University of Houston Mathematics Contest 2019

Algebra 2 Exam

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1. A rectangle is inscribed in a circle. If we sum the squares of the lengths of each edge of the rectangle we get 40. Find the radius of the circle.

   (A) $\sqrt{3}$ (B) $2\sqrt{3}$ (C) $\sqrt{5}$ (D) $2\sqrt{5}$ (E) $3\sqrt{2}$ (F) $6\sqrt{2}$

2. Solve for x in the equation below.

$$10^{-\log_2(\log_{10}(8x^3))} = \frac{1}{100}$$

   (A) $2\sqrt[3]{2}$ (B) $2\sqrt[3]{10}$ (C) $5\sqrt[3]{2}$ (D) $5\sqrt[3]{10}$ (E) $10\sqrt[3]{2}$ (F) $10\sqrt[3]{10}$

3. Suppose that $2^x = 3$. Find 8^{x+1}.

   (A) 15 (B) 27 (C) 54 (D) 108 (E) 144 (F) 216

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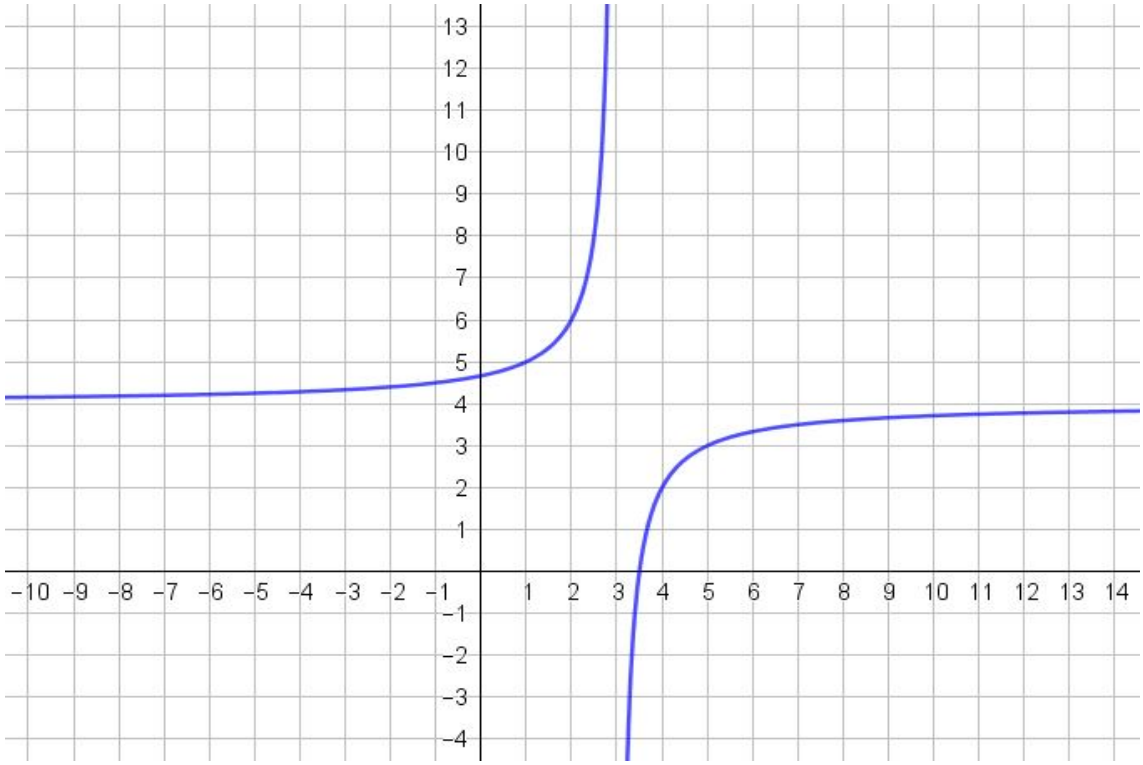
4. If $f(x) = \dfrac{1}{x}$ and the graph of $af(x+b)+c$ is shown below, find the value of $a+b+c$.



