Name: School:

University of Houston Mathematics Contest
Algebra I Exam – Spring 2019

1. Two numbers have sum equal to 85 and difference equal to 37. Find the sum of the squares of the two numbers.

   (A) 8594
   (B) 7225
   (C) 4297
   (D) 3977
   (E) 997
   (F) None of the above

2. Write the expanded form of the following polynomial:

$$(5x + 2)(3x + 1)$$

   (A) $8x + 2$
   (B) $15x^2 + 2$
   (C) $8x^2 + 11x + 2$
   (D) $5x^2 + 8x + 2$
   (E) $15x^2 + 11x + 1$
   (F) None of the above

3. Which of the following is an equation describing the line with x-intercept 2, which is parallel to the line with equation $4x + 2y = 1$?

   (A) $2x + y = 4$
   (B) $4x + y = 2$
   (C) $2x + 4y = 2$
   (D) $4x + 2y = 2$
   (E) $2x - y = 4$
   (F) None of the above

4. Anne is twice as old as Bill, and Bill is 3 years older than Christie. The sum of Anne's and Christie's ages equals 72. What is the difference between Anne's age and Christie's?

   (A) 20
   (B) 24
   (C) 28
   (D) 32
   (E) 38
   (F) None of the above

5. If $2 \cdot 8^x = 2^{100}$, what is the value of x?

   (A) 33
   (B) 50
   (C) 99
   (D) 25
   (E) 300
   (F) None of the above

6. Find the expanded form of the following polynomial:

$$(2x^2 - x + 1)(x^2 + 2x - 2)$$

   (A) $2x^4 + 3x^3 - 3x^2 + x - 1$
   (B) $2x^4 + 2x^3 + 3x^2 + 4x - 2$
   (C) $2x^4 + 2x^3 - 5x^2 + x + 2$
   (D) $2x^4 + 3x^3 - 5x^2 + 4x - 2$
   (E) $2x^4 + 5x^3 + 8x^2 + 4x + 2$
   (F) None of the above

7. Arrange the following list of numbers in increasing order, from left to right:

$$\sqrt{11}, \quad \sqrt{2} + \sqrt{5}, \quad 2\sqrt{3}, \quad (\sqrt{2})^3$$

   (A) $\sqrt{2} + \sqrt{5}, \quad (\sqrt{2})^3, \quad \sqrt{11}, \quad 2\sqrt{3}$
   (B) $\sqrt{2} + \sqrt{5}, \quad (\sqrt{2})^3, \quad 2\sqrt{3}, \quad \sqrt{11}$
   (C) $(\sqrt{2})^3, \quad 2\sqrt{3}, \quad \sqrt{2} + \sqrt{5}, \quad \sqrt{11}$
   (D) $(\sqrt{2})^3, \quad \sqrt{11}, \quad 2\sqrt{3}, \quad \sqrt{2} + \sqrt{5}$
   (E) $(\sqrt{2})^3, \quad \sqrt{11}, \quad \sqrt{2} + \sqrt{5}, \quad 2\sqrt{3}$
   (F) None of the above

8. Find the shortest distance between the parabola $y = 4x^2 + 12x + 10$ and the x-axis.

   (A) $1/2$
   (B) 1
   (C) $3/2$
   (D) 3
   (E) $\sqrt{13}/2$
   (F) None of the above

9. How many real numbers c have the property that there is exactly one solution x to the equation $x^2 + 9 = cx$?

   (A) 0
   (B) 1
   (C) 2
   (D) 3
   (E) 4
   (F) None of the above

10. Let $f(1) = 2$ and, for each positive integer $n \geq 2$, let

$$f(n) = 2^{f(n-1)}.$$

    For example, $f(2) = 2^2 = 4$, and $f(3) = 2^4 = 16$. Find the value of $f(4)/f(3)$.

    (A) 1024
    (B) 512
    (C) 64
    (D) 4096
    (E) 16
    (F) None of the above

11. Find the y-intercept of the quadratic polynomial which passes through the points $(1, 1)$ and $(2, 1)$ and has exactly one real zero.

    (A) $9/4$
    (B) $3/2$
    (C) $9/2$
    (D) 6
    (E) 9
    (F) None of the above

12. One-third of the people from country A claim that they are from country B, and the rest admit they are from country A. One-fourth of the people from country B claim that they are from country A, and the rest admit they are from country B. In a combined census of the two countries, one-half of the total population claimed to be from country A. What is the ratio of the population of country A to that of country B?

