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University of Houston
High School Math Contest – 2013
Algebra II Test

1. Find the mean of the factors of the number 2013.
   A. 672
   B. 454.2
   C. 551.8
   D. 372
   E. 160.33

2. Find the point of intersection of the following lines:

   $3x - y = U$ $\qquad\qquad$ $2x + 2y = H$

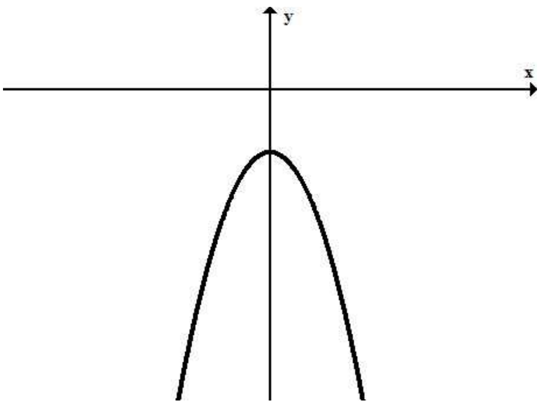
   A. $\left(\frac{1}{8}(3U-2H),\ \frac{1}{8}(2U+H)\right)$
   B. $\left(\frac{1}{8}(2U+H),\ \frac{1}{8}(3H-2U)\right)$
   C. $\left(\frac{1}{4}(2U-H),\ \frac{1}{4}(3H-U)\right)$
   D. $\left(\frac{1}{8}(U+H),\ \frac{1}{8}(3H-5U)\right)$
   E. $(U-H,\ 2U-3H)$

3. Which of the following is a solution to the equation $60x^2 = 42x + 72$?
   A. $-\frac{3}{2}$
   B. $\frac{6}{5}$
   C. $-\frac{1}{2}$
   D. $-\frac{4}{5}$
   E. 2

4. A basket contains 6 black socks and 6 brown socks. The basket is in a dark room so it is impossible to distinguish black and brown socks. Yoshi randomly draws socks out of the basket until he is guaranteed to have a matching pair of socks. How many more socks must he draw to guarantee that he has a pair of brown socks?
   A. 0
   B. 2
   C. 3
   D. 4
   E. 5

5. Find the equation of the line that passes through the point $(-5,-3)$ and the center of the circle $x^2 + y^2 - 6x + 2y - 26 = 0$.
   A. $x - 4y = 7$
   B. $y = 2x + 7$
   C. $-x + 2y = 1$
   D. $y = x + 2$
   E. $-3x + 2y = 9$

6. The function $f(x) = ax^2 + bx + c$ is graphed below. Which of the following statements is true about the coefficients a, b and c?



   A. $a < 0,\ b > 0,\ c > 0$
   B. $a < 0,\ b < 0,\ c < 0$
   C. $a < 0,\ b > 0,\ c < 0$
   D. $a < 0,\ b < 0,\ c > 0$
   E. None of the above

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7. Cube P has sides of length p units. Cube W has sides of length w units. If $p+w=6$ and the sum of the area of one side of Cube P plus the area of one side of Cube W is 22 square units, find the sum of the volume of Cube P plus the volume of Cube W in cubic units.
   A. 90
   B. 216
   C. 132
   D. 108
   E. 69

8. Solve for x: $\frac{1}{\sqrt{x}}+\frac{1}{x+\sqrt{x}}=1$.
   A. $\frac{1}{2}$
   B. 2
   C. 3
   D. 0
   E. 8

9. Recall the complex number i, where $i=\sqrt{-1}$. Write the expression $\frac{3-5i}{-1+7i}$ in the form $a+bi$.
   A. $\frac{3}{50}-\frac{1}{10}i$
   B. $\frac{19}{25}-\frac{13}{25}i$
   C. $-\frac{19}{25}-\frac{8}{25}i$
   D. $-\frac{16}{25}-\frac{13}{25}i$
   E. $\frac{16}{25}-\frac{8}{25}i$

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10. What is the coefficient of the x^2 term in the expansion of $\left(x^2 - \frac{2}{x^2}\right)^7$?