(A) -1 (B) 0 (C) 1 (D) 2 (E) 3 (F) 4

5. The complex number below can be written in standard form, meaning $a+bi$. Find the value of $a-b$.

$$\frac{11+10i}{1+4i}$$

(A) -5 (B) -3 (C) -1 (D) 1 (E) 3 (F) 5

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6. If the solution of the equation

$$-5x - |3 - x| = |1 - 3x| - 2$$

is written in the form $x = \frac{a}{b}$ where $b > 0$ and $\frac{a}{b}$ is a reduced fraction in lowest terms, find $a + b$.

(A) -3 (B) -2 (C) -1 (D) 1 (E) 2 (F) 3

7. A rectangle has the property that when the length and width are each increased by 4 inches the area of the rectangle is made larger by 46 square inches. Determine the perimeter of this rectangle in inches.

(A) 10 (B) 15 (C) 20 (D) 30 (E) 35 (F) 40

8. Solve $\sqrt{3 - x} + x = 2$. If there are multiple solutions, select the largest.

(A) $\frac{5}{2} - \frac{\sqrt{21}}{2}$ (B) $\frac{5}{2} - \frac{\sqrt{53}}{2}$ (C) $\frac{3}{2} - \frac{\sqrt{5}}{2}$

(D) $\frac{5}{2} + \frac{\sqrt{21}}{2}$ (E) $\frac{5}{2} + \frac{\sqrt{53}}{2}$ (F) $\frac{3}{2} + \frac{\sqrt{5}}{2}$

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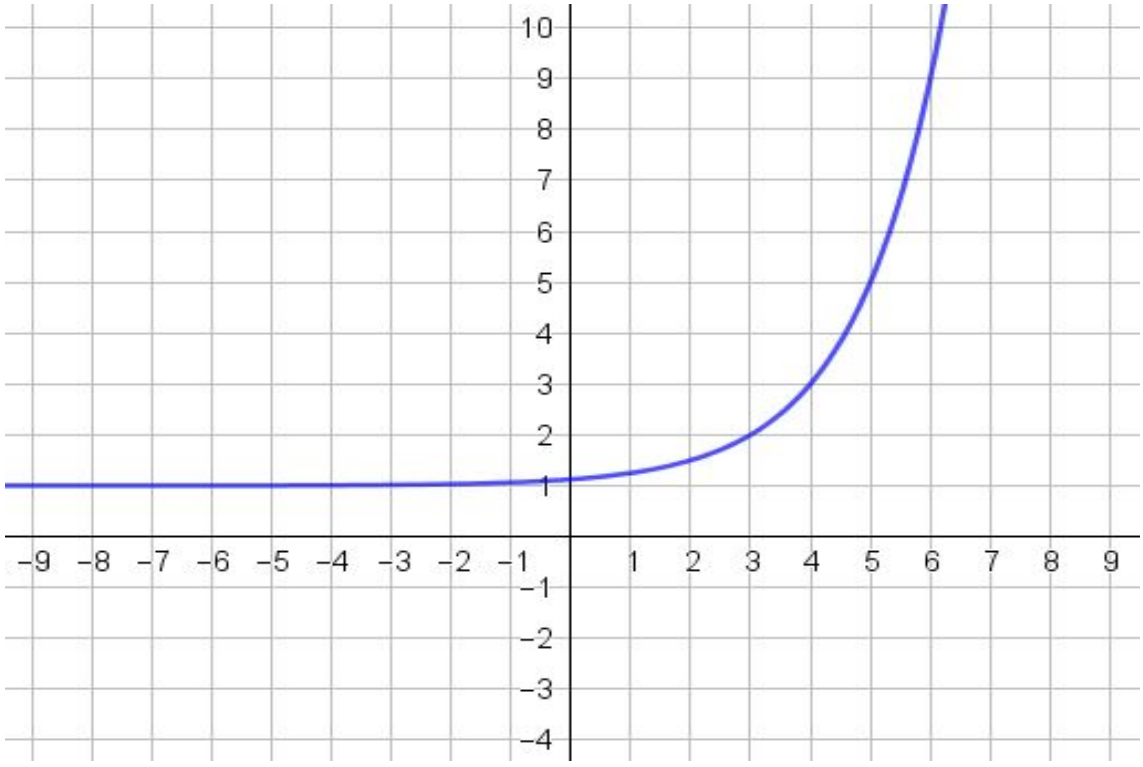
9. The graph of $f^{-1}(x)$ is shown below. Find the domain of the function $g(x)$ defined by $g(x) = f(4 - x)$.



