University of Houston Mathematics Contest 2020

Algebra 2 Exam

Name _____   School: _____

1. Solve for x in the equation below.

$$5^{\log_3(\log_2(27x^3))} = 25$$

(A) $2\sqrt[3]{2}/3$ (B) $\sqrt[3]{2}/3$ (C) $2/3$ (D) $8/3$ (E) 1 (F) $1/\sqrt[3]{3}$

2. Suppose that $5^x = 2$. Find 125^{x+1}.

(A) 1000   (B) 500   (C) 250   (D) 108   (E) 625   (F) 216

3. Given the function $P(x) = x^4 + x^3 - 17x^2 + 15x$ and the fact that $P(1) = 0$, determine the sum of the roots of this polynomial.

(A) 17   (B) -3 (C) -1 (D) 1   (E) $\dfrac{1}{2}$ (F) 5

4. The complex number below can be written in standard form, meaning $a + bi$. Find the value of $a + b$.

$$\frac{3 + 2i}{1 - 5i}$$

(A) -5 (B) $10/26$ (C) $24/26$ (D) $10/24$ (E) $5/2$ (F) $30/26$

5. Solve $x + 5 = \sqrt{2 - x}$.

(A) $\dfrac{-11}{2} - \dfrac{\sqrt{29}}{2}$ (B) $\dfrac{-11}{2} + \dfrac{\sqrt{53}}{2}$ (C) $\dfrac{-11}{2} + \dfrac{\sqrt{23}}{2}$

(D) $\dfrac{-11}{2} + \dfrac{\sqrt{29}}{2}$ (E) $\dfrac{5}{2} + \dfrac{\sqrt{53}}{2}$ (F) $\dfrac{3}{2} + \dfrac{\sqrt{5}}{2}$

6. The inequality below has positive integer solutions. How many of these are even numbers?
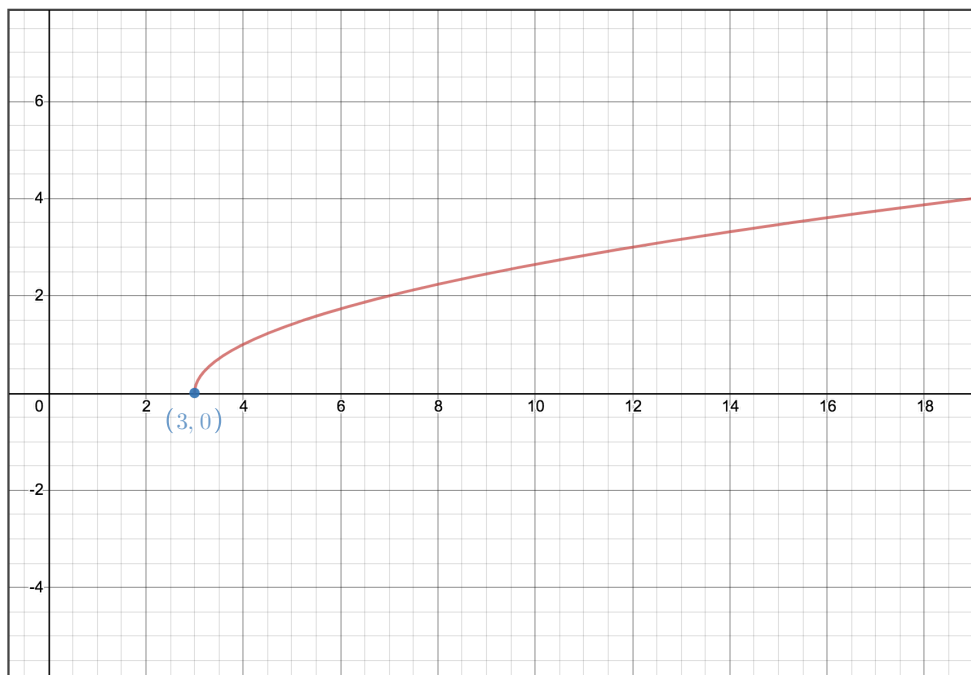
$$5 < |2x - 7| < 13$$

