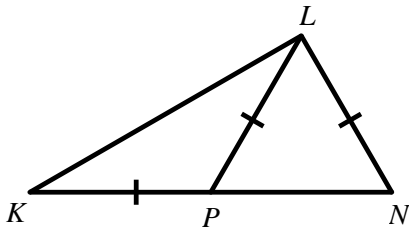**Name:** _____   **School:** _____

University of Houston Mathematics Contest
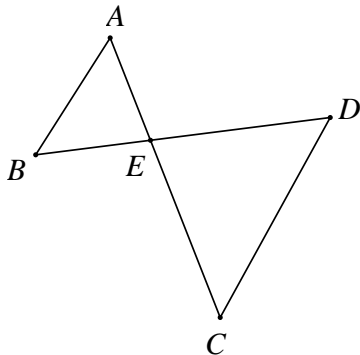# Geometry Exam – Spring 2018

Answer the following. Note that diagrams may not be drawn to scale.

1.   In the figure below, $m\angle LNP = 76°$. Find the measure of $\angle KLN$.

   A. $104°$

   B. $38°$

   C. $66°$

   D. $28°$

   E. $90°$

   F.   None of these

2.   In the diagram below, $\dfrac{AE}{EC} = \dfrac{BE}{ED}$. Which of the following statements is true?

   A. $\triangle ABE \sim \triangle CDE$ by $AA \sim$.
   B. $\triangle ABE \sim \triangle DCE$ by $SAS \sim$.
   C. $\triangle ABE \sim \triangle CDE$ by $SSS \sim$.
   D. $\triangle ABE \sim \triangle DCE$ by $AA \sim$.
   E. $\triangle ABE \sim \triangle CDE$ by $SAS \sim$
   F.   There is not enough information to determine if the triangles are similar.

3.   If a line segment divides two sides of a triangle proportionately, then the line segment is _____ to the third side of the triangle.

   A. proportional   B. similar   C. congruent
   D. parallel   E. perpendicular   F. None of these

4. The angles of a triangle are in a ratio of $1:2:3$. Find the ratio of the sides opposite these angles.

A. $1:\sqrt{3}:2$ B. $1:2:3$ C. $1:\sqrt{2}:3$

D. $1:\sqrt{2}:2$ E. $1:2:\sqrt{5}$ F. Cannot be determined

5. Given the following conditional statement:

If a quadrilateral is a rectangle, then it is reflectionally symmetric.

Determine the truth value of the contrapositive, inverse, and converse, respectively.

A. True, True, True
B. False, True, True
C. False, False, False
D. False, False, True
E. True, True, False
F. None of the above

6. A triangle with side lengths 9, $4\sqrt{6}$, and 11 is what type of triangle?

A. Acute   B. Right   C. Obtuse
D. Concave   E. Equiangular
F. There is not enough information to determine the answer

7. $ADHK \sim TOPL$, where $m\angle A = (5x-6)^\circ$, $m\angle O = (4x-27)^\circ$, $m\angle K = (8x-35)^\circ$, and $m\angle T = 79^\circ$. What shape do $ADHK$ and $TOPL$ represent?

A. Kite   B. Trapezoid   C. Rhombus
D. Rectangle   E. Parallelogram   F. None of these

8. $\overline{DH}$ is an altitude of $\triangle CDE$, where $DE = 8$, $CH = 10$, and $EH = 2$. Find CD.
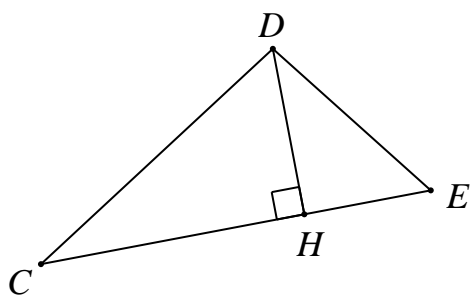
A. $4\sqrt{5}$

B. $2\sqrt{30}$

C. $\sqrt{130}$

D. $4\sqrt{10}$

E. $2\sqrt{6}$

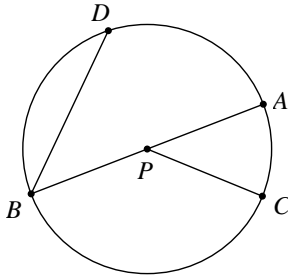F. None of these

9.   Three diagrams are given below.

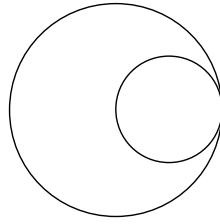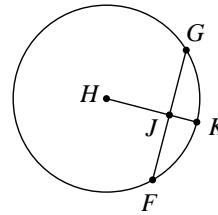**Figure 1**   **Figure 2**   **Figure 3**

Based on the above diagrams, which of the following statements are true?