(A) $(0, \infty)$ (B) $(3, \infty)$ (C) $(5, \infty)$

(D) $(-\infty, 0)$ (E) $(-\infty, 3)$ (F) $(-\infty, 5)$

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10. Given the function $P(x) = 2x^4 + 5x^3 - 8x^2 - 17x - 6$ and the fact that $P(-1) = 0$, determine the sum of the roots of this polynomial.

(A) -5 (B) -3 (C) $-\frac{5}{2}$ (D) $-\frac{3}{2}$ (E) $\frac{1}{2}$ (F) $\frac{3}{2}$

11. A function f is defined on the set of positive integers as follows.

$$f(n) = \begin{cases} n+3 & \text{if } n \text{ is odd} \\ \\ \dfrac{n}{2} & \text{if } n \text{ is even} \end{cases}$$

Suppose k is odd and $f(f(f(k))) = 24$. What is the maximum value for the sum of the digits of k?

(A) 11 (B) 12 (C) 13 (D) 14 (E) 15 (F) 16

12. Simplify the expression below.

$$\sqrt{7 - \sqrt{7 - \sqrt{7 - \sqrt{7 - \cdots}}}}$$

(A) $\frac{1}{2} - \frac{\sqrt{29}}{2}$ (B) $-\frac{1}{2} - \frac{\sqrt{29}}{2}$ (C) $\frac{1}{2} - \frac{\sqrt{6}}{2}$

(D) $\frac{1}{2} + \frac{\sqrt{29}}{2}$ (E) $-\frac{1}{2} + \frac{\sqrt{29}}{2}$ (F) $\frac{1}{2} + \frac{\sqrt{6}}{2}$

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13. The inequality below has integer solutions. How many of these are even numbers?

$$8 < |3x - 2| \leq 31$$

(A) 3 (B) 4 (C) 5 (D) 6 (E) 7 (F) 8

14. Tom pours b two-liter bottles of grape soda into a large container. The soda contains $b\%$ pure grape juice. How many liters of grape juice must be added to the soda in order to quadruple the percentage of grape juice?

(A) $\dfrac{b^2}{100 - 4b}$ (B) $\dfrac{2b^2}{100 - 3b}$ (C) $\dfrac{3b^2}{50 - 2b}$

(D) $\dfrac{3b^2}{100 - 4b}$ (E) $\dfrac{b^2}{50 + 2b}$ (F) $\dfrac{2b^2}{100 + 3b}$

15. Becky and Brian are selling baked goods to raise money for a campus organization. Mr. Garcia purchases 3 cookies, 4 brownies, and 2 cakes for \$49. Mrs. Smith purchases 4 cookies, 2 brownies, and 3 cakes for \$55. Mr. Jones purchases 5 cookies, 3 brownies, and 1 cake for \$46. What is the price of a cake?

(A) \$4 (B) \$5 (C) \$6 (D) \$7 (E) \$8 (F) \$9

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16. Find the y-coordinate of the vertex of the parabola whose graph is given by the quadratic function below.

$$f(x) = -(2x+3)^2 + 18(2x+3) - 76$$

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6 (F) 7

17. It has been suggested that the number of participants in a new club t weeks after creation can be modeled by the function below.

$$P(t) = cb^t$$

Here c and b are positive real numbers. The model says 6 students will be members after 1 week and 96 members will be enrolled 4 weeks after that. At what time t will the club have 24 more members than they had at the club's inception? Express your answer in exact form.

(A) $2\log_2 12$ (B) 2 (C) $\log_2 24$

(D) $2\log_2 3$ (E) 3 (F) $\log_2 27$

18. Find the sum of all solutions of the following equation.

$$\frac{x+2}{x^2-12x+35} + \frac{x+5}{x^2-10x+21} + \frac{x+2}{x^2-8x+15} = 0$$

(A) -3 (B) -2 (C) -1 (D) 0 (E) 1 (F) 2

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19. Find $3x + 5y$, where x and y satisfy the given pair of equations.

