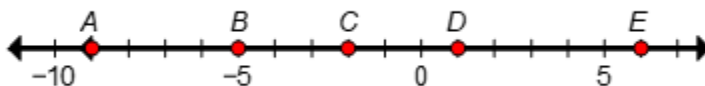


Geometry Exam - University of Houston Math Contest January 30, 2021

1) How many planes pass through any three points?

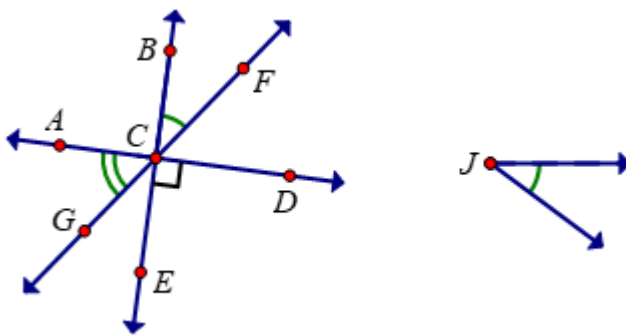
- a) At least one b) Exactly one c) An infinite number d) Exactly six e) Exactly three
f) None of the above

2) Find the distance between A and C using the number line below.



- a) -7 b) -8 c) 8 d) -6 e) 6 f) 7

3) Find $m\angle J$, given that $m\angle ACG = (5x + 2)^\circ$ and $m\angle FCD = (7x - 12)^\circ$.



- a) 143° b) 37° c) 53° d) 7° e) 127° f) None of the above

4) The complement of the supplement of an angle measures 67° . Find the measure of the angle.

- a) 147° b) 157° c) 113° d) 23° e) 33° f) None of the above

5) Which of the following words completes the following statement?

Congruence of angles is _____.

- I. reflexive II. symmetric III. transitive

- a) I and III only b) I only c) I and II only d) I, II and III e) II only
f) II and III only g) III only h) None of the above

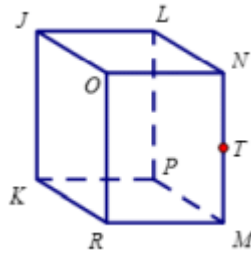
6) Given the following:

- $\angle 1$ and $\angle 2$ are complementary
- $\angle 3$ and $\angle 4$ are complementary
- $\angle 2 \cong \angle 3$

What conclusion can be drawn?

- a) $\angle 1 \cong \angle 3$
- b) $\angle 1 \cong \angle 4$
- c) $\angle 1$ and $\angle 4$ are supplementary
- d) $\angle 1$ and $\angle 4$ are complementary
- e) None of the above

7) Use the following diagram of a right rectangular prism to decide whether the given objects are parallel, skew, intersecting, or neither.

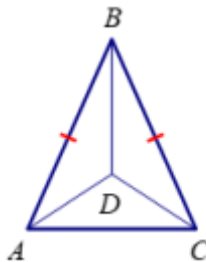


Give the answers to statements i, ii, iii, and iv, respectively.

- i. \overline{ON} and \overline{LP} are _____.
- ii. \overrightarrow{NT} and \overline{RM} are _____.
- iii. \overrightarrow{TN} and line LM are _____.
- iv. Line JK and \overrightarrow{NM} are _____.

- a) skew, neither, skew, skew
- b) skew, intersecting, neither, parallel
- c) skew, neither, neither, skew) intersecting, intersecting, skew, parallel
- e) intersecting, skew, skew, parallel
- f) intersecting, skew, neither, skew

8) In the diagram below, \overrightarrow{BD} and \overrightarrow{AD} are angle bisectors of $\triangle ABC$, and $m\angle BAC = 56^\circ$. Find $m\angle ADB$.



- a) 124°
- b) 28°
- c) 118°
- d) 34°
- e) 68°
- f) None of the above

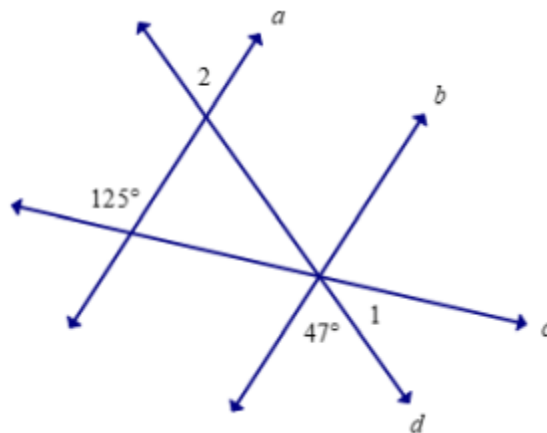
9) Chris is 5 feet, 6 inches tall. He is standing outside and his shadow is 15 feet long. A nearby tree casts a shadow that is 50 feet long. How tall is the tree, in feet?

