

Name: KEY FOR GRADERS School: _____

(Letter answers only are included here. Detailed solutions can be found in a separate document.)

University of Houston High School Mathematics Contest Geometry Exam – Spring 2012

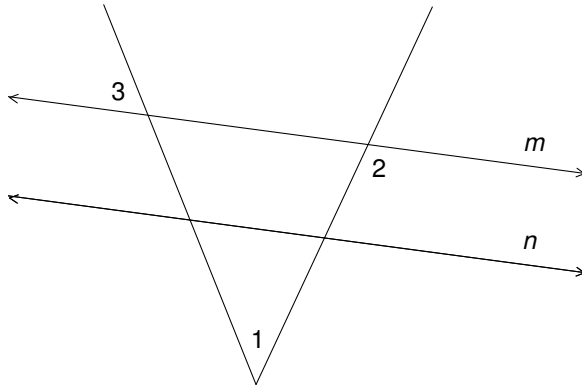
Letter Answers to the Exam:

1	E	15	C
2	E	16	B
3	D	17	C
4	A	18	B
5	B	19	D
6	E	20	D
7	D	21	D
8	C	22	D
9	E	23	E
10	C	24	C
11	B	25	A
12	A	26	C
13	D	27	C
14	A		

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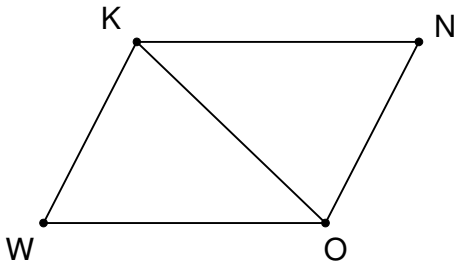
1. Lines m and n are parallel in the figure below. If $m\angle 1 = 31^\circ$ and $m\angle 2 = 102^\circ$, find $m\angle 3$.

Answer: E



- (A) 31°
- (B) 109°
- (C) 78°
- (D) 102°
- (E) 71°

2. In the diagram below, \overline{KW} is parallel to \overline{NO} . What additional information is needed to prove that $\triangle KNO \cong \triangle OKW$ by SAS? Answer: E



- (A) No additional information is needed.
- (B) $\overline{KN} \cong \overline{WO}$
- (C) $\angle N \cong \angle W$
- (D) $\angle NKO \cong \angle WKO$
- (E) $\overline{KW} \cong \overline{NO}$

3. A truck is passing a man who is painting his mailbox, and the man splashes a blob of red paint onto one of the truck's rear tire treads. As the truck continues to move along, the red paint on that tire makes a series of evenly-spaced paint splotches on the road. If the distance between the center of one paint splotch to another is 113 inches, find the approximate diameter of the truck's tire. Answer: D

- (A) 12 in. (B) 18 in. (C) 113 in. (D) 36 in. (E) 56.5 in.

4. The measure of an interior angle of a regular polygon is 9 times the measure of an exterior angle of that polygon. How many sides does the polygon have? Answer: A

- (A) 20 (B) 18 (C) 10 (D) 24 (E) 9

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5. Find the number of diagonals in a regular octagon which do NOT lie on lines of symmetry. **Answer: B**

(A) 12 (B) 16 (C) 20 (D) 8 (E) 32

6. Triangle SCY has vertices $S(1, 7)$, $C(-2, 3)$, and $Y(4, -6)$. Find the slope of median \overline{CD} . **Answer: E**

(A) $\frac{1}{7}$

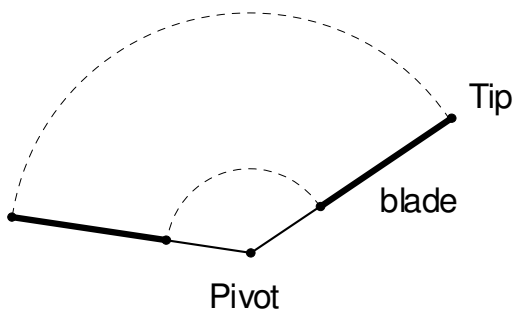
(B) -5

(C) $-\frac{22}{9}$

(D) $-\frac{2}{5}$

(E) $-\frac{5}{9}$

7. A windshield wiper is installed on the back window of a car and is 18 inches long, measured from the pivot point to the tip, as illustrated in the figure below. The rubber blade (the portion which touches the window itself) is 12 inches long. If the wiper traces out an angle of 150° , find the area of the windshield which is cleaned with each full sweep. **Answer: D**