$$2x + 4y = 16$$

$$4x^2 - 16y^2 = 768$$

(A) 26 (B) 28 (C) 30 (D) 32 (E) 34 (F) 36

20. Evaluate and simplify the expression below.

$$\left(\frac{1}{3} - \frac{\sqrt{2}}{3}i\right)\left(\frac{\sqrt{2}}{3} + \frac{1}{3}i\right)\left(-\frac{1}{3} + \frac{\sqrt{2}}{3}i\right)\left(-\frac{\sqrt{2}}{3} - \frac{1}{3}i\right)$$

(A) 1 (B) $\frac{4\sqrt{2}}{81} + \frac{7}{81}i$ (C) $-\frac{7}{81} + \frac{4\sqrt{2}}{81}i$

(D) i (E) $-\frac{4\sqrt{2}}{81} - \frac{7}{81}i$ (F) $\frac{7}{81} - \frac{4\sqrt{2}}{81}i$

21. Find the largest real number x which solves the given equation.

$$\sqrt[3]{x^4 - x^3 + 8x^2 - 12x - 7} = 2 - x$$

(A) 1 (B) $\sqrt{2}$ (C) $\sqrt{3}$ (D) $\sqrt{5}$ (E) $\sqrt{6}$ (F) $\sqrt{7}$

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22. If the expression

$$\frac{\sqrt{x\sqrt[3]{y\sqrt{z}}}}{\sqrt[3]{z\sqrt[3]{y\sqrt{x}}}}$$

is written in the form $x^a y^b z^c$, find the value of $\log_2\left(\frac{1}{a+b+c}\right)$.

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2 (F) 3

23. Determine the value of c which yields exactly 3 distinct solutions for x.

$$-|4-2|x+1|| = -2c$$

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2 (F) 3

24. A second degree polynomial of the form $p(x) = ax^2+bx+c$ has remainder 12 when divided by $x+1$, remainder 9 when divided by $x-2$, and remainder 20 when divided by $x-3$. Determine the value of $a+b+c$.

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5 (F) 6

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25. The table below is called a magic square. If we sum the numbers in any row, column, or diagonal we get the same value. For the given magic square, find the value of E.

19	A	14
10	B	C
D	E	11

(A) 15 (B) 16 (C) 17 (D) 18 (E) 19 (F) 20

26. Given the equation below, which of the following is a valid solution for x?

$$2^{(\log_2(x))^2} + 2x^{\log_2(x)} = 48$$

(A) $\frac{1}{32}$ (B) $\frac{1}{16}$ (C) $\frac{1}{8}$ (D) $\frac{1}{4}$ (E) $\frac{1}{2}$ (F) 1

27. Suppose that $\ln\left(3a^3b\right) = 2x$ and $\ln\left(9ab^2\right) = y$. Find $\ln(a)$ in terms of x and y.

(A) $\frac{2x - y + 2\ln(3)}{6}$ (B) $\frac{2x - y}{6}$ (C) $\frac{2x - y}{5}$

(D) $\frac{4x - y - 4\ln(3)}{5}$ (E) $\frac{4x - y}{6}$ (F) $\frac{4x - y}{5}$

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28. Anna has two jobs after school. She makes \$5 per hour as a babysitter and she makes \$8 per hour cleaning houses. She needs to make at least \$93 per week to pay her bills, but cannot work more than 15 hours in a week. Which of the following is not true?

(A) More than \$45 must come from cleaning each week.

(B) If Anna works more than 10 hours babysitting, she cannot pay her bills.

(C) The number of hours spent babysitting multiplied by 6 plus 7 times the number of hours she works cleaning must be less than 116.

(D) If her babysitting hours plus half her cleaning hours exceeds 7, then Anna will not be able to pay her bills.

(E) It is possible for Anna to pay her bills in a week where she spends more time babysitting than cleaning.

(F) It is not acceptable for Anna to babysit 5 more hours than she spends cleaning in a given week.

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29. For any positive integer n, let $s(n)$ denote the sum of the digits. How many even integers solve the equation below?

$$n + 5s(n) = 2046$$

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4 (F) 5

30. Given the inequality below, express the solution set in the form $\{x \mid a < x < b\}$ where a and b are real numbers.

$$|4x - 1| < |3x + 4| - |2x + 1|$$

Now determine the value of $a + b$.

(A) $\frac{2}{15}$ (B) $\frac{8}{15}$ (C) $\frac{14}{15}$ (D) $\frac{2}{9}$ (E) $\frac{8}{9}$ (F) $\frac{14}{9}$