Create an answer using the numbers associated with the true statements. For example, if only 1, 2, and 5 are true, then the answer is 125.

1. The circles in Figure 2 are concentric.
2. If $\overline{HK} \perp \overline{GF}$ in $\odot H$, then $\overarc{GK} \cong \overarc{KF}$.
3. $\overarc{CB}$ is the intercepted arc for $\angle BPC$.
4. $\overline{HK}$ is a chord of $\odot H$.

5. $\angle DBA$ is an inscribed angle of $\odot P$.
6. $\overarc{BD}$ is a major arc of $\odot P$.
7. $\angle APC$ is an inscribed angle of $\odot P$.

A. 1345   B. 1235   C. 23457   D. 2367   E. 235   F. None of these

10. $\odot T$ is given below with diameter $\overline{EM}$. If $GT = 3$ and $GE = 2$, find GM.
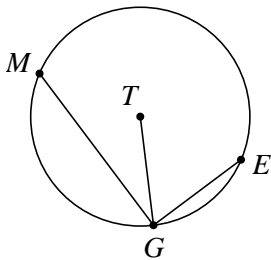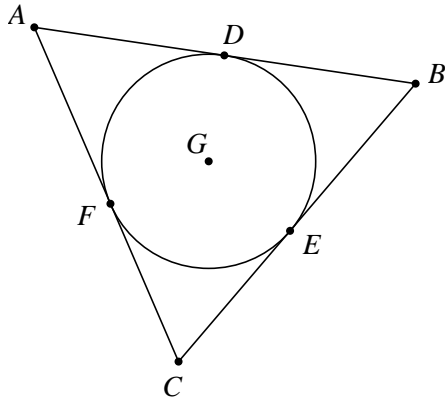

A. $\sqrt{13}$

B. $4\sqrt{2}$

C. $\sqrt{5}$

D. $2\sqrt{10}$

E. 5

F. None of these

11.   Find the area of kite $ABCD$, given that $BC = CD = 10$, $m\angle A = 90°$, and $m\angle C = 60°$.

A. $50\sqrt{3}$ B. $50 + 50\sqrt{3}$ C. $50\sqrt{2}$

D. $25 + 25\sqrt{3}$ E. $50 + 50\sqrt{2}$ F. None of these

12. Segments $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$ are tangent to $\odot G$ at points D, E, and F, respectively. If $AD = 11$, $DB = x+5$, $CE = x-1$, and $EB = 13$, find the length of $\overline{AC}$.
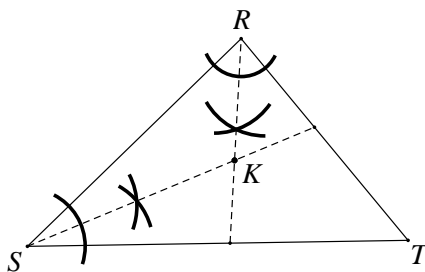

A. 10
B. 14
C. 16
D. 18
E. 22
F. None of these

13. The perimeter of an isosceles trapezoid is 98, and the length of its median is 32. Find the length of one of the legs.

A. 15   B. 34   C. 33   D. 17   E. $\dfrac{47}{16}$ F. None of these

14. A construction has been performed on ΔRST, with all relevant lines and arcs of circles shown. Point K represents the _____ of the triangle.

A. Centroid
B. Orthocenter
C. Circumcenter
D. Incenter
E. Median
F. None of these

15. Quadrilateral $ABCD$ is inscribed in $\odot E$. If $m\angle A = 80°$, $m\angle D = 120°$, and $m\overset{\frown}{BC} = 90°$, find $m\overset{\frown}{AD}$.

