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Calculator Exam – UH Math Contest 2016

Directions: Write your answers on the answer sheet. **DO NOT detach the answer sheet from your exam.** Answers can be given as integers, fractions or in decimal form. The answers which are given in decimal form must be recorded so that they are **accurate to at least 4 places *after* the decimal**.

DO NOT ROUND YOUR ANSWERS until *after* the 4th decimal place!! For example, suppose a question requests the value of sin(2). Your calculator will tell you that sin(2) is 0.9092974268 (assuming your calculator is in radian mode). Examples of correct responses include 0.9092, 0.90929, 0.90923 and 0.90929116. **The response 0.9093 is not correct.**

It does not matter what appears *after* the 4th decimal place, provided the values up to and including the fourth decimal place are correct.

Note: $\cos(23)$ is the cosine of 23 radians. $\cos(23^o)$ is the cosine of 23 degrees. These are different values. If a number does not have a "degree" indicator, then it is assumed to be radians.

Good Luck!!

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1. A point (x, y) is called an integer point if both x and y are integers. Give the number of integer points that lie strictly above the graph of $y = x^2$, and strictly below the graph of $y = 35 + x$.

2. Give the largest solution to $x^2 - x = 35$.

3. Find the value of a so that the system below has a solution.
$$12.31x + 7.24y = -13.63$$
$$-13.16x + 21.22y = 32.67$$
$$x + y = a$$

4. Give the average of the numbers
2.32, 9.613, 11.46, –13.27, 12.72, 21.36, –4.57, 18.61.

5. Give the sum of the reciprocals of the numbers
2.32, 9.613, 11.46, –13.27, 12.72, 21.36, –4.57, 18.61.

6. Give the smallest positive *x*-intercept for the function $f(x) = -3x + 7\sin(x) + \dfrac{3}{x+1}$.

7. Give the distance between the points $(-2, f(-2))$ and $(1, f(1))$ for the function $f(x) = -3x + 7\sin(x) + \dfrac{3}{x+1}$.

8. Give the *y*-intercept of the line that passes through the point $(-2.1, 3.2)$ and is perpendicular to the line that passes through the points $(3.2, 7.1)$ and $(-4.3, 13.8)$.

9. Give the obtuse angle of intersection (in degrees) of the lines $2x + y = 4$ and $3x - 2y = 1$.

10. Give the area of the triangle with vertices (2,–6), (3,13) and (–1,8).

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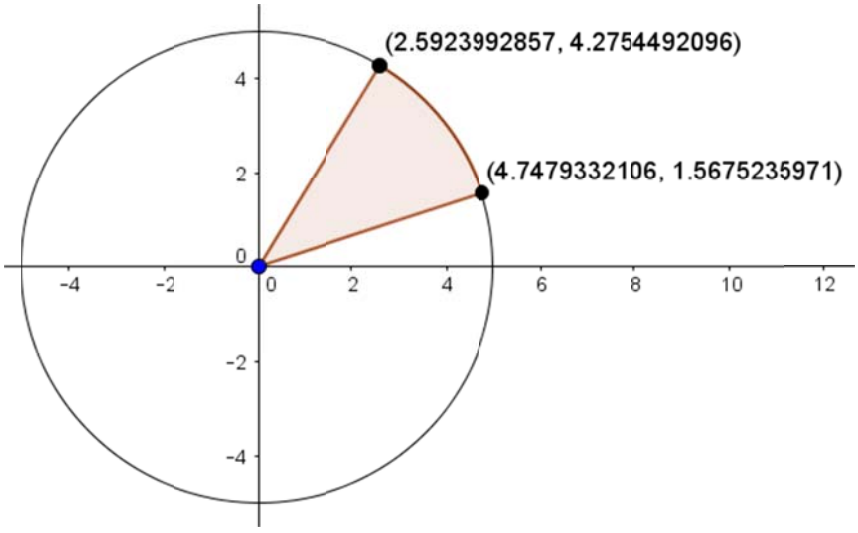
11. The circle of radius 5 centered at the origin is shown below, along with a sector of this circle. Give the area of the sector.



