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University of Houston High School Mathematics Contest
Geometry Exam – Spring 2013

Note that diagrams may not be drawn to scale.

1. Which of the following conditions is NOT sufficient to prove that a quadrilateral is a parallelogram?

(A) The diagonals bisect each other.

(B) Both pairs of opposite angles are congruent.

(C) Both pairs of opposite sides are parallel.

(D) Both pairs of opposite sides are congruent.

(E) One pair of opposite sides is parallel, and the other pair of opposite sides is congruent.

2. **Given:** $\overline{LP}$ is an angle bisector of ΔSTL .

What conclusion follows directly from the given information?

(A) $\overline{SP} \cong \overline{PT}$ (B) $\angle SLP \cong \angle TLP$ (C) $\angle S \cong \angle T$

(D) $\overline{LP} \perp \overline{ST}$ (E) $\angle LPS \cong \angle LPT$

3. **Given:** $\angle 1$ and $\angle 2$ are complementary.

What conclusion(s) follow directly from the given information?

I. $\angle 1$ and $\angle 2$ are adjacent angles.
II. $\angle 1$ is a right angle and $\angle 2$ is a right angle.
III. $m\angle 1 + m\angle 2 = 90$
IV. $m\angle 1 + m\angle 2 = 180$
V. $\angle 1 \cong \angle 2$

(A) V only (B) I and III only (C) III only (D) IV only (E) II and IV only

4. Jennie needs 38 square feet of fabric to use for an art project. When she arrives at the fabric store, she finds that her favorite fabric is only available for purchase by the square yard, and that the store workers are not permitted to sell her a fraction of a square yard. How many square yards of fabric should Jennie purchase?

(A) 3 (B) 12 (C) 4 (D) 13 (E) 5

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5. Suppose that the following statement is true:
If p, then not q.

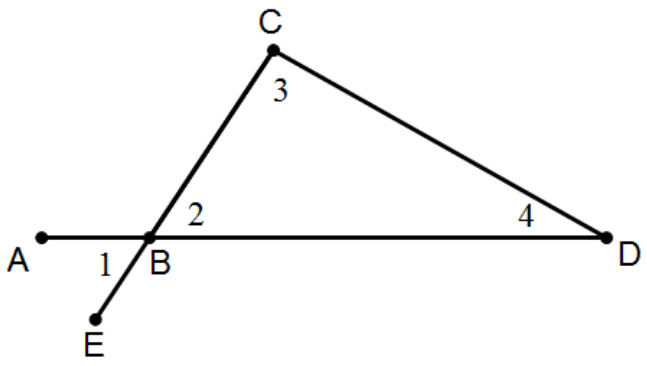
Which of the following must also be true?

I. *If not q, then not p.*
II. *If q, then not p.*
III. *If not p, then q.*
IV. *If not q, then p.*

(A) IV only (B) III and IV only (C) I and III only

(D) II only (E) There is not enough information to make a conclusion

6. In the diagram below, $\angle 1 \cong \angle 4$. Which of the following statements can NOT be proven? (Note: The diagram may not be drawn to scale.)



(A) $\overline{CD} \cong \overline{BD}$

(B) $\angle 2 \cong \angle 4$

(C) $\angle 1 \cong \angle 2$

(D) ΔCBD is an isosceles triangle.

