c) III only
   (d) II, IV
   (e) II only
4. The values of $A$ and $B$ such that

$$f(x) = \begin{cases} Ax^3 + Bx + 2, & x \leq 2 \\ Bx^2 - A, & x > 2 \end{cases}$$

is everywhere differentiable are:

(a) $A = -2, B = -8$
(b) $A = 2, B = 10$
(c) $A = 2, B = 8$
(d) $A = 1, B = 4$
(e) $A = -2, B = 10$

5. \( \lim_{h \to 0} \int_{\frac{\pi}{4}}^{\frac{\pi}{4} + h} \frac{\sin x}{x} \, dx = \)

(a) 0
(b) $\frac{\sqrt{2}}{2}$
(c) 1
(d) $\frac{2\sqrt{2}}{\pi}$
(e) $\frac{\sqrt{2}}{2\pi}$

6. Suppose that $f$ is continuous on $[1, 5]$ and differentiable on $(1, 5)$. Suppose also that $f(1) = 3$ and $f(5) = -1$. Which of the following statements is not necessarily true?

(a) The Mean-Value Theorem applies to $f$.
(b) $f$ is integrable on $[1, 5]$.
(c) There exists a number $c \in (1, 5)$ such that $f'(c) = 1$.
(d) If $k$ is a number between $-1$ and 3, then there exists a number $c \in (1, 5)$ such that $f(c) = k$.
(e) If $c$ is any number such that $1 < c < 5$, then $\lim_{x \to c} f(x)$ exists.

7. If $f'(x) = h(x)$ and $g(x) = x^3 + 1$, then \( \frac{d}{dx}[g(x)] = \)

(a) $h(x^3 + 1)$
(b) $3x^2 h(x^3 + 1)$
(c) $3x^2 h(x)$
(d) $3x^2 h'(x^3 + 1)$
(e) $(x^3 + 1)h(3x^2)$
8. An equation for the normal line to the curve $2x^3 + 2y^2 = 5xy$ at the point $(1, 2)$ is
   (a) $3x - 4y = 2$
   (b) $3x + 4y = -5$
   (c) $3x + 4y = 11$
   (d) $4x - 3y = -2$
   (e) $4x + 3y = 10$

9. The graph of the derivative of a function $f'$ is shown below.

Which of the following is (are) not true? Select all correct answers.

(a) $f$ has a local maximum at $x = 0$.
(b) $f$ is increasing on $[-1, 0]$.
(c) $f$ has a point of inflection at $x = 2$.
(d) $f$ is concave up on $[0, 2]$.
(e) $f$ is concave down on $(a, b)$.

10. A rectangle with one side on the $x$-axis is inscribed in the triangle formed by the lines $y = x$, $y = 0$, and $2x + y = 12$. The maximum area of such a rectangle is:

(a) 6
(b) 3
(c) $5/2$
(d) 5
(e) 7
11. Set \( f(x) = \frac{4}{1 + x^2} \), and let \( H(x) = \int_0^x f(t) \, dt \). The local linearization of \( H \) at \( x = 1 \) is

(a) \( y = 2x \)
(b) \( y = 2x + \pi - 2 \)
(c) \( y = -2x - 4 \)
(d) \( y = 2x + \pi \)
(e) \( -2x + 2\ln 2 \)

12. Given that \( \int_0^2 f(x) \, dx = \frac{8}{3} \), \( \int_1^2 f(x) \, dx = \frac{4}{3} \), and \( \int_0^3 f(x) \, dx = \frac{11}{3} \), find \( \int_3^1 f(x) \, dx \).

(a) \(-5/3\)
(b) 2
(c) 7/3
(d) 5/3
(e) \(-7/3\)

13. The region bounded by \( y = e^x, \ y = 1, \) and the line \( x = 2 \) is rotated about the \( x \)-axis. Which of the following integrals gives the volume of the solid which is generated:

\[ (A) \ \pi \int_0^2 e^{2x} \, dx, \quad (B) \ 2\pi \int_1^2 y(2 - \ln y) \, dy, \quad (C) \ \pi \int_0^2 (e^{2x} - 1) \, dx \]

\[ (D) \ 2\pi \int_0^2 y(2 - \ln y) \, dy, \quad (E) \ \pi \int_0^2 (e^x - 1)^2 \, dx \]

(a) (A) and (D)
(b) (B) and (C),
(c) (A), (D) and (E)
(d) (C) and (D)
(e) (B) only

14. If \( f \) is a continuous function and \( F(x) = \int_0^x \left[ (t^2 + 1) \int_2^t f(u) \, du \right] \, dt \), then \( F''(2) = \)

(a) \( 5f(2) \)
(b) 5
(c) \( 4f(2) \)
(d) \( 5f'(2) \)
(e) \( 4f'(2) \)
15. The work done in lifting an object is the product of the weight of the object and the distance it is moved. A cylindrical barrel 2 feet in diameter and 4 feet high is half full of oil weighing 50 pounds per cubic foot. The work done, in foot-pounds, in pumping the oil to the top of the container is:

(a) $100\pi$
(b) $200\pi$
(c) $300\pi$
(d) $600\pi$
(e) $1200\pi$

16. The curve in the figure shown below is given by $y = \frac{2}{1 + x^2}$. Find the area of the shaded region.

(a) $4 - \frac{\pi}{2}$
(b) $4 - \pi$
(c) $4 - 2\pi$
(d) $4 - \frac{\pi}{4}$
(e) $2\pi - 4$

