

Geometry Exam - University of Houston Math Contest January 29, 2022

EXAM INSTRUCTIONS: Answer the following. Note that diagrams may not be drawn to scale.

1) Suppose that on the planet Moeg, the following statement is true:

If a food is a dessert, then it contains chocolate.

What other statement(s) must be true on the planet Moeg?

- I. If a food contains chocolate, then it is a dessert.
- II. If a food is not a dessert, then it does not contain chocolate.
- III. If a food does not contain chocolate, then it is not a dessert.

- a) I only b) II only c) II and III only d) I and II only
 e) III only f) I, II, and III g) I and III only

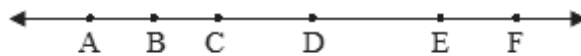
2) \overline{JK} bisects \overline{MN} at point P . If $MP = 3x - 1$ and $MN = 7x - 9$, find the length of \overline{MN} .

- a) 2 b) 10 c) 7 d) 20 e) 5 f) 40

3) Suppose that $\triangle JKL \cong \triangle TPN$, and that the ratio $K : N : J$ is $3 : 5 : 7$. Find $m\angle T$.

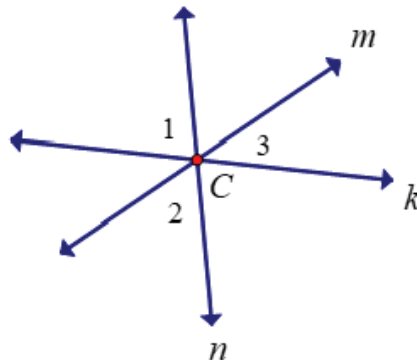
- a) 36° b) 45° c) 84° d) 60° e) 75° f) 105°

4) How many distinct rays can be named using the points given below?



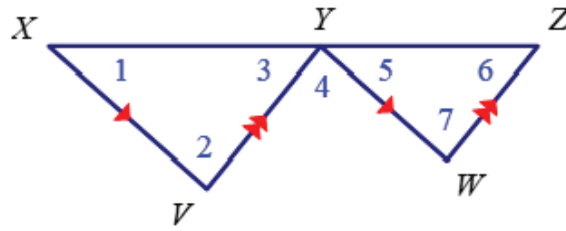
- a) 10 b) 15 c) 20 d) 12 e) 6 f) 30

5) In the diagram below, $m\angle 2 = 64^\circ$. Find y , given that $y = m\angle 1 + m\angle 3 - m\angle 2 - 17^\circ$.



- a) 55 b) 35 c) 25 d) 52 e) 45 f) 116

6) In the diagram below, $\overline{XV} \parallel \overline{YW}$, $\overline{VY} \parallel \overline{WZ}$, $m\angle 2 = 75^\circ$ and $m\angle 6 = 45^\circ$. Find $m\angle 1$.



- a) 45° b) 60° c) 75° d) 15° e) Cannot be determined.

7) A list of numbered statements is found below. Create a number using the numbers associated with the true statements. For example, if only 3, 4 and 6 are true, then your answer should be 346.

1. Some right triangles are acute.
2. Some scalene triangles are isosceles.
3. No isosceles triangles are equilateral.
4. A right triangle can be scalene.
5. No right triangles are equiangular
6. Some right triangles are isosceles

- a) 145 b) 1245 c) 456 d) 45 e) 1235 f) 235
 g) 13456 h) 46 i) 1456 j) 56 k) 146 l) 3456

8) $\triangle FGH$ is isosceles with base \overline{GH} . If $\overline{GH} = (9x - 5)$ feet, $\overline{FG} = (12x - 3)$ feet, and $\overline{HF} = (8x + 13)$ feet, find the perimeter of the triangle.

- a) 611 b) 541 c) 121 d) 107 e) 33.5 f) 16

9) Four angles are formed by a pair of perpendicular lines. If two of those four angles are randomly chosen, which of the following words correctly completes the statement below?

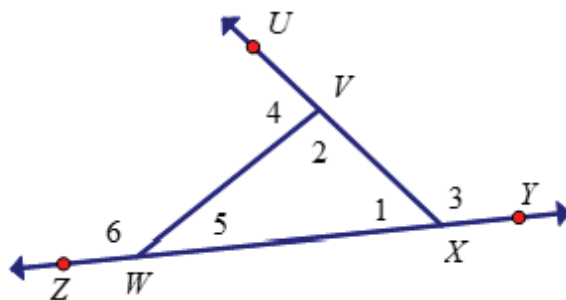
The two chosen angles are _____ angles.

