

1. For how many distinct real coefficients, a , will the system of equations

$$y = ax^2 + 2$$

$$x = ay^2 + 2$$

admit a solution with $y = 1$?

- A. 0
B. 1
C. 2
D. 3
E. 4
2. If $f(e^x) = \sqrt{x}$ for $x \geq 1$ then $f^{-1}(x)$ is
- A. $(\ln x)^2$
B. e^{x^2}
C. $2 \ln x$
D. $e^{\sqrt{x}}$
E. $\sqrt{\ln x}$
3. The decimal $2.0259\overline{259} = 2.0259259259259259\dots$ is equivalent to which of the following?
- A. $\frac{20237}{9990}$
B. $\frac{547}{270}$
C. $\frac{20239}{9999}$
D. $\frac{747}{370}$
E. $\frac{737}{380}$

4. $\cos\left(2\arctan\left(\frac{4}{3}\right)\right) =$

- A. $\frac{1}{8}$
- B. $-\frac{3}{15}$
- C. $-\frac{\sqrt{3}}{2}$
- D. $-\frac{7}{25}$
- E. None of these

5. Determine the sum of the series

$$1 + \frac{1}{1+x^2} + \frac{1}{(1+x^2)^2} + \frac{1}{(1+x^2)^3} + \dots$$

where $x \neq 0$.

- A. $x^2 + 1$
 - B. $\frac{x^2}{1+x^2}$
 - C. x^2
 - D. $1 + \frac{x^2}{1+x^2}$
 - E. $1 + \frac{1}{x^2}$
6. If $y = e^{-5x} + 1$, which of the following lines is an asymptote to the graph of f ?
- A. $y = 1$
 - B. $x = 0$
 - C. $y = 0$
 - D. $y = \ln(-5x + 1)$
 - E. $-5x = \frac{1}{y}$

7. The expression $5\log_{15}(15x) - \log_{15} x^5$ is equivalent to which of the following?

- A. 5
- B. $105x^2$
- C. 105
- D. $5\log_{15} \frac{15}{x}$
- E. $5\log_{15}(4x)$

8. How many solutions are there of the equation

$$\cos^2 x = \cos x$$

where x satisfies $0 \leq x \leq 2\pi$?

- A. No solutions
- B. 1
- C. 2
- D. 3
- E. More than 3 solutions

9. Given $f(x) = \frac{x}{x-1}$, find an expression for $f(3x)$ in terms of $f(x)$

- A. $\frac{3f(x)}{3f(x)-1}$
- B. $\frac{3f(x)}{3f(x)-3}$
- C. $3f(x)-1$
- D. $\frac{3f(x)}{2f(x)+1}$
- E. $\frac{3f(x)}{2f(x)-1}$

10. The fundamental period of the function defined by $f(x) = 2 - 3\cos^2\left(\frac{\pi x}{3}\right)$ is
- A. 2π
 - B. $\frac{\pi}{3}$
 - C. 3
 - D. 6
 - E. 6π
11. If $af(x_1) + bf(x_2) = f(ax_1 + bx_2)$ for all real numbers x_1, x_2, a, b , which of the following could define the function f ?
- A. $f(x) = ax + b$
 - B. $f(x) = \ln x$
 - C. $f(x) = \frac{1}{x}$
 - D. $f(x) = 5x$
 - E. $f(x) = x^4$
12. Determine the value of $\tan(2n)$ when $\pi < n < 2\pi$ and $\cos(n) = -\frac{1}{2}$?
- A. $\sqrt{3}$
 - B. $\frac{\sqrt{2}}{2}$
 - C. $\frac{1}{2}$
 - D. -1
 - E. None of these

13. Convert the following equation from Polar coordinates to Cartesian coordinates:

$$r = \frac{4}{1 + \sin \theta}$$

- A. $y^2 = 4x$
- B. $x^2 - 4 = y$
- C. $x^2 = -8(y - 2)$
- D. $y^2 = 8x$
- E. $y = 8x + 4$

14. If the domain of the function $y = f(x)$ is $[0,1]$, then what is the domain of the function

$$f\left(x + \frac{1}{4}\right) + f\left(x - \frac{1}{4}\right) ?$$

- A. $[0,1]$
- B. $(0,1)$
- C. $\left[0, \frac{1}{2}\right]$
- D. $\left[\frac{1}{4}, \frac{3}{4}\right]$
- E. $\left(\frac{1}{4}, \frac{3}{4}\right)$

15. If $g\left(\frac{3+2x}{4}\right) = 1-x$, for all real numbers x , then $g\left(\frac{7z-8}{4}\right) =$.

A. $-\frac{13+7z}{4}$

B. $\frac{13}{2} - \frac{7z}{2}$

C. $\frac{7-13z}{2}$

D. $\frac{7+13z}{2}$

E. $7z+13$

16. Solve for x :

$$\left(\frac{1}{3}\right)^{2x-3} = 3^{5x}$$

A. no solution

B. $x = \frac{1}{2}$

C. $x = -1$

D. $x = -2$

E. $x = \frac{3}{7}$

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17. Given that

$$\ln 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \dots$$

It can be determined that

$$-2 + 1 - \frac{2}{3} + \frac{2}{4} - \frac{2}{5} + \frac{2}{6} - \dots$$

is equal to

- A. $-2\ln 2$
- B. $-\ln 2$
- C. $2\ln 2$
- D. $\ln 3$
- E. $-3\ln 3$

18. If $f(x+2) - f(x) = 12$, find $f^{-1}(x+2) - f^{-1}(x)$.

- A. 6
- B. $\frac{1}{6}$
- C. 3
- D. 2
- E. $\frac{1}{3}$

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19. Simplify: $\sin x \cos\left(\frac{\pi}{2} - x\right) \csc x \cot x - \sin\left(\frac{\pi}{2} - x\right)$