A. –35
B. 448
C. -70
D. -280
E. 35

11. Simplify

$$\frac{\dfrac{k^3 - yk^2 - y^2k + y^3}{k^2 - yk - nk + ny}}{\dfrac{2yk + 3nk + 2y^2 + 3ny}{2yk - 3n^2 + 3nk - 2ny}}$$

A. -1
B. 1
C. $y+k$
D. $y-k$
E. $k-y$

12. What percentage of the interval $(-2,2)$ is a solution to the inequality $|x+1|+|x-2|<4$?

A. 100%
B. 87.5%
C. 80%
D. 75%
E. 66.7%

13. The two roots of the equation $x^2 - 31x + c = 0$ are prime integers. Find the smallest possible positive integer value of c.

A. 2
B. 56
C. 31
D. 58
E. 33

14. The graphs of the functions $f(x)=4^{\frac{x-8}{2}}$ and $g(x)=8^{-x+2}$ intersect at the point (x,y). Find the product xy.

A. $\frac{\sqrt{2}}{2}$
B. $\frac{7\sqrt{2}}{4}$
C. $\frac{7\sqrt{2}}{64}$
D. $\frac{1}{16}$
E. $\frac{\sqrt{2}}{32}$

15. How many 2 digit numbers are divisible by both of their digits?

A. 9
B. 14
C. 15
D. 18
E. 21

16. The function $f(n)$ defined for $n=1,2,3,4,\ldots$ by $f(n+1)=\frac{f(n)-3}{2}$ and $f(1)=7$. Find $f(4)$.

A. $\frac{-1}{2}$
B. $\frac{-7}{4}$
C. -2
D. $\frac{-5}{4}$
E. $\frac{-7}{2}$

17. Which of the following intervals is a subset of the domain of the function $f(x)=\sqrt{\log_3(1-x)-1}$?
    A. $-1<x<1$
    B. $-3<x<0$
    C. $1\le x\le 6$
    D. $-4\le x\le -2$
    E. $-2<x<0$

18. The real number $\sqrt{12+\sqrt{140}}$ may be represented as $\sqrt{a}+\sqrt{b}$ for two integers a and b with $a>b$. Find $a-b$.
    A. 2
    B. 3
    C. 7
    D. 1
    E. 4

19. Given a real number x, the greatest integer function $\lfloor x\rfloor$ assigns to x the largest integer that is less than or equal to x. For example, $\lfloor 3.6\rfloor=3$ and $\lfloor -1/3\rfloor=-1$. Which of the following statements are true for all real numbers x?

    $P:\lfloor 4x\rfloor=4\lfloor x\rfloor$

    $Q:\lfloor x+2\rfloor=\lfloor x\rfloor+2$

    $R:\lfloor x+y\rfloor\ge\lfloor x\rfloor+\lfloor y\rfloor$

    $S:\lfloor x+y\rfloor=\lfloor x\rfloor+\lfloor y\rfloor$

    $T:\lfloor -x\rfloor=-\lfloor x\rfloor$

    A. All statements are true
    B. P, Q, R, S
    C. S and Q
    D. R and Q
    E. P, Q, R

20. The complex numbers 5, $2-3i$ and $3-2i$ are roots of the polynomial $p(x)$, which has real coefficients. What is the smallest possible degree of $p(x)$?

A. 3
B. 4
C. 5
D. 6
E. 7

21. Triangle *CAT* has vertices $C(3,-8)$, $A(6,-1)$, and $T(-5,2)$. Find the slope of the altitude from point *T*.

A. $-\frac{13}{19}$

B. $-\frac{3}{7}$

C. $\frac{11}{3}$

D. $\frac{4}{5}$

E. $-\frac{3}{11}$

22. Simplify $\log_4\left(32\sqrt{2}\right)$.

A. $\frac{11}{2}$
B. $\frac{5}{4}$
C. $\frac{11}{4}$
D. $8\frac{1}{2}$
E. $\frac{5}{8}$

23. In the xy-plane, how many lines whose x-intercept is a positive prime number and whose y-intercept is a positive integer pass through the point $(4,3)$?