A. 90°   B. 120°   C. 50°   D. 170°   E. 200°   F. None of these

16.   There are several numbered statements below. Create a number using the numbers associated with the true statements. For example, if only 2, 3 and 5 are true, then your answer should be 235.

   1. A convex pentagon can have three acute angles.
   2. A convex quadrilateral can have three acute angles.
   3. A convex hexagon can have four acute angles.
   4. A convex octagon can have three right angles.
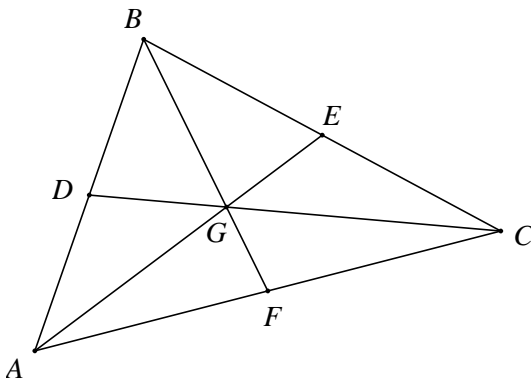   5. A convex nonagon can have four right angles

   A. 124   B. 245   C. 1245   D. 2345   E. 125   F. None of these

17.   In the diagram below, $\overleftrightarrow{KL} \parallel \overleftrightarrow{HP} \parallel \overleftrightarrow{GJ}$. PJ is 3 times greater than KH, LP is 6 units greater than KH, and HG is 2 units smaller than PJ. Find LP.
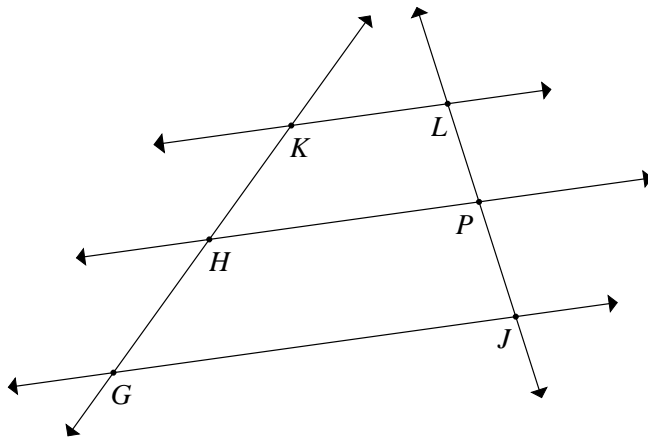
   A. 2

   B. $7 + \sqrt{7}$

   C. $\dfrac{27}{4}$

   D. $\dfrac{3}{4}$

   E. $\dfrac{1}{4}$

   F. None of these

18.   In the diagram below, medians $\overline{AE}$, $\overline{BF}$, and $\overline{DC}$ intersect at point G. If $DG = 3x - 2$, $GE = x + 4$, and $AG = 5x + 2$, find the length of $\overline{GC}$.

   A. 8

   B. 12

   C. 6

   D. 17

   E. 4

   F. None of these

19.   Find the area of $\triangle ABC$ with vertices $A(1,-3)$, $B(-3,2)$, and $C(4,5)$.

   A.   24.5

   B.   17.5

   C.   18.5

   D.   23.5

   E. $18.\overline{6}$

   F.   None of these

20.   Isosceles $\triangle ABC$ has angle bisector $\overline{BD}$ drawn from vertex angle B. If $BD = 20\sqrt{14}$ and the perimeter of $\triangle ABC$ is 280, find the value of $\left| \dfrac{1}{2}\cdot BC - DC \right|$.
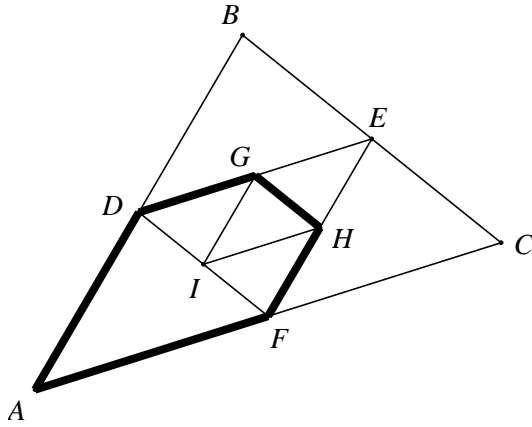
   A.   40