(E) $\overline{CB} \cong \overline{CD}$

7. If the radius of a cylinder is tripled, its volume becomes _____ times as large.

(A) 9 (B) 3 (C) 6 (D) 27 (E) $3\sqrt{3}$

8. A sector of a circle has an area of 45π cm^2, and its central angle measures 18°. Find the perimeter of the sector.

(A) 60π cm (B) 63 cm (C) $(50+50\pi)$ cm (D) 30 cm (E) $(60+3\pi)$ cm

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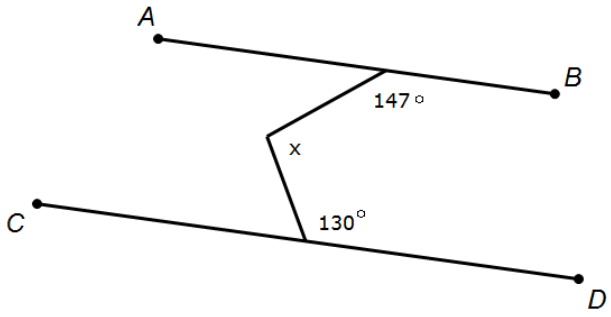
9. Kathy's umbrella is hemispherical in shape and is made of x square inches of waterproof fabric. (Assume no overlap or gaps.) Kathy is standing on a sidewalk outside of a store, waiting for the store to open, and she sees storm clouds overhead. She puts up her umbrella in order to keep dry, and continues to stand in one place as it begins to rain. Assuming that there is no wind and the rain is falling perpendicular to the ground, how large is the area of dry pavement under Kathy's umbrella, in square inches?

(A) x^2 (B) x (C) $\frac{x}{4}$ (D) $2x$ (E) $\frac{x}{2}$

10. In the diagram below, $\overline{AB}$ is parallel to $\overline{CD}$. Use the given angle measures to find the value of x.



(A) 33°
(B) 163°
(C) 17°
(D) 83°
(E) 97°

11. A right square pyramid has a slant height of 8 cm, and its base edge measures 12 cm. Find the volume of the pyramid.

(A) $288\sqrt{7}$ cm^3 (B) 384 cm^3 (C) $96\sqrt{7}$ cm^3 (D) 480 cm^3 (E) 1152 cm^3

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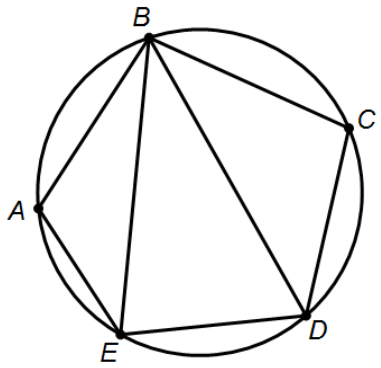
12. P is between T and W, and W is the midpoint of $\overline{KP}$. If PT = 12 and the ratio of WT to PK is 7 : 10, find the length of $\overline{PW}$.

(A) 6 (B) 60 (C) 42 (D) 30 (E) 24

13. Pentagon ABCDE is inscribed in the circle below. If $m\angle A = 130^\circ$ and $m\angle C = 70^\circ$, find $m\angle EBD$.



(A) 100°

(B) 60°

(C) 20°

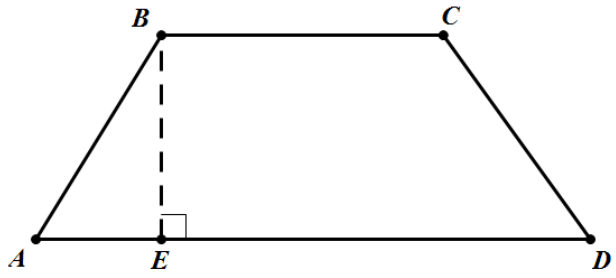
(D) 110°

(E) 40°

14. Trapezoid ABCD is shown below with bases $\overline{BC}$ and $\overline{AD}$.

$$BC = 9 \text{ cm},\ AE = 4\sqrt{3} \text{ cm},\ m\angle A = 60^\circ,\ \text{and}\ m\angle D = 45^\circ$$

Find the perimeter of ABCD.