A. 0
B. 2
C. 4
D. 3
E. 1

24. If $\dfrac{x-1}{x^2-x-6}=\dfrac{A}{x-3}+\dfrac{B}{x+2}$, find $A-B$.

A. $\dfrac{1}{5}$
B. 3
C. $\dfrac{3}{4}$
D. $-\dfrac{3}{5}$
E. $-\dfrac{1}{5}$

25. Simplify:

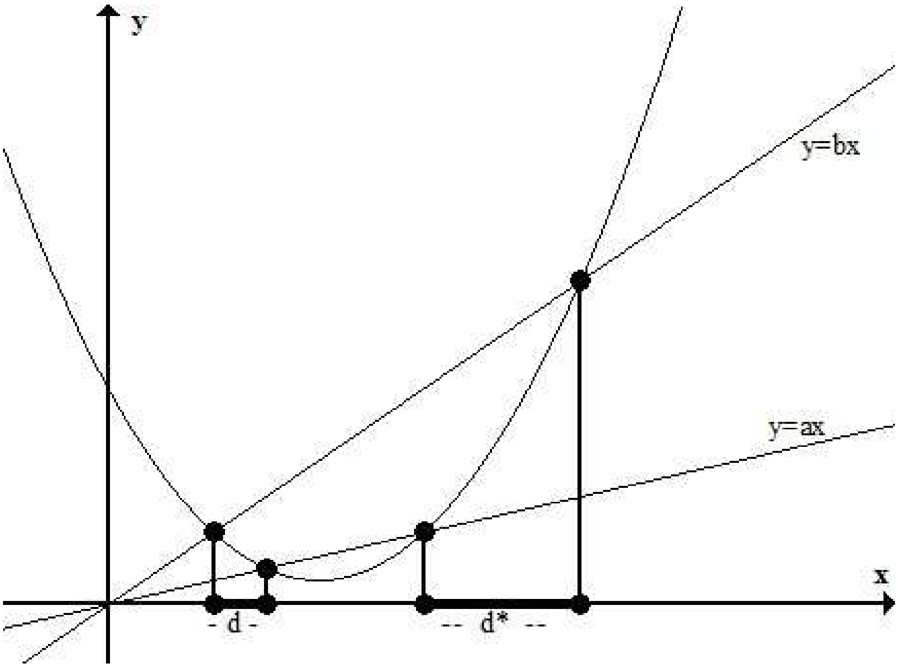
$$\sqrt{\frac{3^{x+7}+3\cdot 3^{x+9}}{3\cdot 3^{x+7}-3^{x+6}}}$$

A. $\dfrac{\sqrt{3}}{2}$
B. $\dfrac{\sqrt{42}}{2}$
C. $\sqrt{21}$
D. $\dfrac{4\sqrt{3}}{3}$
E. $\sqrt{15}$

26. For all positive integers n, the factorial function $n!$ is defined as $1! = 1$, $2! = 2*1! = 2*1$ and $n! = n*(n-1)!$. Find the smallest positive integer k such that $\frac{(k!)(k!)}{(k+6)!}$ is an integer.

   A. 23
   B. 47
   C. 89
   D. 119
   E. 120

27. Graphed below are the functions $y = x^2 + mx + p$, $y = ax$ and $y = bx$ with $a < b$. The lines $y = ax$ and $y = bx$ each intersect the parabola at two points. The points of intersection are projected onto the x-axis to give four points that are the endpoints of the two intervals on the x-axis of lengths d and d* shown. Find $d^* - d$.



   A. $b - a - 2m$
   B. $\sqrt{(m-b)^2 - 4p} - \sqrt{(m-a)^2 - 4p}$
   C. $b - a$
   D. $(b-a) + \sqrt{(m-b)^2 - 4p} - \sqrt{(m-a)^2 - 4p}$
   E. $4p - 2(b-a)$

28. Housepainters Terri, Juan and Sam are painting a neighborhood house. Terri can paint the whole house in 6 hours. Juan can paint the house in 8 hours. If all three work together, the house will be painted in 2 hours. How long would it take Sam to paint the house by himself?
    A. 4 hours and 48 minutes
    B. 4 hours and 50 minutes
    C. 4 hours and 40 minutes
    D. 5 hours
    E. 4 hours

29. A farmer sells 3 cattle to buy 4 sheep and has a surplus of $200. The farmer then sells 5 sheep to buy 4 cattle, but he is $400 short. Find the price of 2 cattle and 3 sheep.
    A. $2600
    B. $1425
    C. $2400
    D. $275
    E. $1150

30. Let x, y and z be real numbers which satisfy the equations below.

$$x+\frac{1}{yz}=\frac{1}{5} \qquad y+\frac{1}{xz}=\frac{-1}{15} \qquad z+\frac{1}{xy}=\frac{1}{3}$$

Find $\frac{z-y}{x-z}$.

A. $\frac{-1}{3}$
B. -2
C. $\frac{1}{2}$
D. -3
E. $\frac{7}{15}$