    (A) $1/2$
    (B) 1
    (C) $3/2$
    (D) 2
    (E) $5/2$
    (F) None of the above

13. Find the y-intercept of the parabola that passes through $(-1, 5)$, $(1, 0)$, and $(4, 0)$.

    (A) 1
    (B) 2
    (C) $5/2$
    (D) 3
    (E) 4
    (F) None of the above

14. Two opposite edges of a square lie on the lines $y = -x/2 + 1$ and $y = -x/2 + 6$. What is the area of the square?

    (A) 15
    (B) $10\sqrt{3}$
    (C) 20
    (D) $10\sqrt{5}$
    (E) 25
    (F) None of the above

15. Suppose that $x + y = 14$ and $8^x - 16^y = 0$. Find $x^2 + y^2$.

    (A) 98
    (B) 100
    (C) 116
    (D) 130
    (E) 148
    (F) None of the above

16. A sequence $a_1, a_2, \ldots$ is defined by setting $a_1 = 1$ and requiring that

$$a_n = 1 + \frac{1}{a_{n-1}} \quad \text{for} \quad n \geq 1.$$

For example, $a_2 = 2, a_3 = 3/2$, and so on. What is a_{10}?

(A) 2/5
(B) 8/5
(C) 10/9
(D) 21/13
(E) 89/55
(F) None of the above

17. Find the largest integer k with the property that 2^k is a divisor of the number

$$(8^2 - 6^2)(7^2 - 5^2) \cdots (4^2 - 2^2)(3^2 - 1^2).$$

(A) 6
(B) 10
(C) 12
(D) 16
(E) 32
(F) None of the above

18. Simplify the expression

$$\frac{1}{\sqrt{0} + \sqrt{1}} + \frac{1}{\sqrt{1} + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \cdots + \frac{1}{\sqrt{2018} + \sqrt{2019}}.$$

(A) 2018
(B) 2019
(C) $\sqrt{2019} - \sqrt{2018}$
(D) $\sqrt{2018}$
(E) $\sqrt{2019}$
(F) None of the above

19. Find the area of the triangle bounded by the three lines with equations $y = x$, $y = 1+x/2$, and $y = 9 - x/2$.

    (A) 2
    (B) 4
    (C) 6
    (D) 8
    (E) 10
    (F) None of the above

20. The sum of two real numbers x and y is 7 and their product is 5. Find the value of x^2+y^2.

    (A) 39
    (B) 40
    (C) 41
    (D) 42
    (E) 43
    (F) None of the above

21. If two real numbers $x > y$ have sum 10 and product 20, what is the value of $2x/y$?

    (A) $3 - \sqrt{5}$
    (B) $5 - \sqrt{5}$
    (C) $3 + \sqrt{5}$
    (D) $5 + \sqrt{5}$
    (E) $5 + \sqrt{5}$
    (F) None of the above

22. In the land of Jyok, there are three units of currency: the Blok, the Clok, and the Dlok. You are told that 2 Bloks and 5 Cloks is worth 52 Dloks, while 3 Bloks and 4 Cloks is worth 57 Dloks. How much is 4 Bloks and 3 Cloks worth?

    (A) 54 Dloks
    (B) 56 Dloks
    (C) 58 Dloks
    (D) 60 Dloks
    (E) 62 Dloks
    (F) None of the above

23. Find the y-intercept of the line which is perpendicular to the line $y = 2x$ and which touches the parabola $y = -x^2 + (3/2)x - 3$ at exactly one point.

(A) -2
(B) -1
(C) 0
(D) 1
(E) 2
(F) None of the above

24. Suppose that $y - x = 4$ and $4^x + 2^y = 192$. Find the value of $x^2 + y^2$.

(A) 16
(B) 26
(C) 40
(D) 58
(E) 80
(F) None of the above

25. How many distinct real numbers x satisfy the equation

$$x^4 - x^2 - 1 = 0?$$

(A) 0
(B) 1
(C) 2
(D) 3
(E) 4
(F) None of the above

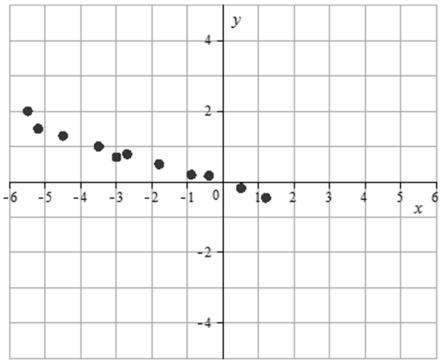
26. The sum of three consecutive odd integers is 381. If a and b represent the largest and smallest of the three integers, respectively, find the value of $5a - 3b$.

(A) 270
(B) 238
(C) 262
(D) 246
(E) 268
(F) None of the above

27. The stopping distance of a car varies directly as the square of the car's speed. If a car takes 240 feet to stop when traveling at 40 miles per hour, what is the stopping distance, in feet, when traveling at 60 miles per hour?