- a) 237.6 b) $18\sqrt{3}$ c) 19.8 d) $15\sqrt{5}$ e) 220 f) None of the above

10) The letters $J, K, L, M, N,$ and O are used to label the six vertices of a convex hexagon (not necessarily in that order). Find the probability that \overline{KM} is a diagonal.

- a) $\frac{1}{2}$ b) $\frac{3}{5}$ c) $\frac{2}{3}$ d) 1 e) $\frac{4}{5}$ f) $\frac{5}{6}$

11) If $a \parallel b$ in the diagram below, find the value of $m\angle 1 - m\angle 2$.



- a) 47° b) 125° c) 78° d) 31° e) 70° f) None of these

12) $\triangle JKL$ is an isosceles triangle with base \overline{JL} . If $JL = 4$ yards and the perimeter of the triangle is 42 feet, find the length of \overline{KL} , in feet.

- a) 5 b) 15 c) 34 d) 18 e) 52 f) None of these

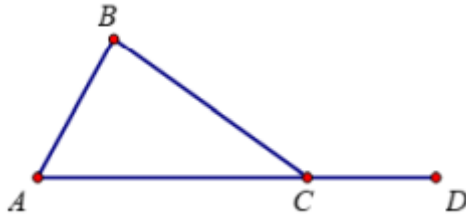
13) In $\triangle HJM$, $m\angle H = (3x + 18)^\circ$, $m\angle J = (5x - 10)^\circ$, and $m\angle M = (4x + 4)^\circ$.

A list of numbered statements is found below. Based on the given information about $\triangle HJM$, create a number using the numbers associated with the true statements. For example, if only 3, 5 and 7 are true, then your answer should be 357.

- | | |
|---|------------------------------------|
| 1. $\triangle HJM$ is an obtuse triangle. | 2. $\triangle HJM$ is equilateral. |
| 3. $\triangle HJM$ is an acute triangle. | 4. $\triangle HJM$ is equiangular. |
| 5. $\triangle HJM$ is a right triangle. | 6. $\triangle HJM$ is isosceles. |
| | 7. $\triangle HJM$ is scalene. |

- a) 57 b) 16 c) 15 d) 37 e) 2346 f) 24
 g) 17 h) 234 i) 56 j) 36 k) 246 l) 35

14) In the diagram below, $m\angle A = (2x + 35)^\circ$ and $m\angle BCD = (5x + 5)^\circ$.



Which of the following statements is true?

- a) $x \leq 34$ b) $x = 10$ c) $x = 20$ d) $10 < x < 35$ e) $11 < x < 34$
 f) There is not enough information to draw a conclusion.

15) Mrs. Jones instructed her geometry students to draw a quadrilateral so that the ratio of the measures of the angles, for four consecutive vertices, is $x : y : x : z$, where $x \neq y$ and $x \neq z$.

Alan drew a rectangle.

Betty drew a kite which contained no right angles.

Charlie drew a rhombus which contained no right angles.

Demi drew a square.

Eduardo drew a parallelogram which contained no right angles.

Fariba drew an isosceles trapezoid.

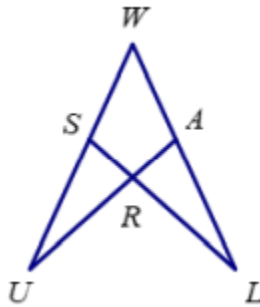
Who gets credit for following Mrs. Jones' instructions?

1. Alan 2. Betty 3. Charlie 4. Demi 5. Eduardo 6. Fariba

Create a number using the numbers associated with the true statements. For example, if only 1 and 3 are true, then your answer should be 13.

- a) 14 b) 2 c) 156 d) 123456 e) 6 f) 12345 g) 35 h) 146 i) 2356 j) 235

16) In the diagram below, $\overline{WS} \cong \overline{WA}$ and $\angle U \cong \angle L$. Which congruence theorem or postulate can be used to prove that $\triangle SWL \cong \triangle AWU$?



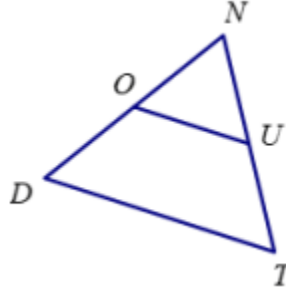
- a) Side-Side-Angle b) Angle-Side-Angle c) Angle-Angle-Side
 d) Side-Angle-Side e) Side-Side-Side f) The triangles are not necessarily congruent.