(A) $75\pi \text{ in}^2$ (B) $135\pi \text{ in}^2$ (C) $60\pi \text{ in}^2$ (D) $120\pi \text{ in}^2$ (E) $25\pi \text{ in}^2$

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8. Circle A is circumscribed about equilateral triangle DEF, and Circle A is inscribed in equilateral triangle RST. Find the ratio of the area of the triangle RST to the area of triangle DEF. **Answer: C**

(A) $\sqrt{3}:1$ (B) 2:1 (C) 4:1 (D) 3:1 (E) $\sqrt{3}:2$

9. Triangle RBY has vertices $R(1, -2)$, $B(-3, 4)$, and $Y(8, 1)$. Classify triangle RBY according to its sides and angles. **Answer: E**

(A) Isosceles, obtuse
(B) Scalene, right
(C) Scalene, acute
(D) Isosceles, right
(E) Scalene, obtuse

10. Use the following statements to answer this question. **Answer: C**

Statement A: If two angles are not congruent, then they are not right angles.

Statement B: If a quadrilateral is rotationally symmetric, then it is a parallelogram.

Statement C: If a quadrilateral is a rectangle, then it is a square.

Which of the following statements are TRUE?

- I. The inverse of Statement A
II. The contrapositive of Statement B
III. The converse of Statement C

(A) I and II only (B) II only (C) II and III only
(D) III only (E) I, II and III

11. A ladder is leaning against the top of a building. The angle of elevation from the base of the ladder to the top of the building is 54° . If the base of the ladder is 16 feet from the base of the house, find an expression which represents the length of the ladder. **Ans: B**

(A) $16\sin(54^\circ)$ (B) $\frac{16}{\cos(54^\circ)}$ (C) $16\tan(54^\circ)$

(D) $16\cos(54^\circ)$ (E) $\frac{\sin(54^\circ)}{16}$

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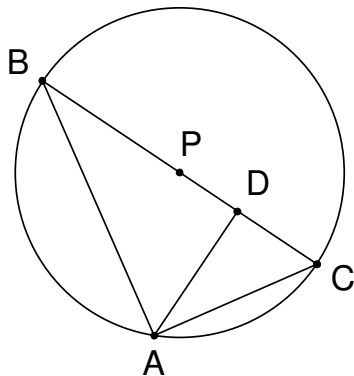
12. A right prism has eight faces, and its base is a regular polygon with side length 8 cm. If the prism has a height of 10 cm, find its volume. **Answer: A**

- (A) $960\sqrt{3}$ cm³
(B) $320\sqrt{3}$ cm³
(C) $160\sqrt{3}$ cm³
(D) 640 cm³
(E) 1280 cm³

13. In circle T, chords \overline{AB} and \overline{CD} intersect at point E. If $AE = 2x$, $CE = x+1$, $BE = x+7$ and $ED = 3x+6$, find all possible measures for x. **Answer: D**

- (A) $x=3$ (B) $\frac{-2+\sqrt{39}}{5}$ (C) $x=6$ (D) $x=2$ or $x=3$ (E) No solution

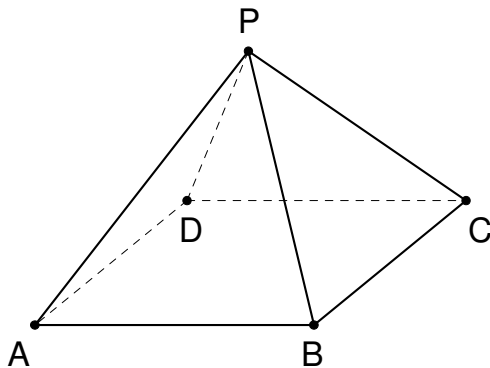
14. Triangle ABC is inscribed as circle P as shown in the diagram below. If $\overline{AD} \perp \overline{BC}$, $AD = 2\sqrt{21}$, and $DC = 6$, find the length of \overline{PD} . **Answer: A**



- (A) 4 (B) 5 (C) 6 (D) 7 (E) 10

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15. A right pyramid is shown below with square base ABCD and apex P. If all of the lateral faces are equilateral triangles, find the measure of angle APC. **Answer: C**



