UNIVERSITY OF HOUSTON MATHEMATICS CONTEST 2020

ALGEBRA I EXAM

1. If $\dfrac{1+2+3+\cdots+n}{n} = 8$, then find the value of n.

    A) 10
    B) 15
    C) 20
    D) 25
    E) 30

2. Arrange the following list of numbers in decreasing order, from left to right:

$$2^{600}, \quad 3^{500}, \quad 4^{400}, \quad 5^{300}$$

    A) $5^{300}, 4^{400}, 3^{500}, 2^{600}$
    B) $5^{300}, 4^{400}, 3^{500}, 2^{600}$
    C) $2^{600}, 3^{500}, 5^{300}, 4^{400}$
    D) $4^{400}, 3^{500}, 5^{300}, 2^{600}$
    E) $4^{400}, 2^{600}, 3^{500}, 5^{300}$

3. In a farm, there are 60 more chickens than ducks, and 5 times as many chickens as ducks. How many chickens are there?

    A) 50
    B) 75
    C) 100
    D) 125
    E) 65

4. On New Year's first day of 2016, Rui got a jar of jellybeans. On each day he ate the same number of jellybeans. He counted 560 on January 31 before eating any and he counted 380 on March 16 before eating any. Find the number of the jellybeans Rui had initially in the jar.

   A) 684
   B) 650
   C) 680
   D) 740
   E) 676

5. On the last history test, at least one student scored each of the grades A, B, C, D, and F. If 8 students got an A, 15 got a C or higher, 12 got a B or lower, and only one got a D, find the percentage of the students who got an F.

   A) 15%
   B) 20%
   C) 25%
   D) 10%
   E) 4%

6. Two vertices of a triangle are at A(1, 5) and B(5, 5) in the coordinate plane. The third vertex, C, is the translation of point B three units down and one unit left. What is the area of triangle ABC?

   A) 8
   B) 10
   C) 6
   D) 25
   E) 12

7. Sabine purchased some $7 baseballs and some $11 softballs. She paid a total of $215. What is the greatest number of softballs she could have purchased?

   A) 3
   B) 26
   C) 4
   D) 17
   E) 18

8. If two fair six-sided dice are rolled, what is the probability that the sum of the numbers rolled is divisible by 5?

   A) 1/9
   B) 7/36
   C) 1/36
   D) 5/36
   E) 2/9

9. If $x = \dfrac{1}{\sqrt{7}} \cdot \dfrac{11}{\sqrt{21}} - \dfrac{4}{\sqrt{7}} \cdot \dfrac{1}{\sqrt{21}}$, then find $\dfrac{1}{x^2}$.

   A) 1/3
   B) 3
   C) 1/21
   D) 21
   E) 7/3

10. The first six terms of an arithmetic sequence are 12, w, x, 33, y, z.
    What is the value of w + z?

    A) 66
    B) 47
    C) 45
    D) 59
    E) 26

11. If $\frac{3x^2-6}{2} = \sqrt{3},$ then the value of $\frac{50}{x^2-2}$ can be expressed in the simplest cardinal
    form as $a\sqrt{b}$. What is the value of ab?

    A) 150
    B) 100
    C) 200
    D) 75
    E) 25

12. If $a\ and\ b$ are positive integers and $a^2 + 6ab + 9b^2 = 100$, what is the sum of all
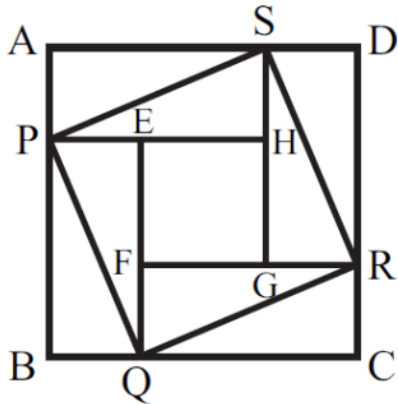   possible values of b?

    A) 6
    B) 12
    C) 10
    D) 7
    E) 11

13. In the figure, square ABCD is divided into eight congruent right triangles and square EFGH, as shown. If the area of square ABCD is $1156\ cm^2$ and the area of square PQRS is $676\ cm^2$, what is the area of square EFGH?

A) 480 B) 120 C) 556 D) 500 E) 196

14. The table shows some values of a linear function $f(x) = mx + b$, for real numbers $m\ and\ b$. What is the value of m?

x	$f(x)$
16	29
20	22
24	15
28	8

A) $-7/4$
B) $7/4$
C) $-4/7$
D) $4/7$
E) -5

15. Let f be a function such that $f(xy) = f(x)/y$ for all positive real numbers x and y.
If $f(500) = 3$, then evaluate $f(600)$.