17) If \overline{WX} bisects \overline{UZ} at point C , which of the following statement(s) are true?

- I. $\overline{UX} \cong \overline{XZ}$ II. $UC = CZ$ III. $\angle UXC \cong \angle ZXC$

- a) I and II only b) I, II, and III c) III only d) I only
 e) II and III only f) II only g) I and III only h) None of the above

18) In the diagram below, \overline{OU} is a midsegment of $\triangle DNT$. If $DO = 3x + 1$, $DT = 8x + 3$, $UT = 2x + 4$, and the perimeter of $\triangle DNT$ is $20x + 5$, find the value of $DT + ON - NU$.



- a) $\frac{27}{7}$ b) 25 c) 45 d) 27 e) 36 f) None of the above

19) In $\triangle ELK$, $EL = 4$, $LK = 10$, and the length of \overline{EK} is a whole number. Find the sum of all possible values of EK .

- a) 85 b) 35 c) 49 d) 90 e) 70 f) None of the above

20) If all of the diagonals are drawn for a regular convex dodecagon (a 12-sided polygon) and one of the diagonals is randomly selected, what is the probability that the selected diagonal is on a line of symmetry?

- a) $\frac{2}{11}$ b) $\frac{1}{18}$ c) $\frac{4}{9}$ d) $\frac{2}{9}$ e) $\frac{1}{9}$ f) None of the above

21) The sum of the measures of all but one of the interior angles of a convex n -gon is 1541° . If the remaining angle measures x° , find the value of $2n + 3x$.

- a) 255 b) 259 c) 251 d) 289 e) 265 f) None of the above

22) Given the following conditional statement:

If a quadrilateral is a square, then it is a rectangle.

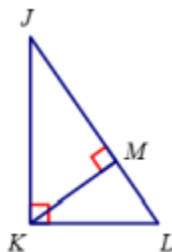
Determine the truth value of the inverse, contrapositive, and converse, respectively.

- a) False, False, False b) False, False, True c) True, True, False
 d) False, True, False e) True, True, True f) True, False, True

23) An isosceles trapezoid has base angles that measure 60° . If the perimeter of the trapezoid is 54 cm and the median of the trapezoid measures 17 cm, find the area of the trapezoid, in square centimeters.

- a) $85\sqrt{2}$ b) 85 c) $85\sqrt{3}$ d) $170\sqrt{3}$ e) 170 f) None of the above

24) In the diagram below, $JK = 2\sqrt{22}$ and $ML = 3$. Find KM .

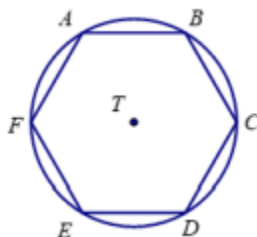


- a) $4\sqrt{6}$ b) $2\sqrt{6}$ c) 8 d) 12 e) 11 f) None of the above

25) Given quadrilateral $ABCD$ (not shown) with diagonal \overline{BD} , where $\overline{AB} \perp \overline{AD}$, $AB = AD = 7$, $BC = 5$, and the perimeter of $ABCD$ is 27. Classify $\triangle BCD$ according to its angles.

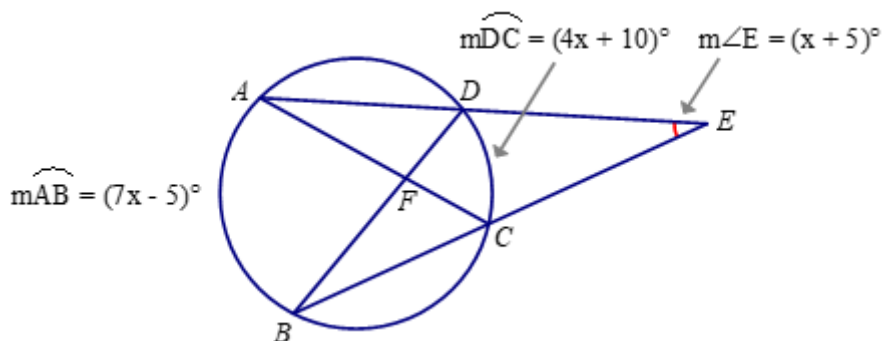
- a) Right b) Obtuse c) Acute d) There is not enough information to solve the problem.