12. Suppose $a_0 = 16{,}383$, and define $a_1, a_2, a_3, \ldots$ using the formula

$$a_n = \begin{cases} \dfrac{a_{n-1}}{2}, & \text{if } a_{n-1} \text{ is even} \\ 3a_{n-1} + 1, & \text{otherwise} \end{cases}$$

Give a_{1000}.

13. A number M is written in base 5 as 430122. Write M in base 10.

14. $f(x) = \frac{3}{2}x^3 + \frac{3}{4}x\sin(x) - 2x + 3$. Give $f(2.14)$.

15. The function $f(x) = ax^2 + bx + c$ has a graph that passes through the points $(-1.2, 2.1)$, $(2.3, 7.2)$ and $(5.2, -3.6)$. Give the value of b.

16. For each positive integer n, define

$$a_n = \begin{cases} \dfrac{1}{n}, & n \text{ is even} \\ \dfrac{2}{3n-1}, & n \text{ is odd} \end{cases}$$

Give the value of $a_1 + a_2 + a_3 + \cdots + a_{1000}$.

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17. We say that a ten number binary sequence is a symmetric binary sequence if and only if it has the form

$$a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}$$

with each a_i either 0 or 1, and

$$a_1 = a_{10}, a_2 = a_9, a_3 = a_8, a_4 = a_7, a_5 = a_6$$

We associated the value

$$\sum_{i=1}^{10} a_i 2^{i-1}$$

with this ten number symmetric binary sequence. Give the sum of the values associated with all ten number symmetric binary sequences.

18. The number π can be approximated by using the formula

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots$$

Give the approximation resulting from this formula if only the first 100 terms are used on the right hand side.

19. The parametric plot of $(\sin(t) + \cos(5t), \cos(t) - \cos(4t))$ divides the *xy* plane into several distinct regions. Give the number of regions.

20. Give the value of $x + y + z$ for which

$$\begin{pmatrix} 2.1x - 3.2y + 4.3z = 6.1 \\ -3.4x + 5.2y - 6.2z = 7.4 \\ 7.1x + 11.2y + 18.3z = 4.5 \end{pmatrix}$$

21. Set $a_1 = 1$, $b_1 = 4$, $c_1 = 0$, $a_2 = 5$, $b_2 = 3$ and $c_2 = 1$. Let $(x, y) = (a_3, b_3)$ be the solution to the system

$$\begin{pmatrix} a_1 x + b_1 y = c_1 \\ a_2 x + b_2 y = c_2 \end{pmatrix}$$

and set $c_3 = \cos(c_2)$. For $n \geq 3$, let $(x, y) = (a_{n+1}, b_{n+1})$ be the solution to the system

$$\begin{pmatrix} a_{n-1} x + b_{n-1} y = c_{n-1} \\ a_n x + b_n y = c_n \end{pmatrix}$$

and set $c_{n+1} = \cos(c_n)$. Give the value of $c_n + c_{n+1} + c_{n+2}$ for $n \geq 30$.

22. The graph of $3x + 2y = 1$ intersects the graph of $x^4 + y^4 = 1$ in more than one point. Give the average of all of the *x* and *y* coordinates of points of intersection.

23. Samantha gives $1000 to an investor who guarantees her that at the end of each year, 4% of the balance at the start of the year will be added to her account, minus an administrative fee of $7. How many years must pass before Samantha's initial investment triples?

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24. A circular dial has numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, and it sits in the center of a circular clock. On the first day of 2016, the dial is positioned so that the numbers on the dial line up with the numbers on the clock. On the second day of 2016, the dial is turned 2 positions in the clockwise direction, so that the number 1 on the dial is lined up with 3 on the clock, the number 2 on the dial is lined up with 4 on the clock, etc. On the third day of 2016, the dial is turned 3 positions clockwise from its position on day 2. On the fourth day of 2016, the dial is turned 4 positions clockwise from its position on day 3. This pattern continues through day 366. How many days do the numbers on the dial align with the numbers on the clock, during the 2016 year?

25. Each date during the 2016 year can be used to form a single number. For example, January 9 can be written as 109, March 27 can be written as 327, October 3 can be written as 1003, and December 31 can be written as 1231. How many of these numbers are not divisible by any of 2, 3, 5 or 7?