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Calculator Exam – 2014

Directions: Write your answers on the answer sheet. **DO NOT detach the answer sheet from your exam.** Answers can be given as integers, fractions or in decimal form. The answers which are given in decimal form should be recorded so that they are **accurate to at least 4 places *after* the decimal**.

DO NOT ROUND YOUR ANSWERS until *after* the 4th decimal place!! For example, suppose a question requests the value of sin(2). Your calculator will tell you that sin(2) is 0.9092974268 (assuming your calculator is in radian mode). Examples of correct responses include 0.9092, 0.90929, 0.90923 and 0.90929116. **The response 0.9093 is not correct.**

It does not matter what appears *after* the 4th decimal place, provided the values up to and including the fourth decimal place are correct.

Note: Problems 11 and 17 reward students for correct answers to previous problems.

Good Luck!!

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1. Give the integer multiple of 7 which is closest to $\frac{3{,}127-2(364-19{,}321)^2}{26}-6$.

2. $f(x)=x^2-3x+2$ and $g(x)=x+\frac{1}{f(x)}$ have two points of intersection. Give the sum of the x and y coordinates of these two points.

3. Give the distance from the point $(-3,2)$ to the vertex of the parabola $y=x^2-\frac{3}{2}x-7$.

4. Give the sum of the first 100 numbers in the sequence 2.2, 4.4, 6.6, 8.8, 10.10, 12.12, 14.14, 16.16, 18.18, ….

5. Give the largest root of the equation $\frac{1}{10}x^5-4x^4+3x^3-2x^2+x+20=0$.

6. Give the value of x associated with the solution to $\begin{cases}13x-53y=12\\ 37x+41y=1\end{cases}$.

7. The function $f(x)=x^3+4x+12$ is invertible. Give $f^{-1}(7.1)$.

8. Give the smallest integer value of the function $f(x)=\frac{1}{61}x^4-x^3-2x+7$.

9. Give the sum of the prime factors (including repetition) for 224551579230.

10. Let $f(x)=\frac{2x+1}{3x+4}$. Give the 23rd value in the sequence $f(1), f(f(1)), f(f(f(1))),\ldots$.

11. Give the average of the correct answers to problems 1 through 10.

12. A loan was given to a woman on June 1, 1961 for \$1,000. The terms of the loan caused the amount owed to increase by 0.5% on the first of each month, starting with July 1, 1961. Fractions of cents were rounded up each time interest was charged on the loan, so that a balance of \$1,037.6312 becomes \$1,037.64. The woman did not make any payments on the loan until 1983, when she started (in January and each subsequent month) paying the smaller of \$200 or the balance due. Her payments continued in this manner, and she eventually pays off the loan. The final payment made by the woman was x dollars and y cents. Give the value of x.

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13. Give the sum of the first 205 positive integers which are not integer multiples of 2, 3 or 7.

14. Find the *x*-coordinate of the fixed point of the function $f(x) = \dfrac{2x+3}{12x+17}$ between 0 and 1.

15. The function in problem 14 has more than one fixed point. Give the average of the *x*-coordinates of the fixed points.

16. Give the smallest *x*-intercept of the parabola that passes through the points (-1,3), (1,6) and (5,-12).

17. Give the average of the correct answers to problems 12 through 16.

18. Consider the sequence of numbers $a_1, a_2, a_3, \ldots$ where $a_i = \dfrac{(-1)^{i+1}}{i}$ for $i = 1,2,\ldots$. Suppose a person moves in order through this sequence, only considering the positive terms, until the first time that the sum of these terms is larger than 1.7. Call this sum S_1, and omit the numbers that were used to create a new sequence. Note that the new sequence should start with $-\dfrac{1}{2}$. Then move in order through the terms of the new sequence, only considering the negative terms, until the first time that the sum of S_1 and these negative terms is less than 1.7. Can this new amount S_2, and omit the numbers that were used from the sequence to create a new sequence. Repeat the steps above on the new sequence creating $S_3, S_4, \ldots$ until the first time that $\left|1.7 - S_k\right| < 10^{-3}$. Give the value of S_k.

19. A point *P* lies in the interior of the first quadrant, on the circle of radius 1 centered at the origin. A line segment *L* is sketched from the point (1,0) to *P*, and *Q* is the midpoint of this line segment. The line through (0,0) and *Q* intersects the circle of radius 2 centered at (0,0) at a point *R* in the first quadrant, and the acute angle between this line and the line through points *R* and *P* has a measure of 14 degrees. Give the *x*-coordinate of *P*.

20. Suppose *B* and *C* are positive integers larger than 1. Three sequences are created as follows. First, we set $A_0 = 0$, $B_0 = B$ and $C_0 = C$. Then, for $i = 1, 2, \ldots$ we let $B_i = 2B_{i-1}$, C_i be the integer part of $\dfrac{C_{i-1}}{2}$, and $A_i = \begin{cases} 1, & \text{if } C_i \text{ is odd} \\ 0, & \text{otherwise} \end{cases}$. If $B = 2{,}731$ and $C = 7{,}342$, then given the value of $A_1B_1 + A_2B_2 + A_3B_3 + \cdots$.

21. Three integers are chosen at random from the interval [1,10]. Examples of choices could be {1,1,7} or {2,8,10}. Give the probability that the sum of the squares of these values is at least 94.

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22. Consider the points (0,0), (1,0), (2,0), (0,1), (1,1), (2,1), (0,2), (1,2) and (2,2). Give the sum of the distances between all possible pairs of these points.

23. There are 50 receivers that are numbered 1 through 50, and there are 3 transmitters, labeled A, B and C. All of the odd numbered receivers can receive a signal from transmitter A, receivers 1 through 25 can receive a signal from transmitter B, and all of the receivers that are integer multiples of 3 can receive a signal from transmitter C. In addition, all of the receivers are capable of communicating with one another. Each receiver can have a state of either 0 or 1, and when it receives a signal from a transmitter, its state changes according to the rules of the transmission signal. The signals from transmitter A and transmitter C cause the state of a receiver to change. The signal from transmitter B causes receiver k to exchange its state with receiver $51-k$. Initially, the receivers all share the common state of 0. Then signals start coming from the transmitters in the order A, then B, and then C, with this triple of signals sent 100 times. Give the sum of the final states of the receivers.

24. Referring to problem 23, how many times does receiver 37 reach the state 1?

25. A triangle is created with vertices $A=(-1,3)$, $B=(2,5)$, and a point C on the parabola $y=x^2$ with *x* coordinate strictly between -1 and 2. Give the *x*-coordinate for C so that $\angle ACB$ is a right angle.

26. **Tie Breaker –** Give the average of the correct answers to problems 1 through 25.