26) Regular hexagon $ABCDEF$ is inscribed in circle T . If the length of \widehat{BC} is 4π cm, find the area of the hexagon, in square centimeters.



- a) $144\sqrt{3}$ b) $532\sqrt{3}$ c) $108\sqrt{3}$ d) $216\sqrt{3}$ e) $864\sqrt{3}$ f) None of the above

27) A circle is below with measures as shown. Find $m\angle AFB$.



- a) 110° b) 140° c) 25° d) 57.5° e) 85° f) None of the above

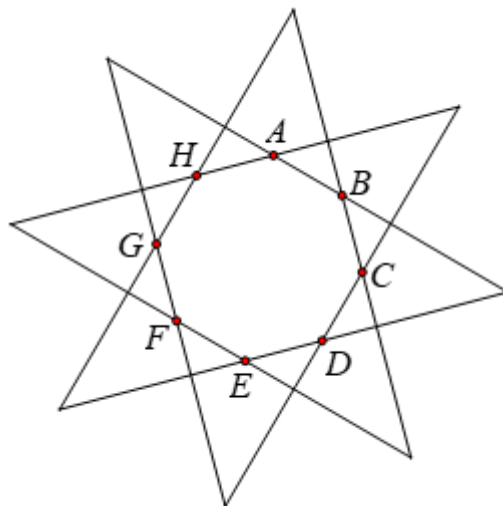
28) A right rectangular pyramid has base edges 18 cm and 10 cm. If the height of the pyramid is 12 cm, find its lateral area, in square centimeters.

- a) 672 b) 384 c) 420 d) 364 e) 720 f) None of the above

29) The volume of a spherical piece of dark chocolate is 288π . It is then sliced into two hemispheres, and one of these hemispheres is fully covered in a thin layer of white chocolate. What is the total surface area of the white chocolate coating?

- a) 288π b) 72π c) 216π d) 432π e) 108π f) None of the above

30) An eight-pointed star (known as an octagram) is shown below. Regular octagon $ABCDEFGH$ can be found in the middle of the diagram. The sides of the octagon are extended until they intersect to form the points of the star. If $AB = 10$ cm, find the area of the entire eight-pointed star, in square centimeters.

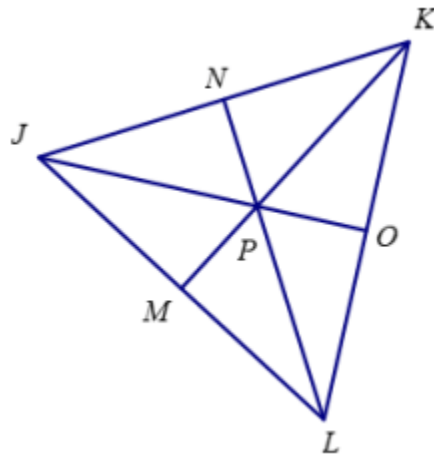


- a) $1400 + 1400\sqrt{2}$ b) $300 + 200\sqrt{2}$ c) $800 + 600\sqrt{2}$
d) $1400 + 1000\sqrt{2}$ e) $900 + 600\sqrt{2}$ f) None of the above

31) Triangle CAT has vertices $C(1, -5)$, $A(3, 5)$, and $T(-7, -5)$ and is inscribed in circle D . If circle D has center (m, n) and radius r , find the value of $m - n - r^2$.

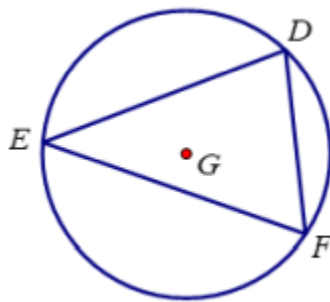
- a) -56
b) -52
c) -50
d) $-\frac{418}{9}$
e) -55
f) None of the above

32) Triangle JKL is shown below with its medians intersecting at point P . If $JM = OL$, $JK = 12$, and $MP = \sqrt{14}$, find the length of \overline{ML} .



- a) $3\sqrt{6}$ b) $\sqrt{42}$ c) $\sqrt{66}$ d) 6 e) $4\sqrt{5}$ f) None of these

33) Circle G is circumscribed about $\triangle DEF$ as shown below. If the diameter of the circle measures 18 cm and $DE = 12$ cm, find $\tan(\angle EFD)$.



- a) $\frac{\sqrt{5}}{2}$ b) $\frac{2}{3}$ c) $\frac{\sqrt{5}}{3}$ d) $\frac{4\sqrt{5}}{5}$ e) $\frac{2\sqrt{5}}{5}$ f) None of these