A. $\frac{\sin^2 x}{\cos x} - \cos x$

B. $\sin x - \cos x$

C. 1

D. 0

E. None of these

20. The angle of rotation which will ensure the absence of the xy term when the equation $2x^2 + \sqrt{3}xy + 3y^2 - 5x + 2y = 0$ is transformed is

A. 30°

B. 45°

C. 60°

D. 120°

E. None of these

21. A cubic polynomial, $p(x)$, with integer coefficients such that $2-2i$ and -3 are solutions of the equation $p(x)=7$ is

- A. $x^3 + 3x^2 - x - 11$
- B. $x^3 - 2x^2 + 4x + 25$
- C. $x^3 + 4x^2 - 6x + 24$
- D. $x^3 - x^2 - 4x + 31$
- E. $x^3 - x^2 - 4x + 17$

22. If $(\tan \theta)(\csc \theta - \sin \theta) = 0.3$, then $\sin \theta$ equals

- A. 0.3
- B. $\frac{1}{0.3}$
- C. $\sqrt{0.91}$
- D. 0.7
- E. cannot be determined from the given information

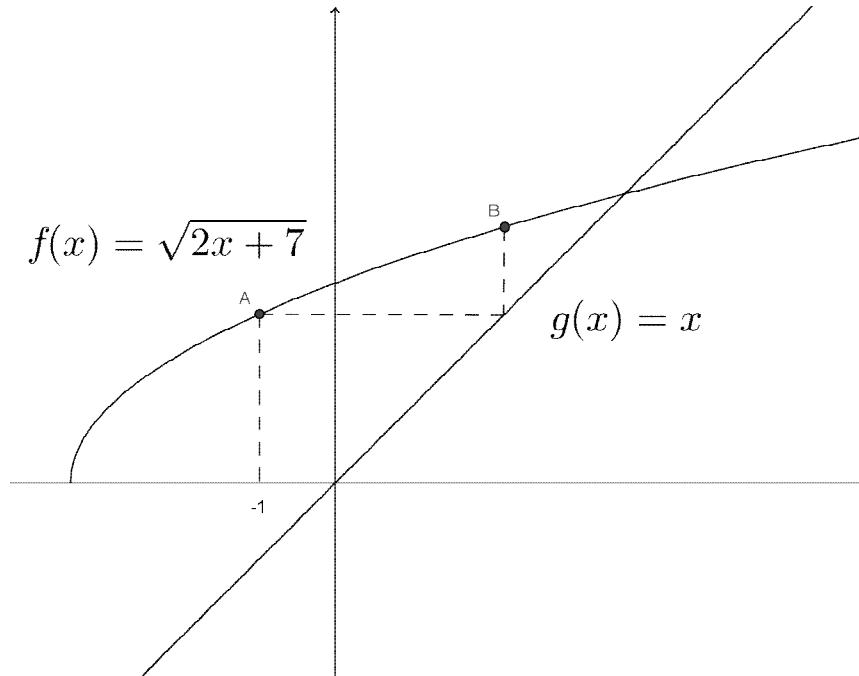
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23. Which of the following statements is true of the graph of the equation $y = \frac{4x^2}{x^2 - 16}$?

- I. It has x – intercept at $-\frac{1}{16}$
 - II. It has vertical asymptotes at $x = \pm 4$
 - III. It has horizontal asymptote at $y = 4$
 - IV. It is symmetric with respect to the y – axis
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- A. I, II, and III only
 - B. II, III, and IV only
 - C. I and IV only
 - D. I, II, and IV only
 - E. All of the statements are true

24. In the figure below, each dashed line is parallel to one of the coordinate axes.



What is the y - coordinate of the point B ?

- A. $\sqrt{5}$
- B. $2\sqrt{5} + 7$
- C. $\sqrt{2\sqrt{5} + 7}$
- D. $\sqrt{\sqrt{5} + 7}$
- E. None of these

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25. Suppose that the domains of f and f^{-1} are both $(-\infty, \infty)$ and that $f^{-1}(10) = 7$. If $f(2t+8) - 6 = 4$, then $t =$

A. -1

B. $-\frac{7}{2}$

C. $\frac{5}{2}$

D. $-\frac{1}{2}$

E. Cannot be determined from the information given.

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Tie – Breaker

A rectangle has dimensions a units by b units with $a < b$. A diagonal divides the rectangle into two triangles. A square, with sides parallel to those of the rectangle, is inscribed in each triangle. Find the distance between the vertices of the squares that lie in the interior of the rectangle (in terms of a and b).