A) 4
B) 2
C) 5/2
D) 1/2
E) 3/2

16. Michelle assigns every letter of the alphabet a different positive integer value.
She finds the value of a word by multiplying the values of its letters together.
For example, if D has a value of 5, and A has a value of 11, then the word DAD
has a value of 5 × 11 × 5 = 275. The table shows the value of some words.
What is the value of the word MATH?

Word	Value
TOTE	18
TEAM	168
MOM	49
HOME	70
MATH	?

A) 19   B) 190   C) 420   D) 84   E) 840

17. If $3^p + 3^4 = 90$, $2^r + 44 = 76$ and $5^3 + 6^s = 1421$, find the product $p \cdot r \cdot s$.

A) 27
B) 40
C) 50
D) 90
E) 70

18. Find the area of the triangle formed by the lines

$$y = 2, \qquad y = 5 + x, \quad and \quad y = 5 - x.$$

   A) 9
   B) 10
   C) 9/2
   D) 5
   E) 18

19. There are 81 grid points (uniformly spaced) in the square shown in the diagram below, including the points on the edges. Point P is the center of the square. Given that point Q is randomly chosen from among the other 80 points, what is the probability that line $\overline{PQ}$ is a line of symmetry for the square?
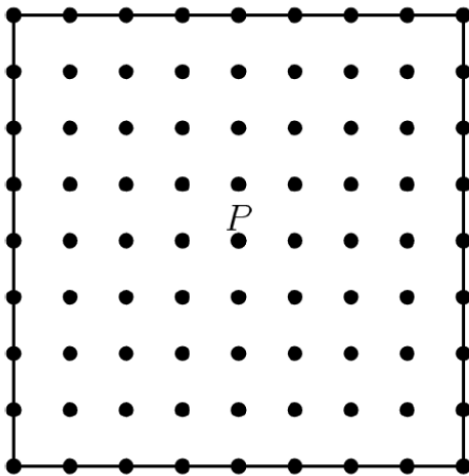
   A) 1/5   B) 1/4   C) 2/5   D) 9/20   E) 1/2

20. Find the greatest prime factor for $27^6 - 9^{16}$.

   A) 19
   B) 7
   C) 31
   D) 5
   E) 17

21. Find the greatest value of n, where 5^n is a factor of the sum $98! + 99! + 100!$.

   A) 26
   B) 25
   C) 24
   D) 23
   E) 22

22. How many different real numbers satisfy the following equation:

$$(x^2 - 5)^2 = 16$$

   A) 0
   B) 1
   C) 2
   D) 3
   E) 4

23. Suppose a, b and c are nonzero real numbers, and $a + b + c = 0$. What are the possible value(s) for

$$\frac{a}{\sqrt{a^2}} + \frac{b}{\sqrt{b^2}} + \frac{c}{\sqrt{c^2}} + \frac{abc}{\sqrt{a^2 b^2 c^2}} \ ?$$

   A) -1 and 1 B) 0 C) $-1, 0$ and 1 D) -2 and 2 E) $-2, 0$ and 2

24. For which value(s) of c does the quadratic function $f(x) = x^2 - 2bx + c$ have a minimum value of 6, given that $|b| \geq 3$?

A) $c \geq 0$ B) $c \geq 1$ C) $6 \leq c \leq 15$ D) $c \geq 9$ E) $c \geq 15$

25. Let $f(x) = a(x + b)(x - 8)$ represent a quadratic function, where a, b are constants. If the function is graphed in the xy-plane, its vertex is $(2, -54)$. Find the value of $a + b$.