- (A) 105°
(B) 120°
(C) 90°
(D) 135°
(E) 150°

16. Given the following information, find the measure of \overline{CR} , in meters. **Answer: B**

$$\triangle BUS \sim \triangle CAR$$

$$BU = (2x - 3) \text{ cm}$$

$$SB = (x + 9) \text{ cm}$$

$$SU = (10x + 150) \text{ mm}$$

$$AR = 800 \text{ cm}$$

$$AC = (x - 4) \text{ m}$$

- (A) 24 (B) 6 (C) 18 (D) 15 (E) 9

17. Given the points $A(-3, 7)$ and $B(5, -11)$, find the equation of the perpendicular bisector of \overline{AB} . **Answer: C**

(A) $y = \frac{4}{9}x + \frac{17}{9}$

(B) $y = -\frac{4}{9}x - \frac{14}{9}$

(C) $y = \frac{4}{9}x - \frac{22}{9}$

(D) $y = -\frac{9}{4}x + \frac{1}{4}$

(E) $y = \frac{4}{9}x - \frac{97}{9}$

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18. A circle passes through the points $(6, 7)$, $(-6, -5)$ and $(2, 11)$. Find the radius of the circle. **Answer: B**

(A) $2\sqrt{6}$

(B) $4\sqrt{5}$

(C) $2\sqrt{10}$

(D) $6\sqrt{2}$

(E) $2\sqrt{3}$

19. A museum purchases a small replica of a famous log cabin, which has been constructed using a scale of 1:30. If the great room in the real log cabin has a 200 square foot floor, find the area of that floor in the replica, in square inches. **Answer: D**

(A) 960 in^2

(B) 480 in^2

(C) 2.6 in^2

(D) 32 in^2

(E) 80 in^2

20. Suppose that solids are created using the following pattern:

Value of k :	$k = 1$	$k = 2$	$k = 3$	$k = 4$
Solid formed:	Triangular pyramid	Rectangular prism	Pentagonal pyramid	Hexagonal prism

Let p represent a positive odd number. If the above pattern continues, find the number of edges of the solid formed when $k = 6p$. **Answer D**

(A) $12p + 4$

(B) $18p$

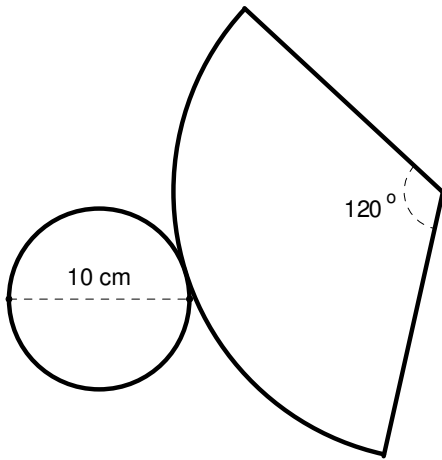
(C) $12p$

(D) $18p + 6$

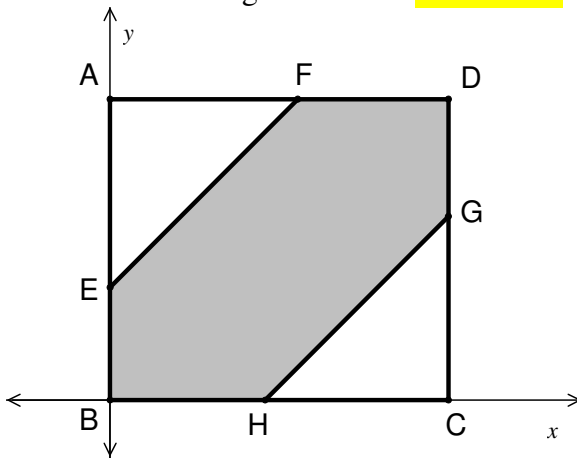
(E) $6p + 2$

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21. The net shown below can be used to form a solid. The net is composed of a circle of diameter 10 cm, which is externally tangent to a sector of a larger circle with central angle 120° . Find the volume of the solid formed from this net. **Answer: D**



- (A) $125\pi \text{ cm}^3$ (B) $\frac{125\pi\sqrt{3}}{3} \text{ cm}^3$ (C) $250\pi \text{ cm}^3$
 (D) $\frac{250\pi\sqrt{2}}{3} \text{ cm}^3$ (E) $\frac{125\pi\sqrt{35}}{3} \text{ cm}^3$
22. Rectangle ABCD is drawn on the coordinate plane and is formed by the lines $x=0$, $y=0$, $x=10$, and $y=7$. Points H and G represent the intersection of the rectangle and the line $y=x-4$. Points E and F represent the intersection of the rectangle and the line $y=x+3$. If a point P is randomly chosen within rectangle ABCD, what is the probability that P lies in the shaded hexagon below? **Answer: D**