   B.   65

   C.   20

   D.   15

   E.   5

   F.   None of these

21.   In the circle below, $\overline{AB}$ is a diameter and $\angle DBP$ is half the measure of $\angle DAP$. If $AD = 5$, $DP = \sqrt{21}$, and $CP = \dfrac{8\sqrt{21}}{7}$, find the value of $|AP - PB|$.
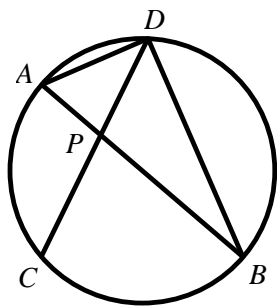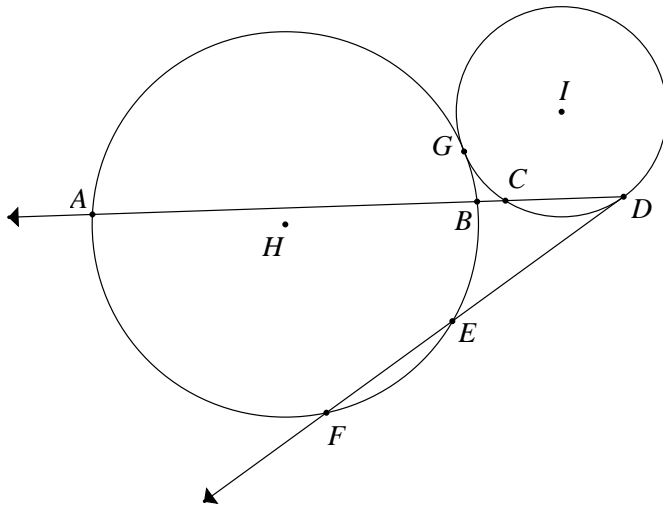

   A. 10   B. 5   C. 2   D. 23   E. $\dfrac{19}{3}$ F. None of these

22. Points *D*, *E*, and *F* are midpoints of the sides of $\triangle ABC$, and points *G*, *H*, and *I* are midpoints of the sides of $\triangle DEF$. Find the area of pentagon *ADGHF*, given that $AB = 7$, $AF = 4$, and $GH = \dfrac{5}{4}$.
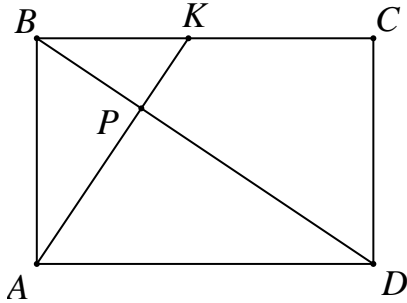
A. $\dfrac{9\sqrt{10}}{4}$

B. $\dfrac{245}{32}$

C. $\dfrac{15\sqrt{3}}{4}$

D. $\dfrac{5\sqrt{33}}{4}$

E. $\dfrac{35\sqrt{3}}{8}$

F. None of these

23. $\odot H$ and $\odot I$ are tangent to each other at point *G*, and $\overrightarrow{DF}$ is tangent to $\odot I$ at point *D*. If $m\overset{\frown}{CD} = 48°$ and $m\overset{\frown}{AF} = 116°$, find $m\overset{\frown}{BE}$.

A. $46°$ B. $96°$ C. $78°$ D. $68°$ E. $48°$ F. None of these

24. Rectangle $ABCD$ is shown below with $AB = 8$ and $AD = 4\sqrt{5}$. $\overline{AK}$ intersects $\overline{BD}$ at point P, and $\overline{AK} \perp \overline{BD}$. Find PK.