A) 4
B) 11/2
C) 6
D) 15/2
E) 5

26. Suppose x is a solution to the quadratic equation $x^2 + 1 = 7x$. What is the sum of x and its reciprocal?

A) $7 + 3\sqrt{5}$
B) $7 - 3\sqrt{5}$
C) 14
D) 7
E) $6\sqrt{5}$

27. The graph shown below is that of the linear function that relates the value V $(in \, \$)$ of a car to its age t, where t is the number of years after 2000. What will be the value of the car in the year 2010?
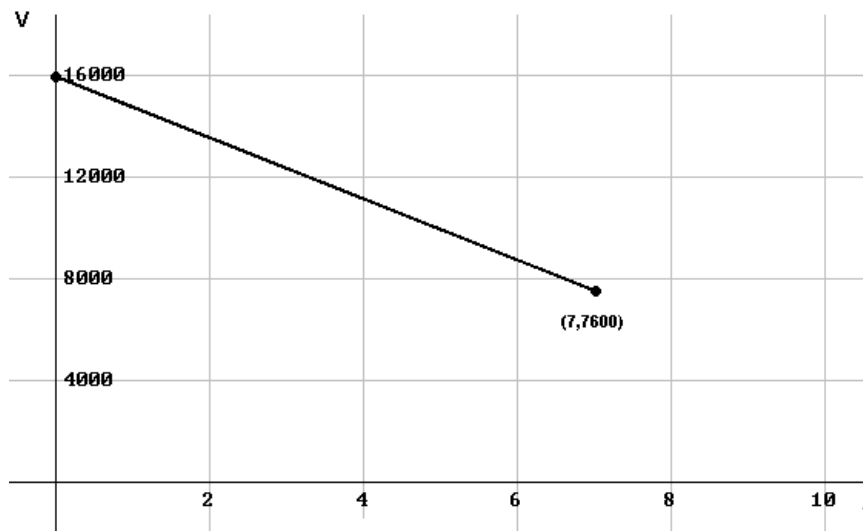

A) 4000 B) 5200 C) 6400 D) 7200 E) 3000

28. The wall clock in Eva's house, which is not accurate, loses time at a constant rate. One day before doing her homework, she notes that her wall clock and her watch (which is accurate) both say 10:00 am. When she finishes her homework, her watch says 10:40 and her wall clock says 10:35. Later that day, Eva loses her watch. She looks at her wall clock and it says 5:00pm. What is the actual time?

A) $5:50pm$ B) $5:55pm$ C) $6:00pm$ D) $6:30pm$ E) $4:35pm$

29. A tortoise challenges a hare to a race. The hare eagerly agrees and quickly runs ahead, leaving the slow-moving tortoise behind. Confident that he will win, the hare stops to take a nap. Meanwhile, the tortoise walks at a slow steady pace for the entire race. The hare awakes and runs to the finish line, only to find the tortoise already there. Which of the following graphs matches the description of the race, showing the distance d traveled by the two animals over time t from start to finish?

(A)

(B)

(C)

(D)

(E)
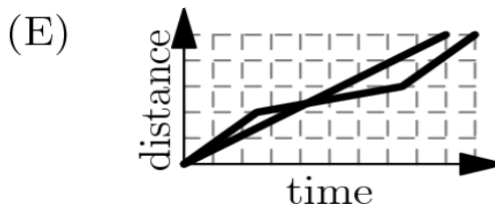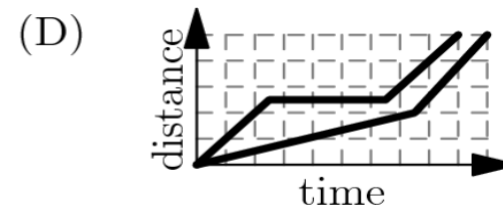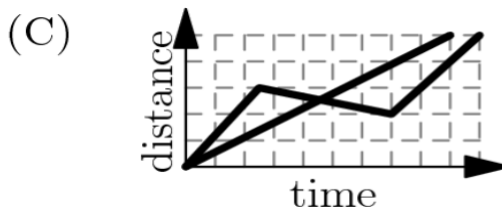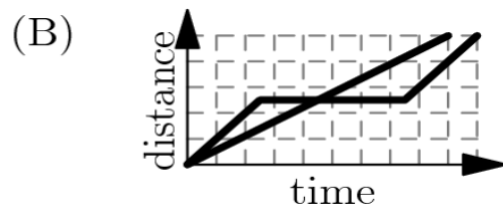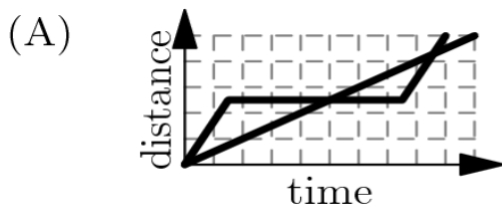
30. Bella begins to walk from her house toward her friend Ella's house. At the same time, Ella begins to ride her scooter toward Bella's house. They each maintain a constant speed, and Ella rides 4 times as fast as Bella walks. The distance between their houses is 2 miles, which is $10{,}560$ feet, and Bella covers $2\frac{3}{4}$ feet with each step. How many steps will Bella take by the time she meets Ella?

A) 704 B) 745 C) 768 D) 867 E) 1056