A list of numbered statements is found below. Create a number using the numbers associated with the choices that correctly complete the statement above. For example, if only 1, 3 and 5 are true, then your answer should be 135.

1. complementary
2. adjacent
3. supplementary
4. right
5. vertical

- a) 234 b) 14 c) 1245 d) 124 e) 34 f) 2345 g) 345 h) 145
 i) None of the above

10) In the diagram below, $m\angle 2 = (6x + 7)^\circ$, $m\angle 3 = (10x - 7)^\circ$, and $m\angle 5 = (2x + 10)^\circ$. Find $m\angle 6 - m\angle 4$.



- a) 55° b) 45° c) 79° d) 65° e) 67° f) 17°

11) In $\triangle RST$, $m\angle R = 61^\circ$ and $m\angle T = 43^\circ$. List the three side lengths of the triangle in order from the greatest to the least measure.

- a) RT, ST, RS b) RS, RT, ST c) ST, RS, RT
d) ST, RT, RS e) RT, RS, ST f) RS, ST, RT

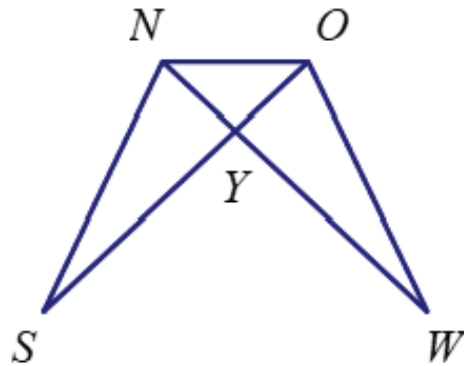
12) Which of the following conditions is NOT sufficient to prove that a quadrilateral is a parallelogram?

- a) Both pairs of opposite angles are congruent.
b) Both pairs of opposite sides are parallel.
c) One pair of opposite sides is congruent, and the other pair of opposite sides is parallel.
d) The diagonals bisect each other.
e) Both pairs of opposite sides are congruent.

13) Find the equation of a line that passes through the point $(2, -5)$ and is perpendicular to the line $6x - 4y = 3$.

- a) $y = -\frac{2}{3}x - \frac{19}{3}$
b) $y = -\frac{2}{3}x - \frac{11}{3}$
c) $y = -\frac{2}{3}x - \frac{4}{3}$
d) $y = \frac{2}{3}x - \frac{19}{3}$
e) $y = -\frac{3}{2}x - 2$
f) None of the above

14) In the diagram below, $\overline{SN} \cong \overline{OW}$ and $\angle WON \cong \angle ONS$. Which theorem or postulate can be used to prove that $\triangle SNO \cong \triangle WON$?



- a) Side-Angle-Side b) Side-Side-Angle c) Side-Side-Side
 d) The triangles are not necessarily congruent
 e) Hypotenuse-Angle f) Angle-Angle-Side g) Angle-Side-Angle

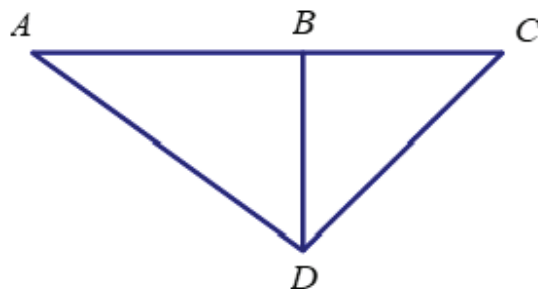
15) If a convex regular hexagon is drawn with its diagonals, find the number of points of intersection of the diagonals in the interior of the hexagon.

- a) 21 b) 12 c) 13 d) 15 e) 19 f) 9

16) Jane has a 14-inch piece of spaghetti and a 6-inch piece of spaghetti. She puts the 6-inch piece onto the table and breaks the 14-inch piece of spaghetti into two pieces. (Assume that all points on the 14-inch piece of spaghetti are equally likely to be the breaking point.) She now has three pieces of spaghetti. What is the probability that they can form a triangle?

- a) $\frac{2}{7}$ b) $\frac{5}{14}$ c) $\frac{1}{2}$ d) $\frac{1}{3}$ e) $\frac{4}{7}$ f) $\frac{3}{7}$

17) In the figure below, \overline{DB} is an altitude of $\triangle ADC$, $\tan(A) = \frac{4}{3}$, and $\sin(C) = \frac{3}{5}$. If all segments in the diagram are whole numbers, find the smallest possible perimeter of $\triangle ADC$.