A. $\dfrac{16}{3}$ B. $\dfrac{8\sqrt{5}}{3}$ C. $\dfrac{10\sqrt{5}}{3}$ D. $\dfrac{16\sqrt{5}}{5}$ E. $\dfrac{32\sqrt{5}}{15}$ F. None of these

25. An unusual dartboard is shown below, in which the smallest circle is inscribed inside an equilateral triangle, which is inscribed inside a regular hexagon (with the vertices of the triangle at the midpoints of the hexagon's nonadjacent sides), which is inscribed inside a circle. The length of $\overparen{AB}$ is 4π. If a dart is thrown at the dartboard, what is the probability of hitting one of the shaded regions? (Assume that all darts thrown are randomly distributed within the outer circle.)
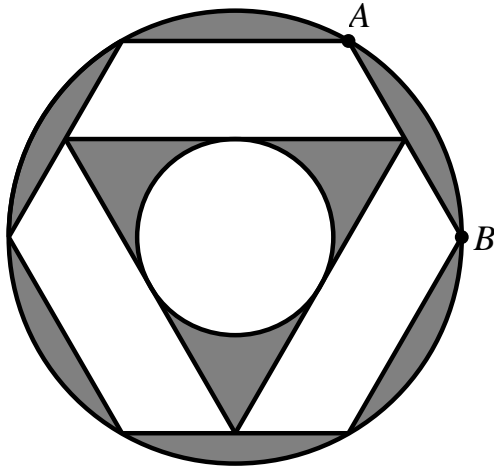
A. $\dfrac{19\pi - 33\sqrt{3}}{16\pi}$ B. $\dfrac{52\pi - 87\sqrt{3}}{64\pi}$ C. $\dfrac{12\pi - 15\sqrt{3}}{16\pi}$

D. $\dfrac{13\pi - 30\sqrt{3}}{64\pi}$ E. $\dfrac{13\pi - 15\sqrt{3}}{16\pi}$ F. None of these

26. The equation of circle P is $x^2 + y^2 + 6x - 2y + 6 = 0$. Circle P is dilated 250% about center of dilation $(-7, 3)$, resulting in circle T. Find the equation of circle T.

  A. $(x+7)^2 + (y-3)^2 = 25$

  B. $(x-7)^2 + (y+4)^2 = 49$

  C. $(x-3)^2 + (y-2)^2 = 25$

  D. $(x+3)^2 + (y-1)^2 = 10$

  E. $(x+7.5)^2 + (y-2.5)^2 = 25$

  F. None of these

27. In a right circular cone, let θ represent the angle formed between the radius of the base and the slant height of the cone. If $\cos(\theta) = \dfrac{3}{5}$ and the volume of the cone is 324π, find the total surface area of the cone.

  A. $72\pi \sqrt[3]{3}$

  B. 135π

  C. 297π

  D. 216π

  E. 24π

  F. None of these

28. The volume of a right square pyramid is eight times the volume of a cube with the same square base. Find the ratio of the lateral area of the pyramid to the lateral area of the cube.

  A. $\sqrt{577} : 4$

  B. $12:1$

  C. $\sqrt{2305} : 4$

  D. $2\sqrt{2} : 1$

  E. $4:1$

  F. None of these

Questions 29-31 are write-in answers rather than multiple choice. Write the answer to each question on the answer sheet in the space provided. All irrational answers (containing radicals or π) should be left as exact answers rather than decimal approximations. All radicals and fractions should be written in simplest form, with no radicals in the denominator. Do not write any units on the answer sheet.

29. $\overline{CT}$ is a median of $\triangle CPK$, where the coordinates of points C, P and T are $C\left(\dfrac{5}{8}, \dfrac{1}{5}\right)$, $P\left(\dfrac{1}{2}, -\dfrac{2}{3}\right)$ and $T\left(\dfrac{9}{20}, -\dfrac{13}{21}\right)$. Find the slope of $\overline{CK}$.

30. $\triangle DAN$ has coordinates $D(-4, 5)$, $A(0, 4)$, and $N(-2, -1)$. $\triangle DAN$ is reflected over the line $y = x - 2$, resulting in $\triangle D'A'N'$. Find the equation of the line that passes through point A' and is perpendicular to the line containing D' and N'. Write your answer in slope-intercept form.

31. Akash has nine straws of the following lengths, in inches: 1, 2, 3, 4, 5, 6, 7, 8, and 9. If he randomly chooses three of the straws, what is the probability that he can form a triangle with those three straws?

END OF EXAM ☺