University of Houston
High School Math Contest – 2014
Algebra II Test

1. For the function $f(x) = \dfrac{\sqrt{4 - log_2(x-1)}}{x^2 - 8x + 12}$, what is the sum of all integers that are in the domain of this function?

   A. 127
   B. 150
   C. 144
   D. 152
   E. 153

2. When the polynomial $\left(\dfrac{x^2}{4} + \dfrac{2}{x}\right)^6$ is expanded and like terms are combined, what is the coefficient of the term x^6?

   A. 15/32
   B. 5/8
   C. 15/8
   D. 15/64
   E. 5/16

3. Given $ln(20) = a$, $ln(15) = b$, $ln(6) = c$, express $ln(160)$ in terms of a, b and c.

   A. $2a + b + 2c$
   B. $2a + 3b - 2c$
   C. $3a + b - c$
   D. $4a - 2b + c$
   E. $2a - b + c$

4. Let $i = \sqrt{-1}$. Which of the following is equivalent to $\left(\dfrac{1+i}{\sqrt{2}} \right)^{2014}$?

A. $\dfrac{1-i}{2}$

B. i

C. $-i$

D. 1

E. -1

5. Let a, b, x, y, z be real numbers satisfying: $x < a, \quad y < b, \quad z > a$.

Which of the following must be true?

    I. $x + y < a + b$

    II. $x - y < a - b$

    III. $y + z > x$

A. I, II and III

B. I and III

C. I and II

D. I only

E. III only

6. The variable x varies directly as the cube of y, and y varies inversely as the square root of z. If x equals 1 when z equals 16, what is the value of z when x equals 8?

A. 4

B. 8

C. 10

D. 1/2

E. 2

7. Find the number of integers that satisfy the following inequality:

$$\left| \left| x^2 + 1 \right| - 2|x| \right| < 2$$

A. 3
B. 6
C. 7
D. 4
E. 5

8. Mark and John take turns in flipping a coin in their respective order. The first one who flips heads will win the game. What is the probability that John wins the game in at most 3 turns?

A. $\dfrac{1}{8}$

B. $\dfrac{1}{64}$

C. $\dfrac{21}{64}$

D. $\dfrac{7}{64}$

E. $\dfrac{7}{8}$

9. Which of the following is a factor of the polynomial
$P(x) = x^5 + 2x^4 - x^3 - 2x^2 + 4x + 8$?

A. $x+3$
B. $x-1$
C. $x-2$
D. $x+1$
E. $x+2$

10. Find the value of x satisfying the equation:

$$\frac{1}{2} log_{16}(2) = log_8(2x) - log_4 \sqrt{2\sqrt{2}}$$

    A. 2

    B. $\sqrt{2}$

    C. $2\sqrt{2}$

    D. 4

    E. $4\sqrt{2}$

11. A and B are two non-empty sets. 1/3 of all elements of A are also elements of B, and 1/4 of all elements of B are also elements of A. If A has a total of 64 subsets, how many elements are in $A \cup B = \{x / x$ in A or B$\}$?

    A. 6

    B. 14

    C. 12

    D. 10

    E. 16

12. What is the **sum** of all possible values of x satisfying the equation

$$\left(\frac{2x-16}{x-4}\right)^2 - \left(\frac{2x-16}{x-4}\right) = 6 ?$$

    A. 1

    B. 2

    C. 4

    D. 6

    E. 10

13. A function f is defined on $\mathbb{Z}^+$ (the set of positive integers) by:

$$f(n) = \begin{cases} 2n-1, & \text{if } n \text{ is even} \\ 2n, & \text{if } n \text{ is odd} \end{cases}$$

Given that $f(f(k)) = 2014$, what is the sum of the digits of the number k ?

A. 7
B. 8
C. 9
D. 10
E. 11

14. Let $i = \sqrt{-1}$. If the reciprocal of $2-4i$ is written in the form $a+bi$ where a and b are real numbers, find the product $a \cdot b$.

A. 1/50
B. 1/25
C. 3/25
D. 4/5
E. -1/25

15. A math teacher divides her class of 6 students in to 2-student teams to organize a math tournament in this class. Each student can only be in one team. How many different sets of 3 teams can be made?

A. 90
B. 180
C. 30
D. 15
E. 12

16. If x represents an integer, find the **sum** of all distinct values of x such that:

$$(x-1)^2(x-4)(x+2)<0$$

A. 4
B. 5
C. 6
D. 7
E. 9

17. Define an operation $a*b$ by $a*b=\dfrac{a}{a-b}$. If $x*y=6$, what is $y*x$?

A. 6
B. $\dfrac{1}{6}$
C. 5
D. -5
E. -4

18. How many integers k satisfy the condition $100 \le k \le 200$ and the condition that k has the same remainder when divided by 6 or by 8?