(A) $\left(54 + 12\sqrt{2} + 4\sqrt{3}\right)$ cm

(B) $\left(30 + 12\sqrt{2} + 12\sqrt{3}\right)$ cm

(C) $\left(30 + 4\sqrt{2} + 4\sqrt{3}\right)$ cm

(D) $\left(180 + 24\sqrt{3}\right)$ cm

(E) $\left(54 + 12\sqrt{2} + 12\sqrt{3}\right)$ cm

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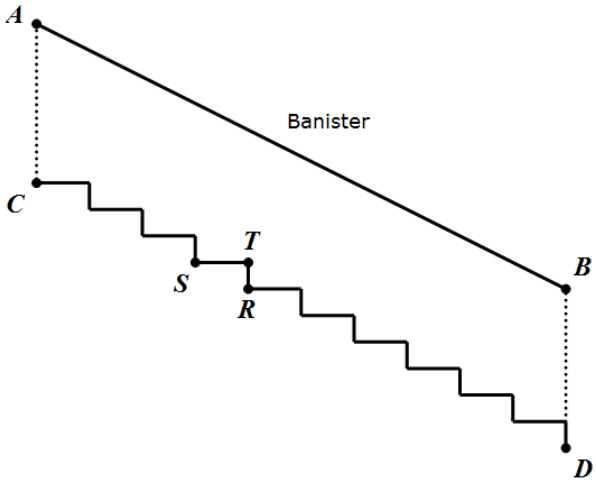
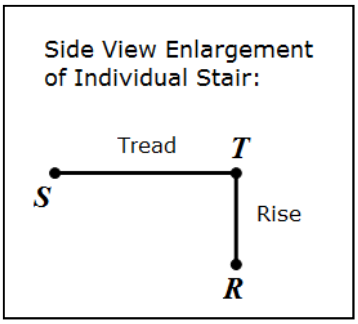
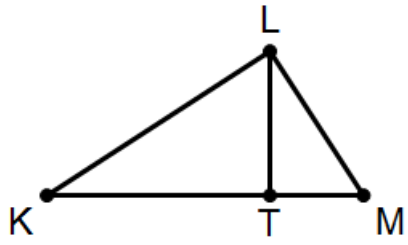
15. The side view of a staircase is shown below, along with a side view enlargement of an individual stair. The banister is represented in the diagram by $\overline{AB}$. Each stair is the same size as any other stair, and the tread of each stair is twice its rise. As would be expected, the banister is installed at a consistent height in relation to each stair, so $\overline{AC} \cong \overline{BD}$. If the banister is 11 feet long, find the tread of each stair, in inches.





(A) $\frac{132\sqrt{5}}{25}$ in. (B) $\frac{44}{5}$ in. (C) $\frac{11\sqrt{5}}{25}$ in. (D) $\frac{66\sqrt{5}}{25}$ in. (E) $\frac{44\sqrt{3}}{5}$ in.

16. Triangle LKM is shown below with altitude $\overline{LT}$. If $m\angle KLM = 90^\circ$, $LM = 6$ and $KM = 9$, find the sine of $\angle KLT$.



(A) $\frac{3}{2}$ (B) $\frac{\sqrt{5}}{3}$ (C) $\frac{2}{3}$ (D) $\frac{3\sqrt{5}}{5}$ (E) $\frac{\sqrt{5}}{2}$

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17. Find the number of sides of a regular convex polygon, given that the measure of one of its interior angles is four times the measure of one of its exterior angles.

(A) 5 (B) 36 (C) 20 (D) 10 (E) 12

18. Triangle *CAT* has vertices $C(3,-8)$, $A(6,-1)$, and $T(-5,2)$. Find the slope of the altitude from point *T*.

(A) $-\frac{13}{19}$

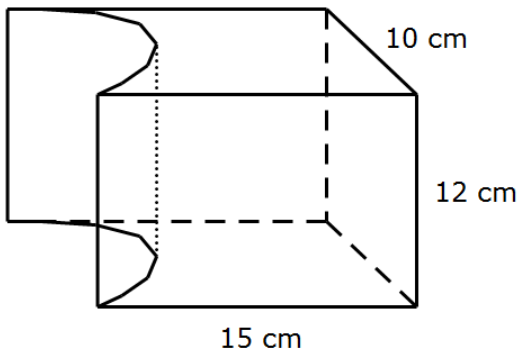
(B) $-\frac{3}{7}$

(C) $\frac{11}{3}$

(D) $\frac{4}{5}$

(E) $-\frac{3}{11}$

19. The figure below represents a right rectangular prism with half of a cylinder removed. (Please overlook the jagged appearance of the semi-cylindrical bases; they should be drawn as smooth curves. ☺) Find the total surface area of the figure.