(A) 2   (B) 1   (C) 5   (D) 7   (E) 6   (F) 8

7. The graph of $f(x)$ is shown below. Define $g(x) = f(x) + 2$. Find the domain of the function $g^{-1}(x)$.

(A) $[0, \infty)$

(B) $[3, \infty)$

(C) $[2, \infty)$

(D) $(-\infty, \infty)$

(E) $(-\infty, 0]$

(F) $(-\infty, 5]$

8. Evaluate and simplify the expression below.

$$\left(\frac{i}{5} - \frac{\sqrt{3}}{5}\right)\left(\frac{1}{5} + \frac{\sqrt{3}}{5}i\right)\left(\frac{1}{5} - \frac{\sqrt{3}}{5}i\right)\left(-\frac{1}{5} - \frac{\sqrt{3}}{5}i\right)$$

(A) 1

(B) $\dfrac{10i}{625}(8 + 2\sqrt{3}i)$

(C) $\dfrac{10i}{625}(8 - 2\sqrt{3}i)$

(D) i

(E) $\dfrac{8i}{625} + \dfrac{8\sqrt{3}}{625}$

(F) $\dfrac{8i}{625} - \dfrac{8\sqrt{3}}{625}$

9. If the expression

$$\frac{\sqrt[3]{z\sqrt{y\sqrt[3]{x}}}}{\sqrt{y\sqrt{z\sqrt{x}}}}$$

is written in the form $x^a y^b z^c$, find the value of $\dfrac{1}{a+b+c}$.

(A) $-72/5$ (B) $-72/23$ (C) $25/72$ (D) 1   (E) $1/12$ (F) $5/72$

10. The table below is called a magic square. If we sum the numbers in any row, column, or diagonal we get the same value. For the given magic square, find the value of E.

2	A	6
9	B	C
D	E	8

(A) 3   (B) 10   (C) 6   (D) 5   (E) 4   (F) 7

11. Given $f(x) = x^2 - 3$ and $g(x) = 3x - 2$ find the value of $f(4) - 3g^{-1}(-1)$.

(A) 28   (B) 16   (C) 14   (D) 12   (E) 20   (F) 21

12. If $|x - 3| = p$ with $x < 3$ find $x - p$.

(A) $3 - 2p$ (B) 3 (C) -3 (D) $3 - p$ (E) $p - 3$ (F) $2p - 3$

13. Suppose the figure below represents the perimeter of a chicken farm. The numbers a and b represent the length of each of the corresponding sides in meters. Here x is an unknown value. If the perimeter of the farm can be written as $P = 6a + 4b$ meters, what is an expression for the area of the farm?
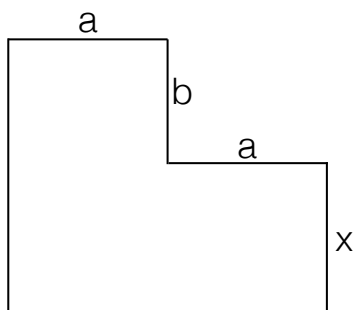
(A) $ab + a(a + b)$ (B) $a(a + 2b)$ (C) $2a(2a + b)$

(D) $a(2a + 3b)$ (E) $4ab + 2a^2$ (F) a(a +2b)

14. How many integers lie in the interval $[-2, 5]$ and the interval of solutions to the equation
$$|3 - |2 - x|| \leq 2x.$$

(A) 5   (B) 4   (C) 6   (D) 7   (E) 0   (F) 1

15. Ivan has two jobs after school. He works as a baby sitter for \$5 an hour and in a convenience store for \$8 an hour. He needs to make at least \$93 per week to pay for school supplies and his lunch, but he cannot work more than 15 hours a week. If b represents the number of hours he works baby sitting and c the number of hours he works at the convenience store, then which of the following inequalities is NOT true?

  (A) $b < 9$

  (B) $3b - 2c \leq 15$

  (C) $c - 5b \geq -39$

  (D) $b - c < 15$

  (E) $b + 2c < 13$

  (F) $c - 2b > -18$

16. What is the sum of all integers that are in the domain of the function

$$f(x) = \frac{\sqrt{2 - \log_3(x - 1)}}{x^2 - 4x - 12}$$

  (A) 55   (B) 49   (C) 48   (D) 54   (E) 44   (F) 10

17. Every morning, Rosa eats 20% of the m&m's that are in a jar in her kitchen. At the end of the second day she notices that there is only 32 left. How many m&m's where originally in the jar?

