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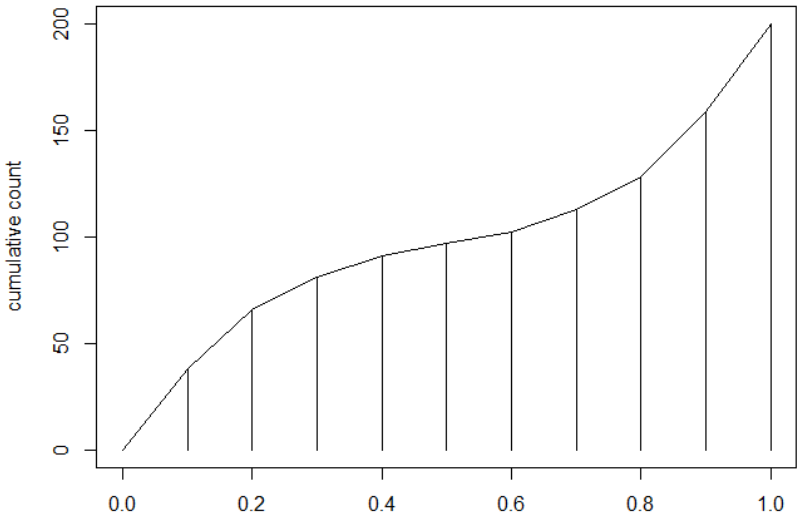
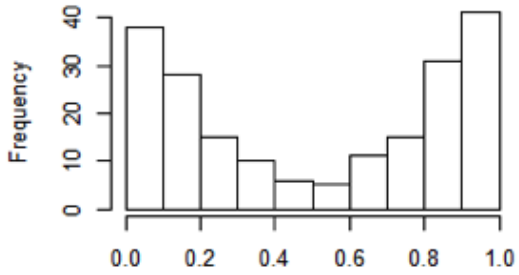
University of Houston High School Mathematics Contest
Statistics Exam – Spring 2016

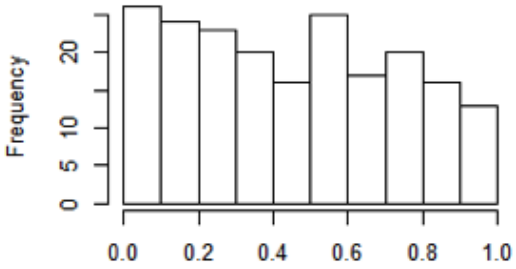
1. The figure below is a cumulative frequency polygon of 200 observations. Which of the histograms represents the same data?



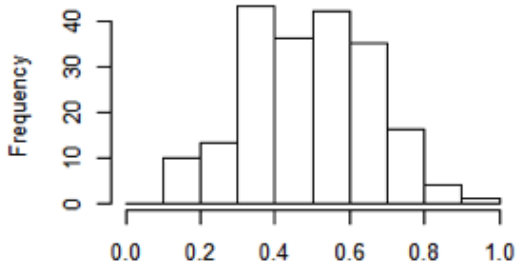
A)



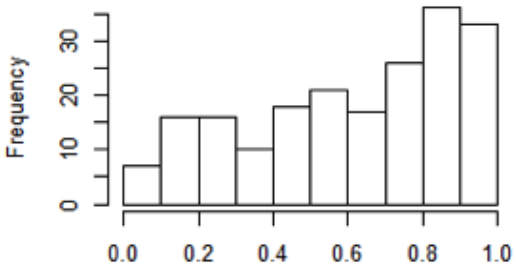
B)



C)



D)



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2. A 400 meter relay team has 4 runners, each running 100 meters. The time taken by each runner to complete his leg of the race is normally distributed with mean 10 sec. and standard deviation 1 sec. The times of the runners are independent. What is the probability that the race is completed in less than 37 seconds? Choose the closest answer.
   A) 0.1215
   B) 0.2215
   C) 0.0668
   D) 0.0228
   E) None of these

3. A candidate for public office conducts a poll to determine her approval rating – the percentage of prospective voters who would vote for her in preference to all other candidates. She wishes to know her approval rating to within 2 percentage points, that is, with an error no greater than 0.02, with probability 90%. To achieve this, she should sample how many prospective voters? Choose the smallest acceptable sample size.
   A) 802
   B) 1692
   C) 1309
   D) 2787
   E) None of these

4. The sample correlation for 27 pairs of observations (x_i, y_i) is -0.311. Assume that the variables x and y are jointly normally distributed. We wish to test the null hypothesis that the true correlation between x and y is 0 against the alternative hypothesis that it is not 0. The p-value is
   A) Less than 1%
   B) Between 1% and 5%
   C) Between 5% and 10%
   D) Greater than 10%