(A) $(900-85\pi)$ cm^2

(B) $(900+35\pi)$ cm^2

(C) $(780-85\pi)$ cm^2

(D) $(1800-150\pi)$ cm^2

(E) $(780+35\pi)$ cm^2

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20. If a polygon is _____, then it will tessellate the plane.

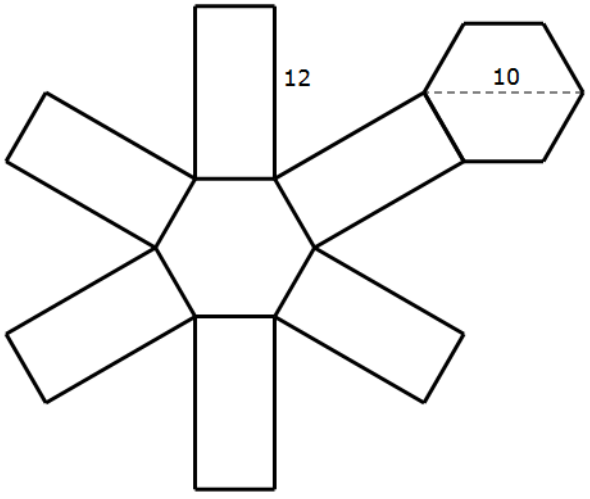
I. rotationally symmetric.
II. reflectionally symmetric.
III. both rotationally and reflectionally symmetric
IV. neither rotationally nor reflectionally symmetric

(A) I only (B) II Only (C) III only (D) IV only (E) None of these

21. The point $B(-5, -2)$ is reflected over the line $y = 6$, and then rotated 90° counterclockwise about the point (3, 4). Find the final image B'' which results from both transformations being applied.

(A) (-7, -4) (B) (-3, -4) (C) (9, 18) (D) (13, 12) (E) (5, 22)

22. Below is a net that can be used to create a solid, formed by six congruent rectangles and two congruent regular hexagons. Find the volume of the solid. Assume that the given measurements are in centimeters.



(A) $3600\sqrt{3}$ cm^3

(B) $1800\sqrt{3}$ cm^3

(C) 900 cm^3

(D) $450\sqrt{3}$ cm^3

(E) $900\sqrt{3}$ cm^3

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23. Find the equation of the line which passes through the centers of the following circles:

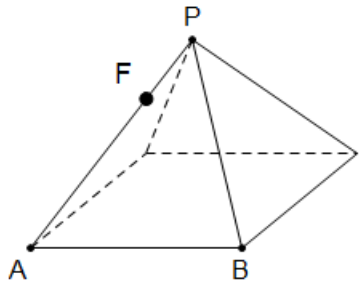
$$x^2 + y^2 - 10x + 4y = 7 \qquad 3x^2 + 3y^2 + 18x - 24y + 48 = 0$$

(A) $y = -\frac{3}{4}x - \frac{7}{4}$ (B) $y = -\frac{4}{3}x + \frac{14}{3}$ (C) $y = 5x - 2$

(D) $y = -\frac{3}{4}x + \frac{7}{4}$ (E) $y = 3x - 4$

24. A square pyramid is shown below with apex P, where points A and B are two vertices of the base, and point F is between P and A. Suppose that the pyramid is then intersected by a plane, where the plane passes through point F, is perpendicular to the base, and is parallel to $\overline{AB}$. Two new solids have been formed by this intersection.

