Name: ________________________________ **School:** ________________________________

University of Houston High School Mathematics Contest
Geometry Exam – Spring 2015

Note that diagrams may not be drawn to scale.

1. A pool has a 4 foot wide sidewalk around it. If the pool is 28 feet long and 15 feet wide, find the outer perimeter of the sidewalk.

(A) 110 ft (B) 118 ft (C) 828 ft (D) 102 ft. (E) 94 ft.

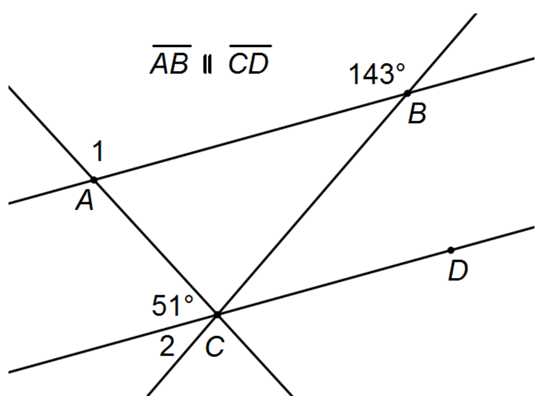
2. A tree trunk had a circumference of 16π inches in 1990. Scientists have found that the cross-sectional area of the trunk of this type of tree increases by 50% every ten years. Assuming this growth rate, what will the tree's circumference be in the year 2030?

(A) 36π in. (B) $18\sqrt{6}\,\pi$ (C) 324π (D) 81π (E) $12\sqrt{6}\,\pi$

3. $\Delta ABC \cong \Delta DEF$, and $\Delta EDF \cong \Delta GHI$. If the ratio of $\angle A : \angle B : \angle C$ is 7:3:5, find the measure of $\angle G$.

(A) 60^o (B) 84^o (C) 12^o (D) 45^o (E) 36^o

4. Find the sum of the measures of angles 1 and 2 in the figure below.



(A) 176^o

(B) 194^o

(C) 221^o

(D) 180^o

(E) 166^o

5. Jackie is making a necklace using a gold chain and 8 identical spherical beads. Each bead has a small hole drilled straight through the center of the sphere in order for the chain to pass through. If the volume of all beads combined is 288π mm^3 prior to the drilling process, and the gold chain is 60 cm in length, how much of the gold chain is showing once the beads are threaded onto the chain?

(A) 12 cm (B) 55.2 cm (C) 36 cm (D) 57.6 cm (E) 50.4 cm

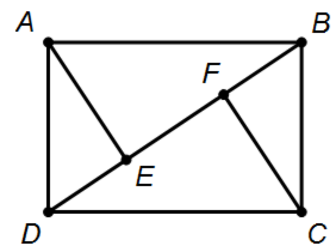
Name: ________________________________ **School:** ________________________________

6. Fiona wants to estimate the height of a light pole. The distance from the ground to Fiona's eyes is 57 inches. She stands 24 feet from the base of the pole, and sights the top of the pole at an angle of elevation of 53 degrees. Find the height of the light pole, in feet.

(A) $\frac{19}{4} + 24 \cdot \sin\left(53^\circ\right)$ (B) $24 \cdot \tan\left(53^\circ\right) + 4.9$ (C) $\frac{24}{\tan\left(53^\circ\right)} + \frac{19}{4}$

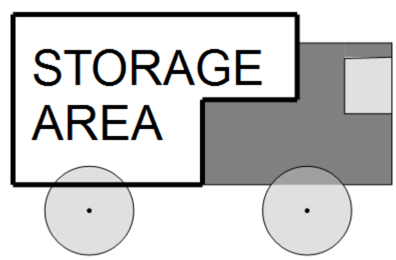
(D) $\cos^{-1}\left(\frac{53}{57}\right) + 24$ (E) $\frac{19}{4} + 24 \cdot \tan\left(53^\circ\right)$

7. In the diagram of rectangle $ABCD$ shown at the right, $\overline{AE}$ and $\overline{FC}$ are both perpendicular to $\overline{DB}$. If $BC = 15$ and $AE = 12$, find the length of $\overline{EF}$.



