Name: _____ **School:** _____

University of Houston Mathematics Contest
Algebra I Exam – Spring 2018

Answer the following.

1. How many real solutions does the equation $|x-3| = |3x+2| - 1$ have?

A. One
B. Two
C. Three
D. Four
E. Five
F. None

2. Your band is planning a concert tour with performances to be given in 5 cities: Houston, Austin, Dallas, New Orleans, and Tulsa. In how many ways can they arrange their itinerary if the 3 performances in Texas must be given consecutively?

A. 720   B. 360   C. 120   D. 12   E. 36   F. 6

3. Give the greatest common factor for $720x^{-\frac{1}{2}} + 1008x^{-\frac{3}{2}}$.

A. $144x^{-\frac{1}{2}}$
B. $24x^{-\frac{1}{2}}$
C. $48x^{-\frac{3}{2}}$
D. $144x^{-\frac{3}{2}}$
E. $48x^{-\frac{1}{2}}$
F. $24x^{-\frac{3}{2}}$

4. Find the least common denominator for $\dfrac{1}{4x^2-16} - \dfrac{7x}{6x^2-24x+24} + \dfrac{3x^2}{x^2-4}$.

A. $6x^2 - 24x + 8$
B. $24x^3 - 48x^2 - 96x + 192$
C. $24x^2 - 96$
D. $12x^6 - 48x^5 - 48x^4 + 384x^3 - 192x^2 - 768x + 768$
E. $6x^4 - 48x^2 + 96$
F. $12x^3 - 24x^2 - 48x + 96$

5. Fifty students were asked, "What is your favorite sport to play?".   The results follow:

22 said Tennis
25 said Hockey
39 said Soccer
9 said Tennis and Hockey
20 said Hockey and Soccer
6 said all three of these sports
4 said none of these sports

How many students said their favorite sport was Tennis or Hockey?

A. 38
B. 32
C. 37
D. 30
E. 25
F. Not enough information is given.

6. Five years ago Kate was 5 times as old as her son. In 5 years her age will be 8 less than three times her son's age at that time. What is the sum of their ages?

A. 52
B. 30
C. 40
D. 31
E. 46
F. Not enough information is given.

7. A secret code contains 6 digits. If a secret code is selected at random, what is the probability that it will have 6 different digits (a code cannot have repeated digits)?

A. $\dfrac{189}{1250}$ B. $\dfrac{21}{100000}$ C. $\dfrac{3}{5}$ D. $\dfrac{1}{10}$ E. $\dfrac{3}{500000}$ F. $\dfrac{1}{6}$

8. A fair coin is tossed and a fair six sided die is tossed. What is the probability that the coin shows heads and the die shows a multiple of three?

A. $\dfrac{1}{4}$ B. $\dfrac{2}{3}$ C. $\dfrac{1}{6}$ D. $\dfrac{1}{2}$ E. $\dfrac{1}{8}$ F. $\dfrac{1}{3}$

9. In the square pictured below, M is the midpoint of AC and N is the midpoint of AB. A point is selected at random in the square. What is the probability that the point lies in the triangle AMN. *Note the diagram may not be drawn to scale.*
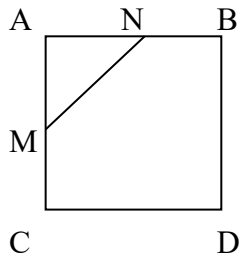
A. $\dfrac{1}{8}$
B. $\dfrac{1}{10}$
C. $\dfrac{1}{4}$
D. $\dfrac{1}{3}$
E. $\dfrac{1}{6}$
F. $\dfrac{1}{2}$

10. The polynomial $f(x) = 2x^4 + 3x^3 - 16x^2 - 17x + 12$ has zeros $x = \dfrac{1}{2}$ and $x = -3$. Find the *product* of the other real zeros of the polynomial.

A. 1
B. -2
C. 2
D. -3
E. 3
F. -4

11. How many real solutions does the equation $\left(x^2 + 1\right)^2 + 2\left(x^2 + 1\right) - 3 = 0$ have?

A. One
B. Two
C. Three
D. Four
E. Five
F. None

12. How many real solutions does the equation $2\left(4^{x+1}\right) = 4\left(8^{x+2}\right)$ have?

A. One
B. Two
C. Three
D. Four
E. Five
F. None

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13. The coefficients a and b of the polynomial $3x^3 + ax^2 + bx + 7$ are real numbers. When the polynomial is divided by $x^2 - 1$ the remainder is $5x + 4$. Find the product of a and b.