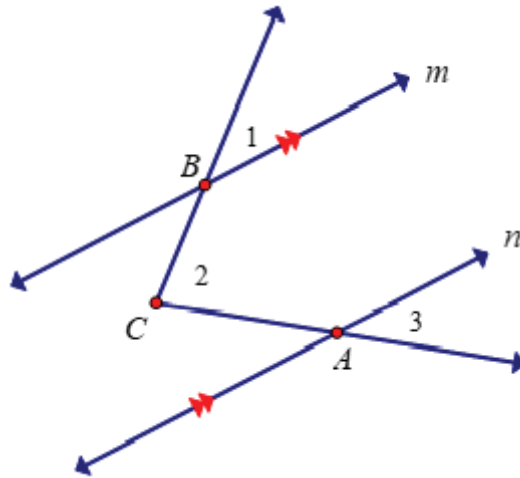


- a) 60 b) 510 c) 16 d) 34 e) 17 f) 120

18) In trapezoid ABCD, the measure of $\angle D$ is 41° , which is 5 times more than twice the complement of $\angle C$. Find the measure of $\angle B$.

- a) 18° b) 139° c) 49° d) 72°
- e) Cannot be determined
- f) 108° g) 131° h) 41°

19) In the figure below, $m \parallel n$, $m\angle 1 = 25^\circ$ and $m\angle 2 = 74^\circ$. Find $m\angle 3$.



- a) 25° b) 106° c) 50° d) 65° e) 74° f) 49°

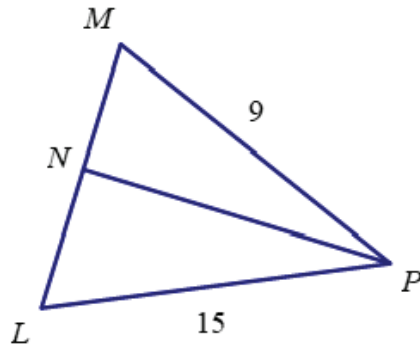
20) One side of a triangle measures 8 inches, and another side measures 14 inches. If the length of the third side is a whole number, how many possible lengths exist for the third side?

- a) 15 b) 7 c) 17 d) 16 e) 6 f) 8

21) ABCDEF is a regular hexagon with side length 10. Find the area of triangle CDE.

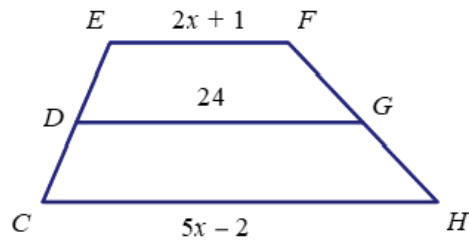
- a) $50\sqrt{3}$ b) 100 c) 50 d) $\frac{375}{2}$ e) $25\sqrt{3}$ f) None of the above

22) If \overline{PN} is an angle bisector of the triangle below and $LM = 20$, find the value of $LN - NM$.



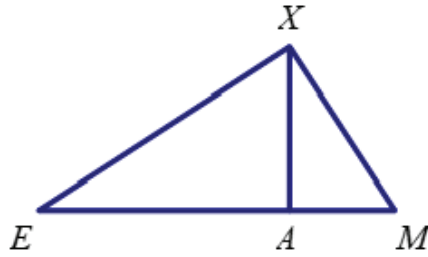
- a) 5 b) 6 c) 0 d) 3 e) 4 f) None of the above

23) Trapezoid CEFH is shown below with median \overline{DG} . Find the value of x .



- a) 6 b) 10 c) 12 d) 9 e) 8 f) None of the above

24) Triangle EXM is shown below with altitude \overline{XA} . If $m\angle EXM = 90^\circ$, $XA = 2\sqrt{3}$ and $EM = 10$, find the product of EA and AM .

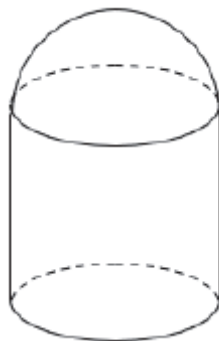


- a) 36 b) $12 + 10\sqrt{13}$ c) 12 d) $38 + 10\sqrt{13}$ e) $2\sqrt{3}$ f) None of the above

25) Cylinder X is similar to cylinder Y. Cylinder X has a surface area of 36 cm^2 , and cylinder Y has a surface area of 64 cm^2 . If the volume of cylinder X is 108 cm^3 , find the volume of cylinder Y, in cubic centimeters.