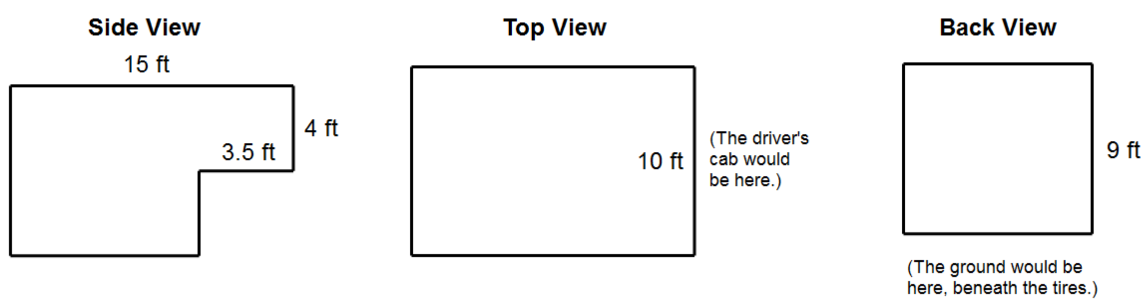
(A) 20 (B) 9 (C) 16 (D) 7 (E) 12

8. A sketch of a moving truck is shown at the right.



The storage area of the truck is the white portion behind the driver's cab, and it contains a "shelf" (shown on the upper right part of the storage area) where smaller boxes can be stored.

Three views are shown below to display the dimensions of the storage area. The top view is the view from above the truck (if you were looking down at the storage area from the sky), and the back view is the view you would see if you were driving behind the moving truck. Note that the driver's cab and wheels are not shown any of the following diagrams.



Find the volume of the storage area, in cubic feet. (Disregard any volume lost from the wheels or from the thickness of the storage area walls.)

(A) 1,350 (B) 1,117.5 (C) 1,210 (D) 1,175 (E) 757.5

9. In spherical geometry, all points are on the surface of a sphere. Assuming that the earth is spherical, answer the following:

Jack is on an expedition and is standing at the North Pole. Jack walks south for 800 feet, then walks west for 400 feet, then walks south for 200 feet, then east for 1300 feet, then walks north for 1000 feet. How far is Jack from where he started, in feet?

(A) $450\sqrt{3}$ (B) $900\sqrt{2}$ (C) 900 (D) 0 (E) 1,700

10. A right triangle is inscribed in a circle. If the circle has an area of 36π cm^2, and one leg of the triangle measures 10 cm, find the area of the triangle, in square centimeters.

(A) $10\sqrt{11}$ (B) 24 (C) 60 (D) $10\sqrt{61}$ (E) $10\sqrt{34}$

11. A dilation is performed on ΔABC using a scale factor of 80%, resulting in the image $\Delta A'B'C'$. A dilation is then performed on $\Delta A'B'C'$, resulting in the image $\Delta A''B''C''$. If $\Delta ABC \cong \Delta A''B''C''$, what scale factor was used in the second dilation?

(A) 180% (B) 125% (C) 120% (D) 25% (E) 20%

12. Find the equation of the line which is tangent to the circle $x^2 + y^2 + 4x - 10y + 16 = 0$ at the point $(1, 7)$.

(A) $3x + 2y = 4$ (B) $2x - 3y = -19$ (C) $x - 12y = -83$
(D) $3x + 2y = 17$ (E) $3x - 2y = -11$

13. A plane is perpendicular to the base of a given solid and passes through the center of its base. A rectangular cross-section is formed. The solid could be a ____________.

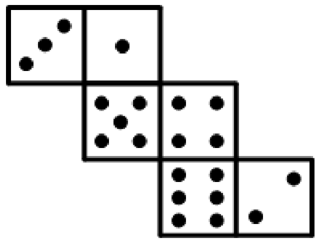
I. right regular triangular prism
II. right rectangular pyramid
III. right circular cylinder
IV. right regular pentagonal prism
V. truncated square pyramid

(A) II or V only
(B) I, III or IV only
(C) I or IV only
(D) III or IV only
(E) I, II, IV or V only

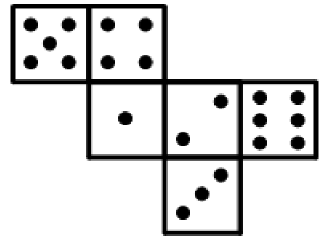
Name: _________________________________ **School:** _________________________________

14. On a standard die, the number of dots on any two opposite faces always adds up to seven. Five nets are shown below. Which net does not form a standard die?