5. Which of the following is not a potential advantage of stratified sampling as compared to simple random sampling?
   A) Stratified sampling may result in more precise estimators of population characteristics.
   B) As a practical matter, stratified sampling may be easier to carry out.
   C) Stratified sampling eliminates bias in estimating population characteristics.
   D) Stratified sampling may yield more information about nonhomogeneous populations.
   E) None of these

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6. Let p_1 and p_2 denote the respective proportions of adult males and females who prefer red wine to white wine when served with chicken Kiev. In a sample of 100 males, 34 preferred red wine. In an independent sample of 150 females, 57 preferred red to white. A 95% confidence interval for $p_1 - p_2$ is:
   A) $(-0.1416, 0.0616)$
   B) $(0.3400, 0.3800)$
   C) $(-0.1416, -0.0616)$
   D) $(-0.1611, 0.0811)$
   E) None of these

7. The distance from a kangaroo rat's birthplace to its first home away from its parents is $c\sqrt{V}$, where c is a constant and V has a chi-square distribution with 2 degrees of freedom. 90% of kangaroo rats live within 50 feet of their parents. 99% of them live within what distance?
   A) 112.64 feet
   B) 70.71 feet
   C) 55.82 feet
   D) 80.13 feet
   E) None of these

8. A random sample of 144 Facebook members had an average age of 31.4 years with a sample standard deviation of 10.1 years. A 90% confidence interval for the mean age of all Facebook members is (choose the closest answer):
   A) (30.02, 32.78)
   B) (29.75, 33.05)
   C) (28.63, 34.17)
   D) (30.32, 32.48)
   E) None of these

9. A sample of 20 men and an independent sample of 20 women were selected by the Labor Department for a comparison of salaries in a particular job category. The subjects of the sample all had similar educational backgrounds and similar levels of experience in their jobs. It was assumed that salaries from both populations are normally distributed and have the same standard deviation. The male subjects had a sample mean salary of $68,755 and a sample standard deviation of $7,340. The female subjects had a sample mean salary of $64,883 with a sample standard deviation of $6,910. To the nearest dollar, a 90% confidence interval for the difference between men's and women's mean salaries is:
   A) (72, 7672)
   B) $(-185, 185)$
   C) (932, 6812)
   D) $(-8146, 15890)$
   E) None of these

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10. The table below shows the prices of 10 houses in thousands of dollars and their sizes in square feet. These houses are all in the same residential area. A 2100 square foot house is offered for sale. A 95% prediction interval for its price is (round to the nearest dollar):

Size	1526	1849	1906	2460	1602	1731	2208	2041	2595	2008
Price	153	202	199	315	148	194	250	167	305	258

A) (215, 257)
B) (182, 289)
C) (205, 266)
D) (169, 302)
E) None of these

11. From a standard deck of 52 playing cards, in how many ways can you get a full house 5 card poker hand? (A full house occurs with 3 of a kind and 2 of a kind.)
A) 312
B) 3456
C) 3744
D) 288
E) None of these

12. A company is hiring for six available positions. The qualified applicant pool consists of five men and seven women. If all applicants have an equal change of being selected, what is the probability that the company will hire an equal number of men and women?
A) 0.379
B) 0.500
C) 0.029
D) 0.243
E) None of these

13. The mean cholesterol level in children is 175 mg/dL with standard deviation 35 mg/dL. Assume this level varies from child to child according to an approximate normal distribution. How high are levels for the highest 2% of all children?
A) Above 231.09 mg/dL
B) Above 246.88 mg/dL
C) Above 182.97 mg/dL
D) Above 195.76 mg/dL
E) None of these

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14. A gas station sells three grades of gasoline: regular, extra and super. These are priced at \$1.75, \$1.99, and \$2.35 per gallon. Let X_1 represent the number of regular gallons of gas sold on a given day, X_2 represent the number of extra gallons of gas sold on a given day, and X_3 represent the number of super gallons of gas sold on a given day. Suppose these are independent random variables with $\mu_1 = 1000,\ \mu_2 = 500,\ \mu_3 = 300,\ \ \sigma_1 = 100,$ $\sigma_2 = 80$, and $\sigma_3 = 50$. Determine the standard deviation of sales revenue on a given day for this station.
    A) \$451.70
    B) \$345.18
    C) \$264.15
    D) \$168.98
    E) None of these

15. A certain MP3 player comes in three configurations: 16 GB (\$80), 32 GB (\$100), and 64 GB (\$120). Let X represent the cost of a single randomly selected purchase of the MP3 player. Suppose X has the distribution given by the table below:

X	\$80	\$100	\$120
$P(X)$	0.2	0.3	0.5

On a particular day, two of these MP3 players are sold. Let X_1 represent the selling price of the first MP3 player sold and let X_2 represent the selling price of the second MP3 player sold. Find the expected value of $\overline{X} = \dfrac{X_1 + X_2}{2}$.
    A) \$106
    B) \$122
    C) \$110
    D) \$100
    E) None of these

16. Suppose we are given a sample of size $n = 49$ from a population whose standard deviation is $\sigma = 20$ and we wish to test the claim:
    $H_0 : \mu = 55$
    $H_a : \mu < 55$
    with an alpha value of $\alpha = 0.07$. For what values of $\overline{x}$, the sample mean, would we reject the null hypothesis of this claim?
    A) 53.29
    B) 49.78
    C) 52.08
    D) 50.78
    E) None of these

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17. Refer to the information given in problem 16. Suppose the population mean was determined to be $\mu = 47$. What is the probability of the Type II error?
    A) 0.093
    B) 0.907
    C) 0.070
    D) 0.930
    E) None of these

18. Suppose a least squares regression equation for a given set of points is found to be $\hat{y} = 10.17 - 2.45x$. Give the residual value corresponding to the point $(8, -10)$.
    A) –9.43
    B) 0.57
    C) –0.57
    D) 9.43
    E) None of these

19. In an experiment to study the effects of illumination level on performance, subjects were timed in both a low level light environment and a high level light environment. The results, recorded in seconds, are:

Subject	1	2	3	4	5	6	7	8	9
Low Light	26	29	32	26	21	41	25	25	27
High Light	18	21	23	20	20	25	16	16	25

Which of the following would be an accurate statement?
    A) Based on 95% certainty, we can conclude completion time is lower in a low level light environment.
    B) Based on 99% certainty, we can conclude completion time is lower in a high level light environment.
    C) Based on 99% certainty, we can conclude completion time is lower in a low level light environment.
    D) Based on 95% certainty, we can conclude there is no difference in completion time in a high level light environment versus a low level light environment.
    E) None of these

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20. M&M Mars, Inc claims that every bag of peanut M&Ms contains 20% brown, 20% yellow, 20% blue, 20% red, 10% orange, and 10% green candies. In order to test this claim, you go buy a 42 ounce bag of peanut M&Ms from the store and count each color. Based on the counts given below, what is the test statistic for your hypothesis test?

Color	Brown	Yellow	Blue	Red	Orange	Green
Counts	195	219	224	180	110	122

A) 9.995
B) 8.169
C) 9.667
D) 7.692
E) None of these

21. The one-sample t statistic for a test of H_0: $\mu = 11$ vs. H_a: $\mu < 11$ based on $n = 13$ observations has the test statistic value of $t = -1.25$. What is the p-value for this test?
A) 0.418
B) 0.882
C) 0.000
D) 0.118
E) None of these

22. The values for Q1 and Q3 for the data set below are Q1=121 and Q3=162. Are there any outliers for this data? If so, what are they?

68	76	96	119	133	147	148	148	149	158	160	163	200	228	236

A) There are no outliers.
B) 68 is the only outlier.
C) The outliers are 228 and 236.
D) 236 is the only outlier.
E) None of these

23. A large-scale study conducted over a one-year period has shown that break-ins at home occur about 4% of the time in the population. The study also shows home security alarms went off 2% of the time when no one was breaking into the home. The security alarm *failed* to go off 1% of the time when someone was really breaking into the home. If an alarm is going off, what is the probability that the house was broken into?
A) 0.6735
B) 0.0588
C) 0.0396
D) 0.5962
E) None of these

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24. Among 20 golden hamster litters recorded, there was a sample mean of $\bar{x} = 7.72$ baby hamsters, with a sample standard deviation of $s = 2.5$ hamsters per liter. Create a 90% confidence interval for the mean number of baby hamsters per liter.
    A) (6.75, 8.69)
    B) (6.80, 8.64)
    C) (7.19, 8.31)
    D) (6.63, 8.82)
    E) Nonne of these

25. In an instant lottery, your chances of winning are 0.2. If you play the lottery five times and outcomes are independent, what is the probability that you win at least once?
    A) 0.2
    B) 0.08192
    C) 0.32768
    D) 0.67232
    E) None of these

26. The weight of a randomly selected bag of corn chips coming off an assembly line is a random variable with mean $\mu = 10$ oz. and standard deviation $\sigma = 0.2$ oz. Suppose we pick four bags at random assume that weight of each of the bags are independent. The combined weight of these four bags is a random variable with a standard deviation (in oz.) of
    A) 0.16.
    B) 0.40.
    C) 0.64.
    D) 0.80.
    E) None of these

END OF EXAM ☺