- (A) $\frac{16}{35}$ (B) $\frac{13}{35}$ (C) $\frac{19}{35}$ (D) $\frac{22}{35}$ (E) $\frac{11}{14}$

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23. When a plane intersects a cube, the intersection of the plane and the cube results in a polygon with _____ sides (depending on the location of the plane in relation to the cube). **Answer: E**

(A) 3 (B) 3 or 4 (C) 3, 4 or 5 (D) 3, 4, 6, or 8 (E) 3, 4, 5, or 6

24. Two semicircles with diameters d_1 and d_2 are drawn along the diameter of a circle, as shown in Figure 1. The diameter is removed in Figure 2, and the circle is divided into regions P and Q. If $d_2 = a \cdot d_1$, where a is a positive constant, find the ratio of the area of region Q to the area of the outer circle. **Answer: C**

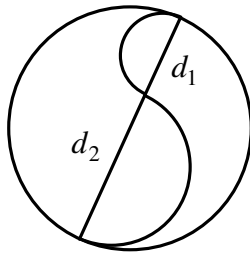


Figure 1

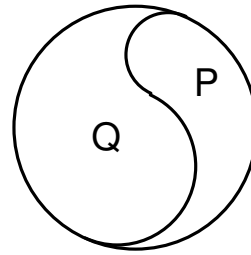
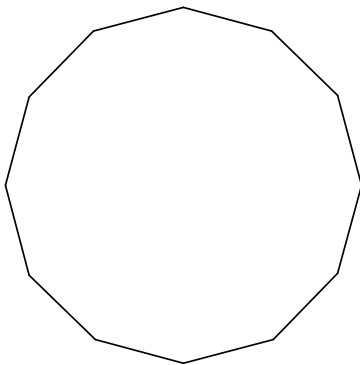


Figure 2

(A) $\frac{1}{a+1}$ (B) $\frac{2}{a+3}$ (C) $\frac{a}{a+1}$ (D) $\frac{a^2}{a^2+3}$ (E) $\frac{a^2+2}{(a+1)^2}$

25. Find the area of a regular dodecagon with side length 10 cm. **Answer: A**



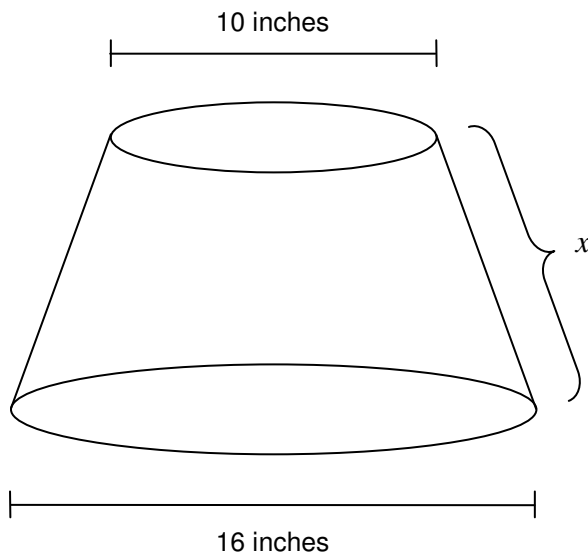
(A) $(600+300\sqrt{3}) \text{ cm}^2$ (B) $(600+140\sqrt{3}) \text{ cm}^2$ (C) $(450+65\sqrt{3}) \text{ cm}^2$
(D) $(700+400\sqrt{2}) \text{ cm}^2$ (E) $(400+75\sqrt{3}) \text{ cm}^2$

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26. Which of the following pentagons will always tessellate the plane? **Answer: C**

- (A) A pentagon with two pair of congruent sides
- (B) A regular pentagon
- (C) A convex pentagon with one pair of parallel sides
- (D) A pentagon with five congruent sides
- (E) None of the above will always tessellate the plane

27. A lampshade is in the shape of a truncated cone. The top opening has a diameter of 10 inches, and the bottom opening has a diameter of 16 inches. If the lampshade is made of $\frac{663\pi}{8}$ square inches of cloth (not counting any overlap in the design), find the slant height of the lampshade, indicated by the value x in the diagram below. **Answer: C**



- (A) 4 in. (B) 13 in. (C) $\frac{51}{8}$ in. (D) $\frac{663}{64}$ in. (E) $\frac{85}{8}$ in

END OF EXAM ☺

(Make sure that the bubbling form is filled in with the letter of each correct answer.)