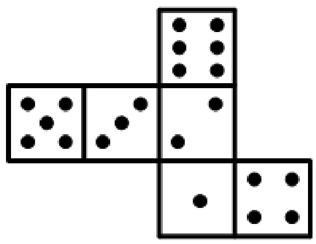
(A)



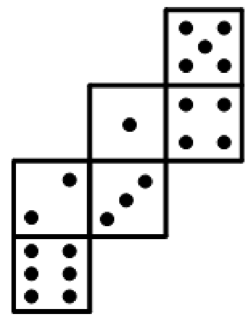
(B)



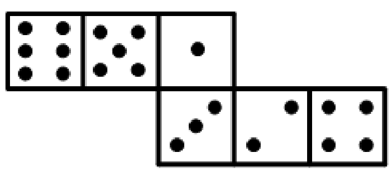
(C)



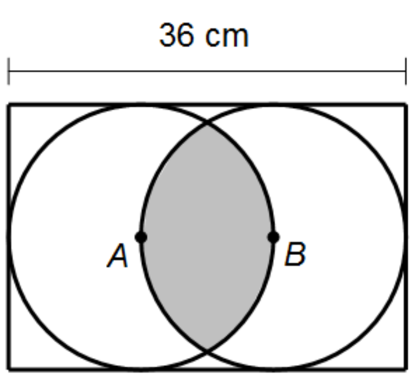
(D)



(E)



15. Two congruent circles are drawn below, where each circle passes through the center of the other circle. The two circles fit inside a rectangle such that each circle is tangent to three sides of the rectangle. The length of the rectangle is 36 cm. If a dart is thrown and has an equal probability of landing anywhere inside the rectangle, what is the probability that the dart lands within the shaded intersection of the two circles?



(A) $\dfrac{4\pi - 3\sqrt{3}}{12\pi}$

(B) $\dfrac{4\pi - 3\sqrt{3}}{72}$

(C) $\dfrac{\pi}{9}$

(D) $\dfrac{2\pi - 3\sqrt{3}}{18}$

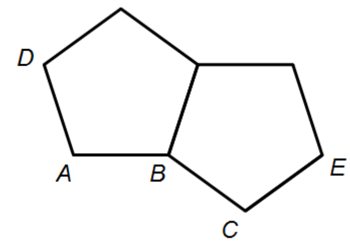
(E) $\dfrac{4\pi - 3\sqrt{3}}{36}$

16. Pyramid A has n faces. Pyramid B has $(n+2)$ faces and $(4n-12)$ edges. Find the sum of the number of edges in Pyramids A and B.

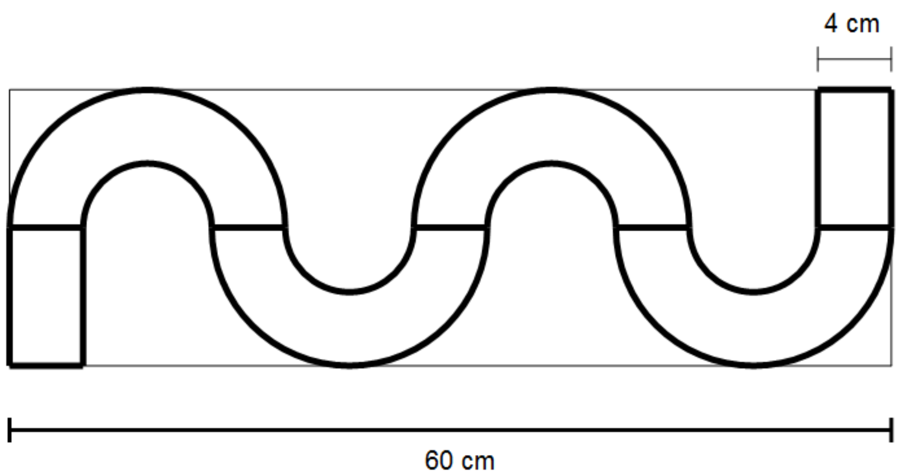
(A) 76 (B) 30 (C) 90 (D) 28 (E) 32

17. Jeanne is playing with congruent regular pentagonal shapes which snap together at their edges. She snaps two pentagons together, forming the design shown below. Next, Jeanne takes out another pentagon and snaps it against side $\overline{DA}$. She then snaps another pentagon against side $\overline{CE}$, and continues to snap pentagons together in this fashion until they meet together to form a design which resembles a flower. What type of polygon (containing sides $\overline{AB}$ and $\overline{BC}$) is formed in the center of this flower-like design?