Of these two new solids, consider only the solid which contains point P. How many edges, vertices, and faces does this new solid have?



(A) 11 edges, 7 vertices, 5 faces

(B) 10 edges, 7 vertices, 6 faces

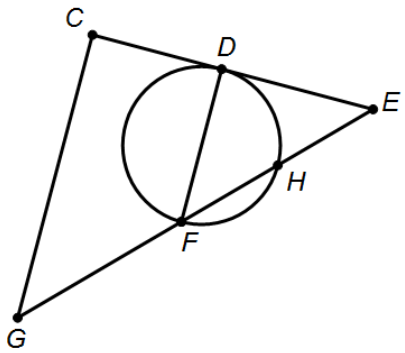
(C) 9 edges, 6 vertices, 6 faces

(D) 11 edges, 7 vertices, 6 faces

(E) 10 edges, 7 vertices, 5 faces

25. $\overline{CE}$ is tangent to the circle below at point D, $\overline{GE}$ is a secant which intersects the circle at points F and H, $\overline{DF}$ is a diameter, and $\overline{DF}$ is parallel to $\overline{CG}$. Given the following information, find the measure of $\overline{CG}$.

$CD = x + 4, \quad DE = x + 2, \quad FH = x + 1, \quad HE = x$



(A) $7\sqrt{5}$ (B) 14 (C) $8\sqrt{7}$ (D) 4 (E) $4\sqrt{5}$

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26. A right circular cone has a height of 9 inches and is shown below. The hole in the base represents a location where water can either enter or drain from the cone.

The cone is placed in the position of Figure 1 and is filled with water at a constant rate. It takes 18 minutes to completely fill the cone.

The cone is then turned upside down, as in Figure 2, and water drains out of the cone at the same rate at which it filled. How long does it take for the water to drain out to the point where the depth of the water is 3 inches above the base?

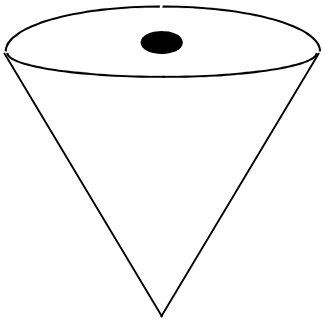


Figure 1

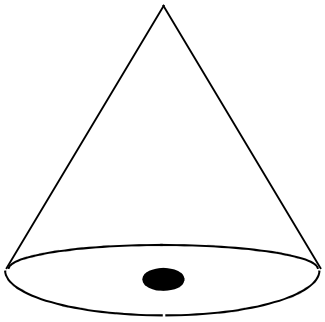


Figure 2

(A) 5 minutes, 20 seconds

(B) 6 minutes

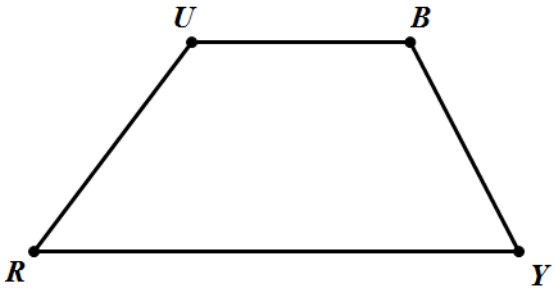
(C) 12 minutes

(D) 12 minutes, 40 seconds

(E) There is not enough information to solve this problem.

27. Trapezoid RUBY is shown below, where $\overline{UB}$ is parallel to $\overline{RY}$. Find the area of the trapezoid, given the following measurements:

$UB = 7$, $UR = 2\sqrt{13}$, $RY = 12$, and $UY = 10$



(A) $19\sqrt{13}$

(B) 38

(C) 57

(D) 95

(E) 76

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28. This question is based on a fractal known as the Sierpinski triangle:

Start with an equilateral triangle, completely shaded as shown in Figure 1.

