1. \( \lim_{x \to 0} \frac{\tan^3 2x}{x^3} = \)

   (a) \( \frac{1}{8} \)
   (b) 4
   (c) 8
   (d) \( \frac{1}{4} \)
   (e) The limit does not exist

2. \( \lim_{x \to 0} \frac{e^{ax^2} - \cos 2x}{x^2} = 8. \) What is \( a? \)

   (a) \( a = 3 \)
   (b) \( a = 6 \)
   (c) \( a = 4 \)
   (d) \( a = 2 \)
   (e) \( a = 1 \)

3. \( \lim_{h \to 0} \frac{\int_{\frac{\pi}{6} + h}^{\frac{\pi}{6}} \frac{\sin x}{x} \, dx}{h} = \)

   (a) 0
   (b) \( \frac{\sqrt{3}}{2} \)
   (c) \( \frac{\pi}{3} \)
   (d) \( \frac{3}{\pi} \)
   (e) \( \frac{6}{\pi} \)
4. The values of $A$ and $B$ such that

$$f(x) = \begin{cases} 
  x^2 - 2, & x \leq 2 \\
  Ax^2 + Bx, & x > 2 
\end{cases}$$

is everywhere differentiable are:

- (a) $a = 1/2$, $B = 2$
- (b) $a = 1/4$, $B = 1/2$
- (c) $a = -3/2$, $B = 2$
- (d) $a = 2$, $B = -1$
- (e) $a = 3/2$, $B = -2$

5. Set $f(x) = x^3 - 6x$. The value of $c$ that satisfies the Mean-Value Theorem for Derivatives on the interval $[0, 5]$ for $f$ is

- (a) $\frac{5}{\sqrt{3}}$
- (b) 0
- (c) $\frac{1}{\sqrt{3}}$
- (d) $\frac{5}{3}$
- (e) $-\frac{5}{\sqrt{3}}$

6. If $f$ and $g$ are twice differentiable and $h(x) = f[g(x)]$, then $h''(x) =$

- (a) $f''[g(x)]g'(x) + f'[g(x)]g''(x)$
- (b) $f''[g'(x)]g'(x)^2 + f'[g'(x)]g''(x)$
- (c) $f''[g'(x)]g'(x)^2 + f'[g'(x)]g''(x)$
- (d) $f''[g(x)]g'(x) + f'[g(x)]g''(x)$
- (e) $f'[g(x)]g'(x)^2 + f'[g(x)]g''(x)$
7. The normal line to the curve $y = 2x^2 - 4x + 2$ at the point $(2, 2)$ has an $x$-intercept at:
   (a) $\left(\frac{5}{2}, 0\right)$
   (b) $(10, 0)$
   (c) $(5, 0)$
   (d) $(2, 0)$
   (e) $\left(-\frac{5}{2}, 0\right)$

8. When the local linearization of $f(x) = \sqrt{4 + \ln(1 + x)}$ near $x = 0$ is used, an estimate of $f(0.08)$ is:
   (a) 1.98
   (b) 2.01
   (c) 2.02
   (d) 2.04
   (e) 2.06

9. Which of the following statements about the function $f(x) = x^4 - 4x^3$ is true?
   (a) The graph of $f$ has no points of inflection and the function has one relative extremum.
   (b) The graph of $f$ has one point of inflection and the function has two relative extrema.
   (c) The graph of $f$ has two points of inflection and the function has two relative extrema.
   (d) The graph of $f$ has two points of inflection and the function has one relative extremum.
   (e) The graph of $f$ has one point of inflection and the function has one relative extremum.

10. The position of a particle moving along the $x$-axis is given by
    $$s(t) = \int_0^t (u^2 - 6u + 8) \, du$$
    for $0 \leq t \leq 8$. For what values of $t$ is the speed of the particle increasing?
    (a) $2 < t < 4$ only
    (b) $t > 4$ only
    (c) $t > 3$ only
    (d) $0 < t < 2$ and $t > 4$
    (e) $2 < t < 3$ and $t > 4$
11. The function $f(x) = x^3 + 2x - 6$ has an inverse. If the graph of $f$ passes through the point $(2, 6)$, then $(f^{-1})'(6) =$
(a) $1/14$
(b) $1/10$
(c) $-6$
(d) $14$
(e) $10$