(A) A regular decagon
(B) A regular hexagon
(C) A regular pentagon
(D) A regular octagon
(E) A regular dodecagon



18. Declan is playing with his train tracks. He takes out four identical curved pieces and two identical rectangular pieces, and arranges them on a rectangular mat, as shown below. The mat is 60 cm long, the outer and inner curves are semicircles, and the track is 4 cm wide at every point. Find the area of Declan's racetrack, in square centimeters.

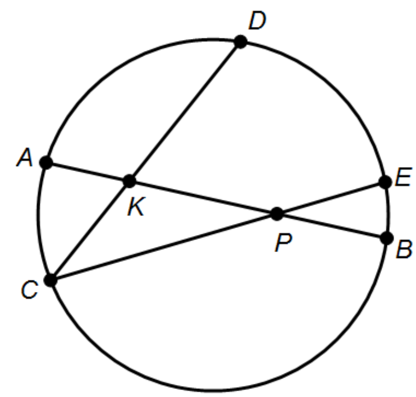


(A) $88\pi+60$ (B) $112\pi+72$ (C) $130\pi+72$

(D) $\dfrac{225\pi}{2}+60$ (E) $162\pi+72$

Name: ______________________________ **School:** ______________________________

19. In the circle at the right:
Chords $\overline{AB}$ and $\overline{CD}$ intersect at point *K*.
Chords $\overline{AB}$ and $\overline{CE}$ intersect at point *P*.
$AK = 4$, $CK = 3$, $CP = 10$, $CE = 16$, and $PB = 5$.
Find the length of $\overline{KD}$.



(A) $\frac{68}{3}$ (B) 20 (C) $\frac{12}{13}$
(D) $\frac{52}{3}$ (E) $\frac{152}{9}$

20. The volume, V, of a right square pyramid is divisible by 12, 21, and 9. The base edges have integer lengths. If $2000 < V < 3000$, and the digits of V are all distinct, find the smallest possible height of the pyramid.

(A) 56 (B) 42 (C) 168 (D) 14 (E) 672

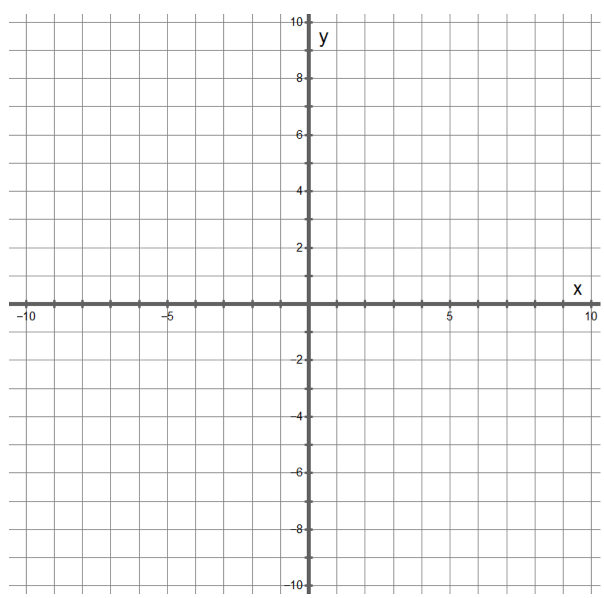
21. ABCDEFGH is a convex regular octagon with $CD = 4\sqrt{2}$. Find the length of $\overline{AE}$.

(A) $4\sqrt{5+4\sqrt{2}}$ (B) $4\sqrt{6+4\sqrt{2}}$ (C) $8\sqrt{2}$

(D) $8+4\sqrt{2}$ (E) $8\sqrt{2+\sqrt{2}}$

22. ΔABC has vertices $A(-6,-1)$, $B(-1,-1)$, and $C(-5,-4)$. If ΔABC is reflected over the line $y = -\frac{1}{2}x + 1$ to form $\Delta A'B'C'$, find the sum $CB' + AC'$.