    (A) 50   (B) 40   (C) 54   (D) 45   (E) 800   (F) 160

18. How many positive integers b have the property that $\log_b 256$ is positive integer.

    (A) 0   (B) 1   (C) 2   (D) 3   (E) 4   (F) 5

19. Clara owns a parking lot where cars and motorcycles can park. She charges $1 a day for motorcycles and $2 a day for cars. One day she has 100 vehicles in her parking lot, which account for 326 wheels. How much money did she make that day?

    (A) 150   (B) 163   (C) 100   (D) 326   (E) 180   (F) 50

20. Evaluate $S = (9 + 4\sqrt{5})^{1/3} + (9 - 4\sqrt{5})^{1/3}$.

   (A) 9/2   (B) 3   (C) 3/2   (D) 18   (E) 18/5   (F) 6

21. Find the equation of the line that passes through the center of the circles

$$x^2 + y^2 - 6x - 4y = 12$$
$$x^2 + y^2 - 2x = 9$$

   (A) $y = 2/3x$ (B) $y = 5 - x$ (C) $y = 2$

   (D) $y = x + 1$ (E) $y = x - 1$ (F) $y = 2x - 1$

22. Suppose $\ln(10) = a$, $\ln(15) = b$ and $\ln(6) = c$. Then we can write $\ln(250)$ as

   (A) $b + c - 2a$ (B) $2b - c + 2a$ (C) $b - c + 2a$
   (D) $b + c - a$ (E) $2a - b + c$ (F) $3a + b + c$

23. Let $i = \sqrt{-1}$. Choose the expression that is equivalent to

$$\left(\frac{1+i}{\sqrt{2}}\right)^{2018}$$

(A) i (B) 1   (C) 1/2   (D) $-i$ (E) -1 (F) $i/2$

24. Suppose $f(x+3) = \dfrac{\sqrt[3]{2x-16}}{2}$. Find $f^{-1}(1/2)$

(A) 17/2   (B) 11/2   (C) 9   (D) 8   (E) 9/2   (F) 15/2

25. Suppose

$$A = \cfrac{1}{1 - \cfrac{1}{1 - \cfrac{1}{1-x}}} \qquad\qquad B = \cfrac{1}{2 - \cfrac{2}{2 - \cfrac{1}{1+x}}}.$$

What is the sum of the two solutions to the equation $|AB| = 5$?

(A) 2   (B) 0   (C) 3/2   (D) 1/2   (E) 1   (F) -1

26. A colony of bacteria that grows in a petri dish doubles in size every 20 minutes. At 9am on a certain day the petri dish is covered completely with bacteria. At what time was only 6.25% of the dish covered?

   (A) 8am   (B) 7:40am   (C) 11am   (D) 8:20am   (E) 7:20am   (F) 7am

27. How far apart are the vertices of the parabolas $y = x^2 - 2x + 4$ and $x = y^2 + 4y + 3$?

   (A) $\sqrt{5}$ (B) $\sqrt{13}$ (C) $\sqrt{15}$ (D) $\sqrt{29}$ (E) 4   (F) 3

28. Let $f(x)$ be a linear function and $g(x)$ a quadratic function with only one x intercept. These functions intersect at the points $(5, 16)$ and $(-2, 9)$. Find the value of $f^{-1}(3) + g(2)$

   (A) 15   (B) 16   (C) -7 (D) -5 (E) -6 (F) 3

29. A 20% concentrate is to be mixed with a mixture having a concentration of 60% to obtain 80 lt. of a mixture with a concentration of 30%. How much of the 20% concentrate and the 60% concentrate are needed?

    (A) 60 lt. of 20% concentrate and 20 lt. of 60% concentrate.

    (B) 20 lt. of 20% concentrate and 60 lt. of 60% concentrate.

    (C) 24 lt. of 20% concentrate and 56 lt. of 60% concentrate.

    (D) 56 lt. of 20% concentrate and 24 lt. of 60% concentrate.

    (E) 30 lt. of 20% concentrate and 50 lt. of 60% concentrate.

    (F) Non of the above

30. Find the product abc if the numbers a, b, c satisfy the following system of equations.

$$a - 2b + 3c = -11$$
$$-a + 3b = 8$$
$$2a - 5b + 5c = -23$$

    (A) 12   (B) -24 (C) 9   (D) 24   (E) -6 (F) 6