A. 21
B. 25
C. 26
D. 28
E. 30

19. If $f(x+4)=\dfrac{\sqrt[3]{9x-18}}{3}$, then what is $f^{-1}(1)$?

A. 9
B. 5
C. 12
D. 3
E. 14

20. Simplify the expression: $\left(\dfrac{\sqrt{3} \cdot 4^x - \sqrt{3}}{4^x - 2^x} \right) \sqrt{\dfrac{4^{2x+1} + 4^{2x}}{4^x - 4^{x-2}}}$

A. $\sqrt{3}\left(2^{x+1} + 2\right)$

B. $2^{x+1} + 16$

C. $4 \cdot 2^x + 1$

D. $\sqrt{3}\left(4^x - 1\right)$

E. $2^{x+2} + 4$

21. Mary can complete 2/3 of a task in 6 days; Sarah can complete 1/3 of the same task in 5 days. If Sarah and Tom work together, they can complete this task in 6 days. Tom started to work on this task on Monday; Mary and Sarah joined him the next day to complete this task all together. On which day will they complete the task?

A. Wednesday
B. Thursday
C. Friday
D. Saturday
E. Sunday

22. A line passing through the origin also passes through the center of the ellipse $2x^2 + y^2 - 8x + 6y = -13$. What is the equation of this line?

A. $2y = x$
B. $3y = 2x$
C. $2y = 3x$
D. $3y = -2x$
E. $2y = -3x$

23. Sally's teacher asks a simple probability question:

"There are *m* red and *n* black marbles in an urn. Three marbles are drawn in succession, without replacement. Given that the first marble is black, what is the probability that second marble is red and third marble is black?"

When Sally asks what the numbers *m* and *n* are, her teacher gives a hint:
"The sum of the first *m* positive odd integers is 144. The sum of first *n* positive even integers is 210."

What is the answer to this probability question?

A. $\dfrac{7}{25}$

B. $\dfrac{11}{24}$

C. $\dfrac{11}{48}$

D. $\dfrac{13}{50}$

E. $\dfrac{11}{25}$

24. Let $A = \dfrac{1}{1-\dfrac{1}{1-\dfrac{1}{x+1}}}$ and $B = \dfrac{2}{2-\dfrac{2}{2-\dfrac{2}{x+2}}}$.

If $|AB| = 4$, what is the **sum** of all possible values of x?

A. -3

B. 1

C. -3

D. -2

E. 2

25. The line $x + 2y - 8 = 0$ intersects the y-axis at point A and line $4x + 2y - 20 = 0$ intersects the y-axis at point B. Let C be the point where these lines intersect. What is the area of the triangle ABC?

A. 12
B. 6
C. 24
D. 18
E. 20

26. The real numbers c, b, a form an arithmetic sequence and $a \geq b \geq c \geq 0$. Consider the quadratic equation $ax^2 + bx + c = 0$ which has only one root. What is this root?

Note: An arithmetic sequence is a sequence of numbers such that the difference between the consecutive terms is constant. For example, 5, 7, 9 form an arithmetic sequence and the common difference is 2.

A. $-1 - \sqrt{3}$
B. $-1 + \sqrt{3}$
C. $-4 + \sqrt{3}$
D. $2 - \sqrt{3}$
E. $-2 + \sqrt{3}$

27. A cyclist enters a tunnel and when he is 1/5th of the way, he hears a train whistle behind him. The tunnel is not wide enough; so he has to get out. Being a mathematician, he calculates that if he turns back, he will barely get out of the tunnel on time. If he continues biking in the same direction, he will still barely make it out of the tunnel. We know that this cyclist travels 2 miles in 10 minutes. What is the speed of the train?

A. 60 mph
B. 20 mph
C. 36 mph
D. 24 mph
E. 48 mph

28. James, Ken and Melanie started working on a math booklet on the same day. Each worked at a constant rate. James solved 12 problems per day and finished the booklet 3 days after Ken did. Melanie solved 3 more problems than Ken did every day, and finished 2 days before Ken. How many days did it take for Melanie to finish this booklet?

A. 18
B. 15
C. 12
D. 10
E. 9

29. If (x, y) is a point with integer coordinates that satisfies the following inequalities, how many **distinct** possible values does $x + y$ have?

$$\begin{cases} x^2 + (y-1)^2 \geq 1 \\ y < -2|x| + 6 \\ y > 0 \end{cases}$$

A. 5
B. 6
C. 7
D. 8
E. 10

30. The average of 3 positive integers is 5 and the average of their reciprocals is $\dfrac{17}{72}$.

If the product of these numbers is 96, what is the sum of the two smallest numbers on this list?

A. 5
B. 6
C. 7
D. 8
E. 9