12. A particle moves along the ellipse $3x^2 + 2y^2 - 4y = 3$ so that $dy/dt = 3$ at all times $t$. The speed of the particle when it is at the point $(1, 2)$ is:
(a) $\sqrt{14}$
(b) $\sqrt{13}$
(c) $5$
(d) $2\sqrt{3}$
(e) $4$

13. The average value of $f(x) = x \ln x$ on $[1, e]$ is:
(a) $\frac{e^2 + 1}{4}$
(b) $\frac{e^2 + 1}{4(e + 1)}$
(c) $\frac{e + 1}{4}$
(d) $\frac{e^2 + 1}{4(e - 1)}$
(e) $\frac{2e^2 + 1}{4(e - 1)}$

14. Given that $\int_{0}^{2} f(x) \, dx = \frac{8}{3}$, $\int_{1}^{2} f(x) \, dx = \frac{4}{3}$, and $\int_{0}^{3} f(x) \, dx = \frac{11}{3}$, find $\int_{3}^{1} f(x) \, dx$.
(a) $-\frac{5}{3}$
(b) $2$
(c) $\frac{7}{3}$
(d) $\frac{5}{3}$
(e) $-\frac{7}{3}$
15. \( \int e^{2x} \sqrt{e^x - 3} \, dx = \)

(a) \( \frac{2}{5}(e^x - 3)^{5/2} + 2(e^x - 3)^{3/2} + C \)
(b) \( \frac{4}{5}(e^x - 3)^{5/2} + \frac{2}{3}(e^x - 3)^{3/2} + C \)
(c) \( \frac{5}{2}(e^x - 3)^{5/2} - \frac{3}{4}(e^x - 3)^{3/2} + C \)
(d) \( \frac{2}{5}(e^x - 3)^{5/2} + \frac{2}{3}(e^x - 3)^{3/2} + C \)
(e) \( \frac{1}{5}(e^x - 3)^{5/2} + (e^x - 3)^{3/2} + C \)

16. The region bounded by \( y = e^x, \ y = 1, \) and the line \( x = 2 \) is rotated about the \( y \)-axis. Which of the following integrals gives the volume of the solid which is generated:

\( (A) \) \( \pi \int_0^2 e^{2x} \, dx \), \( (B) \) \( 2\pi \int_0^2 x(e^x - 1) \, dx \), \( (C) \) \( \pi \int_0^2 \left( e^{2x} - 1 \right) \, dx \)

\( (D) \) \( 2\pi \int_1^e y(2 - \ln y) \, dy \), \( (E) \) \( \pi \int_1^e \left( 4 - \ln^2 y \right) \, dy \)

(a) (A) and (D)
(b) (B) and (E),
(c) (A), (D) and (E)
(d) (C) and (D)
(e) (B) only

17. If \( f \) is a continuous function and \( F(x) = \int_0^x \left[ (t^2 + 2t) \int_t^2 f(u) \, du \right] \, dt \), then \( F''(2) = \)

(a) \( 4f(2) \)
(b) \( 4f'(2) \)
(c) \( -4f'(2) \)
(d) \( -8f(2) \)
(e) \( 8f(2) \)

18. A cylindrical container 2 feet in diameter and 4 feet high is half full of oil weighing 50 pounds per cubic foot. The work done, in foot-pounds, in pumping the oil to an outlet one foot above the top of the container is:

(a) \( 100\pi \)
(b) \( 250\pi \)
(c) \( 400\pi \)
(d) \( 500\pi \)
(e) \( 800\pi \)
19. \[ \int_{4}^{\infty} \frac{2 \, dt}{16 + t^2} = \]
(a) \( \pi/2 \)
(b) \( \pi/8 \)
(c) \( \pi/16 \)
(d) \( \pi/4 \)
(e) Divergent