17. The function $f(x) = x^3 + 2x - 9$ has an inverse. If the graph of $f$ passes through the point $(2, 3)$, then $(f^{-1})'(3) =$

(a) 14
(b) $1/29$
(c) $1/12$
(d) $1/14$
(e) 29
18. If \( x = 2 \sin \theta \), then \( \int_1^2 \frac{x^2}{\sqrt{4 - x^2}} \, dx \) is equivalent to

(a) \( 4 \int_1^2 \sin^2 \theta \, d\theta \)
(b) \( 2 \int_0^{\pi/2} \sin \theta \tan \theta \, d\theta \)
(c) \( 2 \int_{\pi/6}^{\pi/2} \frac{\sin^2 \theta}{\cos^2 \theta} \, d\theta \)
(d) \( 4 \int_0^{\pi/2} \sin^2 \theta \, d\theta \)
(e) \( 4 \int_{\pi/6}^{\pi/2} \sin^2 \theta \, d\theta \)

19. The general solution of the differential equation \( \frac{dy}{dx} = \frac{1 - 2x}{y} \) is a family of:

(a) straight lines
(b) circles
(c) ellipses
(d) parabolas
(e) hyperbolas

20. Which differential equation which has the slope field

(a) \( \frac{dy}{dx} = \frac{5}{y} \)
(b) \( \frac{dy}{dx} = \frac{5}{x} \)
(c) \( \frac{dy}{dx} = \frac{x}{y} \)
(d) \( \frac{dy}{dx} = 5y \)
(e) \( \frac{dy}{dx} = x + y \)
21. The curve \( r = 1 - 2 \sin \theta \) is shown in the figure. The area enclosed by the inner loop of the curve is:

(a) \( \pi - \frac{3\sqrt{3}}{2} \)
(b) \( \pi - 3\sqrt{3} \)
(c) \( 3\sqrt{3} - \pi \)
(d) \( \pi + \frac{3\sqrt{3}}{2} \)
(e) \( \frac{1}{2} \left( 3\pi - 3\sqrt{3} \right) \)

22. The length of the graph of \( f(x) = \ln \sec x \), \( 0 \leq x \leq \pi/3 \) is:

(a) \( 3 + \sqrt{2} \)
(b) \( \ln \left( 2 + \sqrt{3} \right) \)
(c) \( \ln \left( \sqrt{3} \right) \)
(d) \( 2 + \sqrt{3} \)
(e) \( \ln \left( \frac{1 + \sqrt{3}}{2} \right) \)

23. A particle moves along the parabola \( x = 3y - y^2 \) so that \( \frac{dy}{dt} = 3 \) at all times \( t \). The speed of the particle when it is at the point \( (2, 1) \) is:

(a) \( 2\sqrt{2} \)
(b) \( 3 \)
(c) \( \sqrt{13} \)
(d) \( 3\sqrt{2} \)
(e) \( 2\sqrt{3} \)

24. If 40 grams of a radioactive substance decays to 20 grams in two years, then, to the nearest gram, the amount left after 3 years is:

(a) 11
(b) 12
(c) 14
(d) 16
(e) 17
25. A curve in the plane is defined by the parametric equations: \( x = e^{2t} + 2e^{-t} \), \( y = e^{2t} + e^t \). An equation for the line tangent to the curve at the point where \( t = \ln 2 \) is:

(a) \( 10x - 7y = 8 \)
(b) \( 5x - 6y = -11 \)
(c) \( 5x - 3y = 7 \)
(d) \( 7x + 10y = -8 \)
(e) \( 3x - 2y = 3 \)

26. If a block of ice melts at the rate of \( \frac{72}{2t + 3} \) cm\(^3\)/min, then the closest approximation to the amount of ice which melts during the first three minutes is:

(a) 16 cm\(^3\)
(b) 22 cm\(^3\)
(c) 31 cm\(^3\)
(d) 40 cm\(^3\)
(e) 68 cm\(^3\)

27. Which infinite series converge(s)?

(I) \( \sum_{n=1}^{\infty} \frac{4^n}{n!} \)  
(II) \( \sum_{n=1}^{\infty} \frac{2^{3n}}{3^{2n}} \)  
(III) \( \sum_{n=1}^{\infty} \frac{4n^3}{n^4 + 1} \)

(a) I only
(b) I and II
(c) II only
(d) II and III
(e) I, II and III

28. The radius of convergence of the power series \( \sum_{n=1}^{\infty} \frac{n!}{n^n} x^n \) is:

(a) 0
(b) \( \frac{1}{e} \)
(c) 1
(d) \( e \)
(e) \( \infty \)
29. The function $f$ is infinitely differentiable, $f(2) = 4$, and

$$f^{(n)}(2) = \frac{(n-1)!}{3^n} \quad \text{for all} \quad n \geq 1.$$ 

The interval of convergence of the Taylor series for $f$ is:

(a) $-\infty < x < \infty$
(b) $-3 < x < 3$
(c) $-1 < x < 5$
(d) $0 \leq x \leq 4$
(e) $-1 \leq x < 5$

30. Using two terms of an appropriate Maclaurin series, estimate $\int_{0}^{1} \frac{1 - \cos x}{x} \, dx$. 

(a) $\frac{19}{96}$
(b) $\frac{23}{96}$
(c) $\frac{1}{4}$
(d) $\frac{25}{96}$
(e) undefined; the integral is improper