(A) $5+\sqrt{10}$
(B) $\sqrt{74}+\sqrt{178}$
(C) $7\sqrt{2}+\sqrt{85}$
(D) 29
(E) $\sqrt{85}+\sqrt{130}$



Name: ______________________________ School: ______________________________

23. A right rectangular pyramid with base dimensions of 12 cm x 8 cm is sliced parallel to the base so that the volume of the truncated pyramid is $\frac{7}{8}$ of the volume of the original pyramid. The height of the truncated pyramid is 5 cm. Find the total surface area of the truncated pyramid, in square centimeters.

(A) $15\sqrt{29}+120$

(B) $4\sqrt{34}+6\sqrt{29}+24$

(C) $120+12\sqrt{34}+18\sqrt{29}$

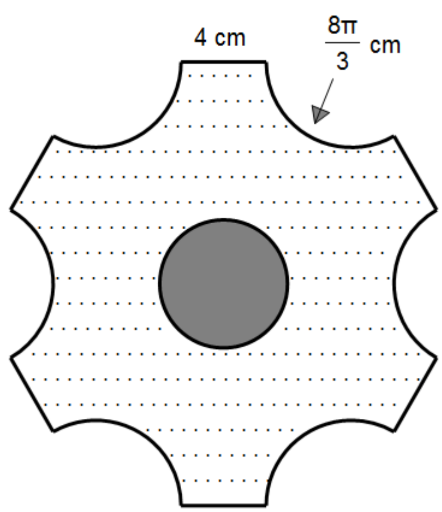
(D) 280

(E) $24\sqrt{29}+16\sqrt{34}+96$

24. The figure at the right is formed from a regular hexagon which has a sector carved out at each vertex, and a circle cut out from its center. Each of the six flat edges measures 4 cm. The dark circular cutout in the center has a diameter of 6 cm.

Each of the six arcs measures $\frac{8\pi}{3}$ cm.

Find the area of the figure, in square centimeters.



(A) $432\sqrt{3}-25\pi$

(B) $600\sqrt{3}-137\pi$

(C) $216\sqrt{3}-68\pi$

(D) $216\sqrt{3}-41\pi$

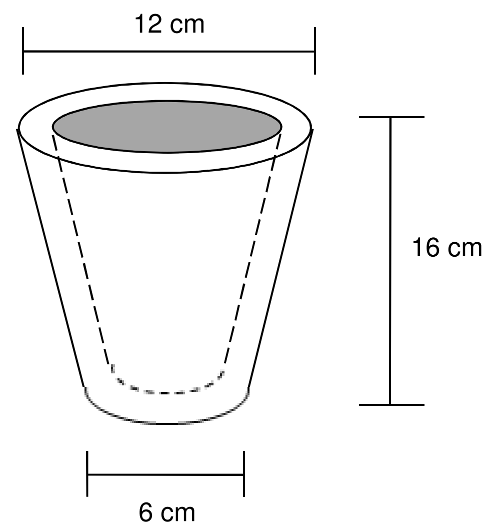
(E) $600\sqrt{3}-164\pi$

Name: ________________________________ **School:** ________________________________

25. Find the slope of the line that passes through the origin, lies in the first and third quadrants, and forms a 75° angle with the x-axis.

(A) $\dfrac{2\sqrt{6}}{3}$ (B) $\sqrt{6}+\sqrt{2}$ (C) $2+\sqrt{3}$ (D) $\dfrac{6}{5}$ (E) 5

26. A metal cup is snugly encased inside a foam insulating sleeve which covers the sides and bottom of the cup. The sleeve extends beyond the radius of the cup by 1 cm at every point on the side of the cup, and also adds 1 cm of insulation between the base of the sleeve and the bottom base of the cup. The insulated sleeve is 16 cm tall, has an outer top diameter of 12 cm and a bottom diameter of 6 cm. Find the diameter of the bottom of the metal cup.



(A) $\dfrac{35}{8}$ cm

(B) 4 cm

(C) $\dfrac{45}{8}$ cm

(D) 5 cm

(E) $\dfrac{22}{5}$ cm

END OF EXAM ☺