(A) $106.\overline{6}$
(B) 600
(C) 260
(D) 540
(E) 360
(F) None of the above

28. A scatter plot is shown below. Which of the following describes the linear correlation coefficient?

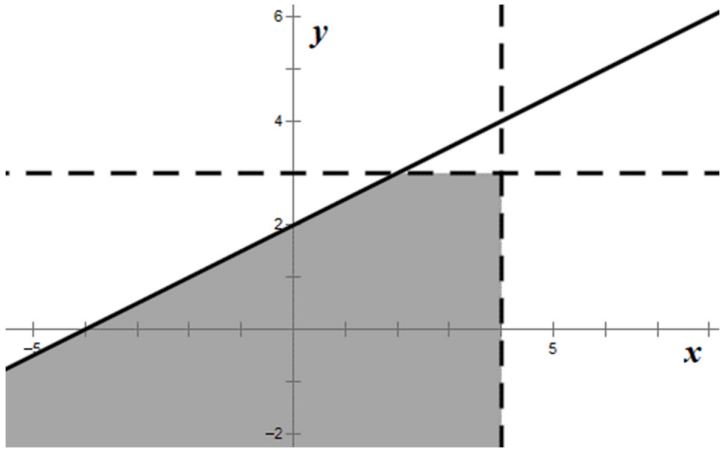


(A) Strong positive correlation
(B) Strong negative correlation
(C) Correlation close to zero
(D) Weak positive correlation
(E) Weak negative correlation
(F) None of the above.

29. The width of a rectangle is 12 less than twice its length, and the perimeter of the rectangle is five times the width. Find the length of the rectangle's diagonal.

(A) 9
(B) 15
(C) $3\sqrt{13}$
(D) 58.5
(E) $\dfrac{12\sqrt{17}}{7}$
(F) None of these

30. Write the system of inequalities that corresponds to the following graph.



(A) $x<3,\quad y<4,\quad 2y-x\le 4$

(B) $x<4,\quad y<3,\quad y\le 2x+2$

(C) $x<3,\quad y<4,\quad x+2y\le 4$

(D) $x<4,\quad y<3,\quad x-2y\ge -4$

(E) $x<4,\quad y<3,\quad 2y-x<4$

(F) None of these

31. Simplify: $\left(\sqrt{3}+\sqrt{5}+\sqrt{7}\right)\left(\sqrt{3}+\sqrt{5}-\sqrt{7}\right)\left(\sqrt{3}-\sqrt{5}+\sqrt{7}\right)\left(-\sqrt{3}+\sqrt{5}+\sqrt{7}\right)$

(A) 61

(B) 9

(C) 59

(D) $15\sqrt{3}$

(E) 29

(F) None of the above

32. Find the domain and range of $f(x)=-2x^2+12x-11$.

(A) Domain: $-\infty<x<\infty$; Range: $y\le -20$

(B) Domain: $x\ge 3$; Range: $y\le 7$

(C) Domain: $-\infty<x<\infty$; Range: $y\le 7$

(D) Domain: $x\le 7$; Range: $-\infty<y<\infty$

(E) Domain: $-\infty<x<\infty$; Range: $y\le -11$

(F) None of the above

33. Let A and B represent the solutions to $20x^2 - 9x - 18 = 0$, where $A > B$. Find the value of $7A + 3B$.

(A) $-87/10$

(B) $-177/20$

(C) $-3/10$

(D) $177/20$

(E) $11/6$

(F) None of the above

34. A certain chemical compound decays at a rate of R% per year. If 4000 units of a substance decays to 2560 units in two years time, find the yearly rate of decay.

(A) 80%

(B) 32%

(C) 20%

(D) 64%

(E) 40%

(F) None of the above

35. Simplify the following expression: $\left(\dfrac{2a^4b^{-2}c^6}{3a^{-2}c^2b^8}\right)^{-\frac{1}{2}}$

(A) $\dfrac{\sqrt{6}\cdot a^3c^2}{3b^5}$

(B) $\dfrac{9a^4}{4b^8c^6}$

(C) $\dfrac{a\sqrt{6c}}{2b^2c^2}$

(D) $\dfrac{\sqrt{6}\cdot b^5}{2a^3c^2}$

(E) $-\dfrac{a^6b^4}{3b^{10}}$

(F) None of the above

36. The following sequence is arithmetic: $x+y,\ \ 5x+2y,\ \ 3x-y,\ \ 6x-2$
Find the 23rd term.

(A) 111
(B) −124
(C) 67
(D) −109
(E) 51
(F) None of the above

37. The fourth term of a geometric sequence is 21 and the ninth term is 5103. Find the sum of the first three terms of the sequence.

(A) 7/4
(B) −6035.4
(C) 91/9
(D) −14
(E) 35/9
(F) None of the above

38. Smithville and Jonesville are 840 miles apart form each other. Train A leaves Smithville at 7:30 a.m. and begins traveling toward Jonesville. At 9:00 a.m., Train B leaves Jonesville and begins traveling toward Smithville. If Train A is traveling at 64 miles per hour and Train B is traveling at 80 miles per hour, at what time will the trains meet?

(A) 2:10 p.m.
(B) 12:36 p.m.
(C) 10:30 a.m.
(D) 12:40 p.m.
(E) 2:06 p.m.
(F) None of the above