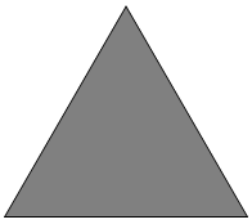

Figure 1

Next, connect the midpoints of each side of the shaded equilateral triangle. This forms a new triangle. Make that new triangle white, so that it appears to be 'removed' from the diagram, as shown in Figure 2.

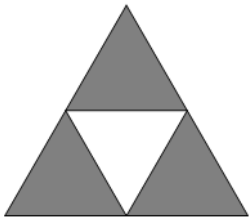

Figure 2

Repeat the previous step on <u>each</u> shaded triangle in Figure 2, resulting in Figure 3.

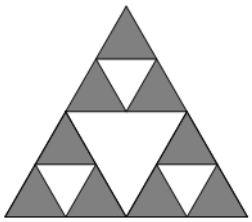

Figure 3

This procedure could continue indefinitely. Assume in this case that we will perform this iteration two more times, creating Figure 4 and Figure 5.

Question: If a dart is thrown at the diagram from Figure 5, what is the probability that the dart will land within one of the white triangles? (Assume that all darts thrown are randomly distributed within the largest outer triangle.)

(A) $\frac{27}{64}$ (B) $\frac{175}{256}$ (C) $\frac{37}{64}$ (D) $\frac{1}{40}$ (E) $\frac{81}{256}$

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29. The coffee mug in Figure 1 is composed of a hemisphere and a cylinder. The height of the cylinder is the same as the height of the hemisphere. Figure 2 shows a coffee stirrer in the same mug, which reaches exactly from the center of the bottom of the mug to the edge of any point of the rim.

If the mug has a volume of $\frac{1215\pi}{8}$ cm^3, find the length of the stirrer.

(Note that the handle is immaterial to the problem except to make it look like a mug. Assume that the mug has some type of base below the hemisphere to keep it from tipping over.)

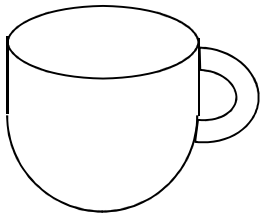


Figure 1

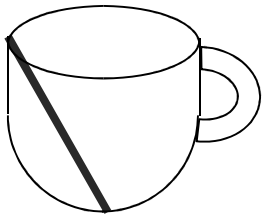
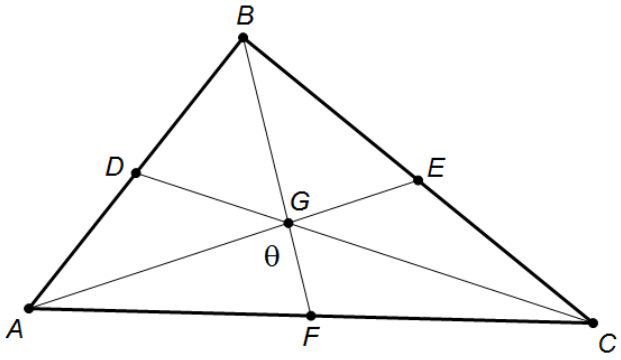


Figure 2

(A) $\frac{9\sqrt{5}}{4}$ cm (B) 9 cm (C) $\frac{9\sqrt{5}}{2}$ cm (D) $\frac{9\sqrt[3]{49}}{7}$ cm (E) $\frac{9\sqrt{3}}{2}$ cm

30. ΔABC is shown below with centroid G. The area of ΔDBG is 33 cm^2, and the length of $\overline{AG}$ is 12 cm. Let θ represent the measure of $\angle AGF$. If the cosine of θ is $\frac{\sqrt{5}}{4}$, find the length of $\overline{BG}$.



(A) $4\sqrt{11}$ cm (B) $2\sqrt{5}$ cm (C) 12 cm (D) $2\sqrt{11}$ cm (E) $\sqrt{11}$ cm

END OF EXAM ☺