Name: ____

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Calculator Exam – 2013

Directions: Write your answers in the table below. **DO NOT detach this sheet from your exam.** Some of the answers below are integers. The answers which are not integers should be recorded so that they are **accurate to at least 4 places *after* the decimal**. **DO NOT ROUND YOUR ANSWERS until *after* the 4th decimal place!!** For example, suppose a question requests the value of sin(2). Your calculator will tell you that sin(2) is 0.9092974268 (assuming your calculator is in radian mode). Examples of correct responses include 0.9092, 0.90929, 0.90923 and 0.90929116. **The response 0.9093 is not correct.** It does not matter what appears *after* the 4th decimal place, provided the values up to and including the fourth decimal place are correct.

Note: Problems 11, 19 and 23 reward students for correct answers to previous problems.

1.		14.	
2.		15.	
3.		16.	
4.		17.	
5.		18.	
6.		19.	
7.		20.	
8.		21.	
9.		22.	
10.		23.	
11.		24.	
12.		25.	
13.			

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1. Give the largest integer smaller than $(219-4173)^2-\frac{3}{2}\sqrt{2843}$.

2. Give the reciprocal of the largest root of $x^4-2x^2-13x=7$.

3. $h(x)=\frac{x}{2}+\frac{2}{x}$. Define $g(x)=h\left(h\left(h\left(h\left(h\left(h(x)\right)\right)\right)\right)\right)$. Find $g(1)$.

4. Determine the number of points of intersection of $f(x)=\sin(10x)$ and $g(x)=\frac{1}{5}x^4$.

5. Given the system $\begin{pmatrix} \frac{12}{19}x-\frac{37}{17}y=21 \\ \frac{11}{15}x+\frac{241}{32}y=116 \end{pmatrix}$. Find $984916x+246229y$.

6. A quadratic function passes through the points (–3,–11), (1,6) and (3,–2). This function has a value at $x=2$ that is a positive rational number. Assume this rational number is written in lowest terms. Give the denominator.

7. Give the integer part of the *x*-intercept of the line of best least squares fit for the data (–3, –11), (1,6) and (3, –2).

8. 211 is a factor of 21432114. Give the sum of the prime factors of 21432114.

9. Give the sum of the first 213 odd integers.

10. Give the smallest integer that is larger than the maximum value of the function $f(x)=\frac{12x^3+3x^2-7}{x^4+2x^2+2}$.

11. Give the average of the answers to problems 1 through 10.

12. Give the *x*-intercept of the function in problem 10.

13. The function $f(x)=2x^3+7x+\sin(2x)$ is invertible. Find $f^{-1}(17)$.

14. There are 3 points of intersection of pairs of the lines $2x+3y=4$, $7x-2y=3$ and $x+y=15$. Give the area of the triangle determined by these three points.

15. Give the repeated root of the equation $-459x^4+2727x^3-5193x^2+3017x+196=0$.

16. Find the sum of the values in the set $\left\{1,\frac{1}{2},\frac{1}{3},\frac{1}{4},\cdots,\frac{1}{500}\right\}$.

17. The *permanent* of a 2 by 2 matrix of the form $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is given by $ad+bc$. Suppose the values a, b, c and d are chosen at random so that each is an integer whose absolute value is less than 4. What is the probability that the *permanent* of the matrix is 0?

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18. Quadratic functions have the form $f(x) = ax^2 + bx + c$. Suppose the values a, b and c are chosen at random so that each is an integer whose absolute value is less than 5, and $a \neq 0$. How many of these functions will not have an x-intercept?
19. Give the average of the answers to problems 17 and 18.
20. A function $f(x)$ has a fixed point at a value x if and only if $f(x) = x$. A fixed point $x = a$ is said to be stable if picking a value x_0 close to a causes the values $x_1 = f(x_0),\ x_2 = f(x_1),\ x_3 = f(x_2), \cdots$ to eventually get closer and closer to a. The function $f(x) = \frac{1}{100}(x-6.45)(x+3.54)(x-2.45)+2.48$ has 3 fixed points. Give the smallest stable fixed point of $f(x)$.
21. A collection of points of the form (a,b) is plotted in the xy-plane, where a and b are integers whose absolute values are smaller than 101. How many of these points lie *above* the line $3x - 17y = 47$?
22. 31 points are placed on the circle of radius 1 centered at the origin so that one of the points is at (1,0), and the remaining points are evenly distributed around the circle. A curve given parametrically by $x = \cos(t) + \frac{1}{10}\sin(13t)$ and $y = \sin(t) + \frac{1}{10}\cos(5t)$ encloses a region in the xy-plane, and a curve given by $x = \cos(t) + \frac{1}{10}\sin(5t)$ and $y = \sin(t) + \frac{1}{10}\cos(13t)$ encloses another region in the xy-plane. How many of the 31 points lie outside both of these regions?
23. Give the average of the answers to problems 21 and 22.
24. The first 9 terms in a sequence are given by 1, 11, 21, 1211, 111221, 312211, 13112221, 1113213211 and 31131211131221. Give the sum of the digits in the 10th term of the sequence.
25. **Tie Breaker: (curve jumping)** Graph the functions $f_1(x) = x\sin(x)$, $f_2(x) = (x^2 - 3x - 7)\cos(x)$, $f_3(x) = \sin(2x)$ and $f_4(x) = \frac{1}{2}$. Create a new function $g(x)$ as follows. For $x \leq -7$, $g(x) = f_1(x)$. The definition of $g(x)$ changes by moving x from smaller to larger values (starting at $x = -7$). As soon as the graph of $g(x)$ intersects one of the $f_i(x)$ functions above, $g(x)$ becomes that function, and remains that function until it intersects one of the other $f_j(x)$ functions above. $g(x)$ then becomes that function, and the pattern continues. Evaluate $g(g(1))$.