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Calculator Exam – 2020 – Version A – 9:00am Exam

1. $f(x) = \frac{3}{2}x^3 - \frac{3}{4}x + 3$. Give $f(3.14)$.

2. The graphs of $f(x) = x^2 + 2x - 1$ and $g(x) = x + \frac{1}{f(x)}$ have 4 points of intersection. Give the sum of the x coordinates of these points.

3. Give the distance from the point $(-3,2)$ to the line $y = 2x - 7$.

4. Solve the system $\begin{cases} 31x + 23y = -12 \\ 43x - 29y = 17 \end{cases}$, and give the value of x.

5. The function $f(x) = x^3 + 16x + 12$ is invertible. Give $f^{-1}(33.21)$.

6. Give the smallest integer value of the function $f(x) = \frac{1}{6}x^4 - 7x^3 - 12x + 7$.

7. Let
$$f(x) = \frac{2x - 1}{x + 4}.$$
Give the 23rd value in the sequence $f(0), f(f(0)), f\left(f(f(0))\right), \ldots$

8. Give the average of the numbers
$$1, -\frac{2}{3}, \frac{4}{5}, -\frac{6}{7}, \frac{8}{9}, \ldots, \frac{100}{101}.$$

9. Give the number of positive solutions to
$$\frac{x}{12} - \cos(4x) = 1.$$

10. Give the sum of the reciprocals of the positive integer values that are smaller than 62,913, and are integer multiples of 5, 7, 11 or 13.

11. Let $p_0 = 4327$, and define
$$p_{n+1} = \frac{p_n}{2} + \frac{7}{2p_n}$$
for $n = 0,1,2,3$. Give p_3.

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12. Give the slope of the line of best least squares fit for the data $(-1,13)$, $(1,-2)$ and $(5,-31)$.

13. A triangle is formed by joining the vertices of the parabolas $y = x^2 - 3x + 7$, $y = -2x^2 - 3x + 2$ and $y = 4 + 15x - 3x^2$. Give the area of the triangle.

14. A point (x, y) is called an integer point if both x and y are integers. Give the number of integer points with positive prime x coordinates that lie strictly above the graph of $y = \frac{1}{2}x^2$, and strictly below the graph of $y = 61$.

15. Give the y-intercept of the line that passes through the point $(-2.1,3.2)$ and is perpendicular to the line that passes through the points $(3.2,7.1)$ and $(-4.3,13.8)$.

16. Give the obtuse angle of intersection (in radians) of the lines $2x - 7y = 13$ and $-13x + 2y = 7$.

17. Give the area of the intersection of the circular disk of radius 3 centered at (1,1) with the rectangle with diagonal vertices $(-3,2)$ and (6,0).

18. A number is written in base 2 as 1100110011. Give this number in base 10.

19. The function $f(x) = ax^2 + bx + c$ has a graph that passes through the points $(1.2,2.1)$, $(2.3,7.2)$ and $(4.2,-2.6)$. Give the maximum value of this function.

20. Give the sum of the positive integers less than 2020 that give a remainder of 3 when divided by 7.

21. $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \cdots + \frac{1}{999} =$

22. Determine the number of roots of the function

$$f(x) = 12\sin\big(15(x+5)\big) + \frac{x^2}{30} + \frac{x}{25}$$

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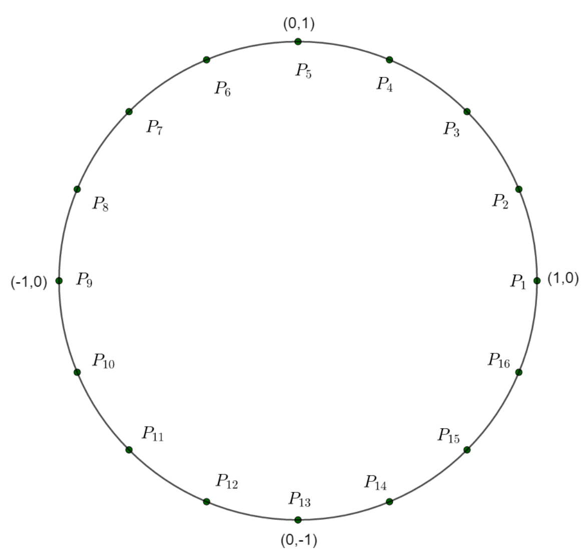
23. A particle moves in the direction of increasing x values, along the line $y = 63 - 2x$, starting from the point associated with $x = -17$, until it comes to a point on the line whose x coordinate is the first prime number. Then it changes direction and moves on a line of slope 1, until it reaches a point where the x coordinate is the next prime number. It then changed direction and moves on a line of slope -1 until it comes to a point where the x coordinate is the next prime number. It then changes direction and moves on a line of slope 1 until it comes to a point where the x coordinate is the next prime number. It then changes direction and moves on a line of slope -1 until it comes to a point where the x coordinate is the next prime number. This pattern continues until the x coordinate is 2020, and the particle stops. What is the total distance traveled by the particle?

24. Let C_1 be the circle of radius 1 centered at the origin, and let C_2 be the circle of radius 2 centered at the origin. 16 equally spaced points are placed on C_1, with the first point $P_1 = (1,0)$, and the other 15 points $P_2, \ldots, P_{16}$ ordered in such a way that they are placed in a counter clockwise fashion around the circle. The image below captures this information.



A similar process is used to create the points $Q_1, \ldots, Q_{16}$ on the circle C_2, starting with $Q_1 = (2,0)$. Give the sum of the absolute values of the x and y coordinates of these 32 points.

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25. Refer to the points created in the previous problem. Create the 16 line segments $\overline{Q_1P_2}$, $\overline{Q_2P_4}$, $\overline{Q_3P_6}$,…, $\overline{Q_8P_{16}}$, $\overline{Q_9P_2}$, $\overline{Q_{10}P_4}$, … , $\overline{Q_{16}P_{16}}$. Give the sum of the lengths of these line segments.

26. **Tie Breaker:** Give the average of the correct answers to problems 1-25. The closest answer to the actual answer breaks the tie.