20. If \( x = 2 \sin \theta \), then \[ \int_{1}^{2} \frac{x^2}{\sqrt{4 - x^2}} \, dx \] is equivalent to
(a) \( 4 \int_{1}^{\pi/2} \sin^2 \theta \, d\theta \)
(b) \( 2 \int_{0}^{\pi/2} \sin \theta \tan \theta \, d\theta \)
(c) \( 2 \int_{\pi/6}^{\pi/2} \frac{\sin^2 \theta}{\cos^2 \theta} \, d\theta \)
(d) \( 4 \int_{0}^{\pi/2} \sin^2 \theta \, d\theta \)
(e) \( 4 \int_{\pi/6}^{\pi/2} \sin^2 \theta \, d\theta \)

21. If \( \frac{dy}{dx} = \frac{2x^3 - 1}{y} \) and \( y = 2 \) when \( x = 1 \), then, when \( x = 2 \), \( y = \)
(a) \( \pm \sqrt{17} \)
(b) \( \pm \sqrt{15} \)
(c) \( \pm 5 \)
(d) \( \pm 2\sqrt{3} \)
(e) \( \pm 2\sqrt{5} \)

22. Which differential equation which has the slope field
(a) \( \frac{dy}{dx} = \frac{3}{x} \)
(b) \( \frac{dy}{dx} = \frac{x}{y} \)
(c) \( \frac{dy}{dx} = 3y \)
(d) \( \frac{dy}{dx} = \frac{3}{y} \)
(e) \( \frac{dy}{dx} = x - y \)
23. The curve \( r = 2 \sin 3\theta \) is shown in the figure. The area enclosed by one loop of the curve is:

(a) \( \pi/6 \)  
(b) \( \pi/2 \)  
(c) \( \pi/3 \)  
(d) \( \pi \)  
(e) \( 2\pi/3 \)

24. A curve in the plane is defined by the parametric equations

\[
x = t^2 + 1, \quad y = \frac{1}{3}t^3 - t + 2, \quad 0 \leq t \leq 3.
\]

Find the length of the curve.

(a) \( 2\sqrt{3} \)  
(b) 12  
(c) 8  
(d) 10  
(e) \( \sqrt{14} \)

25. If 50 grams of a radioactive substance decays to 25 grams in two hours, then, to the nearest gram, the amount left after 5 hours is:

(a) 5  
(b) 7  
(c) 9  
(d) 10  
(e) 12
26. If a block of ice melts at the rate of \( \frac{64}{2t + 3} \) cm³/min, then the closest approximation to the amount of ice which melts during the first three minutes is:

(a) 29 cm³  
(b) 32 cm³  
(c) 35 cm³  
(d) 40 cm³  
(e) 48 cm³

27. Which infinite series converge(s)?

(I) \[ \sum_{n=1}^{\infty} \frac{10^n}{n!} \]  
(II) \[ \sum_{n=1}^{\infty} \frac{2n^2}{\sqrt{4n^5 + 2n + 1}} \]  
(III) \[ \sum_{n=1}^{\infty} \frac{2^{3n}}{3^{2n}} \]

(a) I and III  
(b) I only  
(c) I and II  
(d) III only  
(e) I, II and III

28. Suppose that the power series \( \sum_{k=0}^{\infty} a_k(x - 2)^k \) converges at \( x = 4 \). Which of the following series must be convergent?

I. \( \sum_{k=0}^{\infty} a_k3^k \)  
II. \( \sum_{k=0}^{\infty} a_k \)  
III. \( \sum_{k=0}^{\infty} (-1)^k a_k \)  
IV. \( \sum_{k=0}^{\infty} (-1)^k a_k2^k \)

(a) II only  
(b) I and III  
(c) II, III, and IV  
(d) III and IV  
(e) II and III
29. Using two terms of an appropriate Maclaurin series (Taylor series at \( x = 0 \)), estimate \( \int_{0}^{1} \frac{1 - \cos x}{x} \, dx \).

(a) \( \frac{19}{96} \)
(b) \( \frac{23}{96} \)
(c) \( \frac{1}{4} \)
(d) \( \frac{25}{96} \)
(e) undefined; the integral is improper

30. A function \( f \) is infinitely differentiable and has the property that \( |f^{(k)}(x)| \leq k2^k \) for all \( x \in (-1, 1) \) and all \( k \). Find the least integer \( n \) such that the Taylor polynomial of degree \( n \) in powers of \( x \) for \( f \) approximates \( f(1/4) \) to within 0.0005

(a) 3
(b) 4
(c) 5
(d) 6
(e) 7