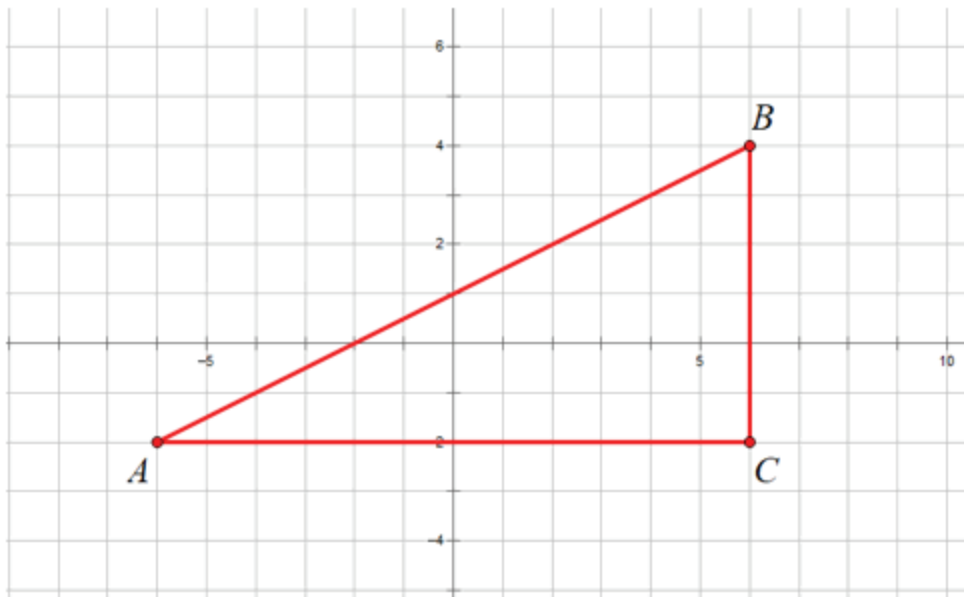
- a) 256 b) 192 c) 144 d) 60.75 e) $\frac{243}{4}$ f) None of the above

26) A silo is used to store corn on a farm. It is composed of a right circular cylinder topped by a hemisphere, both having the same radius. The height h of the silo is 5 times the length of its radius r . If a bumblebee is flying around and is equally likely to land anywhere on the exterior surface of the silo, what is the probability that the bumblebee lands on the hemispherical portion of the silo?



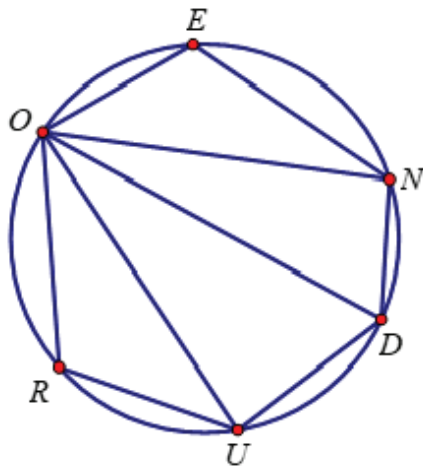
- a) $\frac{1}{4}$ b) $\frac{1}{5}$ c) $\frac{1}{2}$ d) $\frac{2}{11}$ e) $\frac{1}{3}$ f) None of the above

27) Triangle ABC is shown below with vertices $A(-6, -2)$, $B(6, 4)$, and $C(6, -2)$. Reflect triangle ABC over the line $x = 2$ to obtain the image named triangle $A'B'C'$. What is the area of the portion of triangle $A'B'C'$ that lies in the second quadrant?



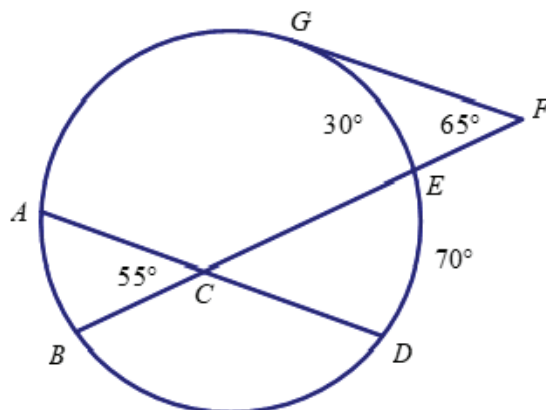
- a) 7 b) 15 c) 12 d) 9 e) 8 f) None of the above

28) Hexagon ENDURO is inscribed in a circle as shown below. If $m\angle E = 110^\circ$ and $m\angle OUD = 85^\circ$, find the degree measure of \widehat{ND} .