A. -8
B. 9
C. 12
D. -6
E. 3
F. -10

14. Simplify the following expression $\dfrac{a + 27a^{-2}}{1 - 3a^{-1} + 9a^{-2}}$.

A. a
B. $a + 3$
C. $a - 9$
D. $a^2 + 9$
E. $(a+3)(a-3)$
F. $a + 9$

15. We use a base-10 number system to add, subtract, multiply, and divide.

$$7031_8$$

Now suppose the numbers 7031 and 5017 are both in base 8, what is -5017_8 ?

A. 2000_8 B. 2023_8 C. 2012_8 D. 2021_8 E. 2016_8 F. 2022_8

16. Let the following points be the vertices of a right triangle with hypotenuse AC.

A(0 , 0), B(2 , 2) and C(-4 , Y)

Determine Y then calculate 2Y + 1.

A. 0   B. -8   C. 1   D. 17   E. 3   F. -10

17. A park has a rectangular field whose length is $\dfrac{7}{5}$ its width. The perimeter of the field is 240 meters. What is the product of its length and width?

A. 4125   B. 1750   C. 1000   D. 2000   E. 2700   F. 3500

18. It took you 3.5 hours to drive from point A to point B. On your way back to point A, you increased your speed by 20 kilometers per hour and it took you 3 hours. What was your average speed for the entire journey? *Round answers to the nearest whole number.*

A. 115 km/hr
B. 100 km/hr
C. 120 km/hr
D. 143 km/hr
E. 129 km/hr
F. 123 km/hr

19. Given the function $f(x) = 3(x+2)^2 x^{-3} - 6(x+2)^{-2}$, for which real values of x is the function undefined?

A. 0   B. -2, 2   C. -2, 0   D. -2, 0, 2   E. 0, 2   F. -2

20. How many real solutions does the equation $\sqrt{x-5} = -2$ have?

A. One   B. Two   C. Three   D. Four   E. Five   F. None

21. For what real value of the constant k does the system of equations $2x - y = 4$ and $6x - 3y = 3k$ have an infinite number of solutions?

A. None   B. 3   C. 1   D. 4   E. 2   F. $\dfrac{1}{2}$

22. The set of points $\{(-4,0),(1,6),(4,3),(x^2,2)\}$ is a relation. For the values of x, if any, that do not make the relation a function, give the product.

A. None   B. 4   C. -2   D. 3   E. -1   F. 6

Questions 23 – 28 are write-in answers rather than multiple choice. Write your answer to each question on the answer sheet in the space provided. Do not write any units on the answer sheet.

23. A circle has equation $x^2 + 3x + y^2 - 4y = 18$. Find its x and y-intercepts. Write your answers as ordered pairs.

24. The points $A(-2,1)$, $B(6,1)$ and $C(-2,7)$ are plotted on the Cartesian coordinate plane. Find the equation of a circle that passes through the three points. *Write your answer in the form* $(x-h)^2 + (y-k)^2 = r^2$.

25. You wish to construct an open-top box having a square base and surface area of 108 square inches with maximum volume. Let x represent the side length of the base.

a. Write an expression in terms of x that describes the volume of the box.

b. Determine the domain of the function in part (a). *Write your answer in interval notation.*

26. An orange grove has an average yield of 36 bushels of orange/tree if tree density is 22 trees/acre. For each unit increase in tree density, the yield decreases by 2 bushels. How many trees should be planted in order to maximize the yield?

27. In the following equation, $f(x) = 3x^2 - 2kx + 2$ k is a real constant. Give the value(s) of k that would make the equation have no real solutions.

28. A power station and a factory are on opposite sides of a river that is 40 meters wide. From a point directly across the river from the power station, the factory is 800 meters downstream. It will cost \$50 per meter to run the cable underwater and \$35 to run the cable on land. It is cost effective to run the power line underwater, diagonally across the river, to a point downstream from the factory, and then construct the remainder of the power line underground. If x is the length of the underground power line, write a function C in terms of x.