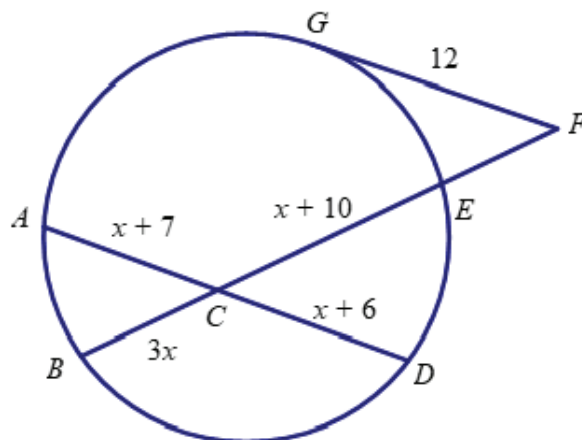
- a) 30° b) 55° c) 97.5° d) 70° e) 42.5° f) None of the above

29) \overline{FG} is tangent to the circle shown below. Find the measure of \widehat{BD} , given that the measure of \widehat{ED} is 70° , the measure of \widehat{EG} is 30° , $m\angle F = 65^\circ$, and $m\angle ACB = 55^\circ$.



- a) 250° b) 125° c) 160° d) 100° e) 110° f) None of the above

30) \overline{FG} is tangent to the circle shown below. Find the length of \overline{EF} , given that $AC = x + 7$, $BC = 3x$, $CE = x + 10$, $CD = x + 6$, and $FG = 12$.

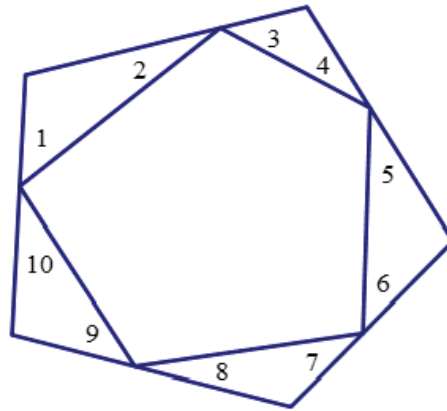


- a) 3 b) 8 c) 4 d) 6 e) 9 f) 12

31) In any right triangular prism, how many pairs of edges are skew?

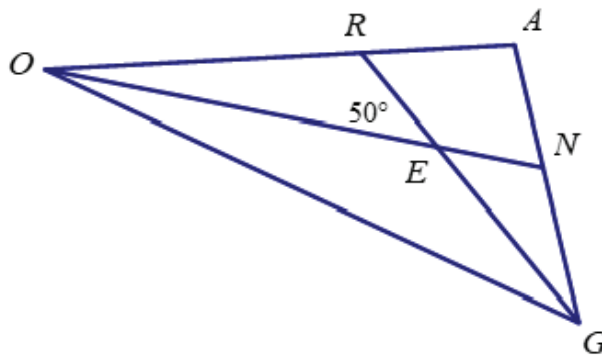
- a) 18 b) 9 c) 12 d) 24 e) 15 f) 6

32) Find the sum of the numbered angles in the diagram below.



- a) 540° b) 900° c) 180° d) 360° e) 720° f) None of the above

33) In $\triangle AGO$, \overline{ON} and \overline{GR} are angle bisectors. If $m\angle REO = 50^\circ$, find $m\angle A$.



- a) 50° b) 80° c) 130° d) 30° e) 100° f) 75°

34) A spherical glass ornament of radius 6 cm has a cube manufactured inside it. What is the volume of the largest cube that can fit inside the ornament, in cubic centimeters?

- a) $216\sqrt{2}$ b) $24\sqrt{3}$ c) $192\sqrt{3}$ d) 1728 e) $54\sqrt{2}$ f) 216

35) **TIEBREAKER QUESTION:** A right pyramid has an equilateral triangle as its base. If the pyramid has slant height 5 cm and base edge $6\sqrt{3}$ cm, find its volume in cubic centimeters.

- a) $27\sqrt{3}$ b) $36\sqrt{3}$ c) $108\sqrt{3}$ d) $45\sqrt{3}$ e) $135\sqrt{3}$ f) None of the above