Calculator Exam - University of Houston 2023 Math Contest
January 28, 2023

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1) Give the largest value of x where the graphs of $f(x) = 6x^3 - 36x^2 - x + 63$ and $g(x) = -19x^2 + 51x + 81$ intersect.
a) 4.7961 b) 4.7784 c) 4.7833 d) 4.7683 e) 4.7572 f) None of these.

2) Use the functions $f(x)$ and $g(x)$ defined in problem 1. Give the average of the y-coordinates of the points of intersection of the graphs.
a) -31.4482 b) -30.8776 c) -30.2136 d) -31.4537 e) -30.3315 f) None of these.

3) Let S be the set of positive integers from 1 to 100. Define S_E to be the set of even numbers in S and S_O to be the set of odd numbers in S. Let SQ_E be the sum of the squares of the numbers in S_E and SQ_O be the sum of the squares of the numbers in S_O. $SQ_E - SQ_O =$.
a) 5051 b) 5054 c) 5049 d) 5052 e) 5050 f) None of these.

4) The lines $y = nx - 1$ for $n = -10, -9, \cdots, 9, 10$ divide the circle of radius 5 centered at $(0, -1)$ into many sectors. Give the number of sectors.
a) 40 b) 32 c) 22 d) 46 e) 42 f) None of these.

5) Refer to the setting of problem 4. Give the area of the smallest sector.
a) 0.1467 b) 0.1296 c) 0.1373 d) 0.1132 e) 0.1473 f) None of these.

6) Refer to the setting of problem 5. Find the average of the areas of the sectors.
a) 1.8823 b) 1.8687 c) 1.8742 d) 1.8831 e) 1.8699 f) None of these.

7) Let S be the set given in problem 3. Define $S_{m,n}$ to be the set containing all points of the form (m, n) where m and n are in S. Find the shortest distance from a point (m, n) in $S_{m,n}$ to the line $y = 0.356x$.
a) 0.0041 b) 0.0036 c) 0.0039 d) 0.0035 e) 0.0037 f) None of these.

8) Define the function $f(x) = \dfrac{x}{2} + \dfrac{4}{x}$. Let $x_0 = 0.1$ and define $x_n = f(x_{n-1})$ for $n = 1, 2, \cdots$. The values of x_n get closer and closer to a value of x as n gets larger. Give this x value.
a) 2.3142 b) 2.8284 c) 2.8436 d) 2.8902 e) 2.7136 f) None of these.

9) Define the function $g(x) = 3.45x(1 - x)$ and let $x_0 = 0.5$. Then create the values $x_n = g(x_{n-1})$ for $n = 1, 2, \cdots$. Give the value of $x_k + x_{k+1} + x_{k+2} + x_{k+3}$ for large values of k.
a) 2.5804 b) 2.5799 c) 2.5801 d) 2.5798 e) 2.5780 f) None of these.

10) Let $g(x)$ be given in problem 9. A fixed point of $g(x)$ is a number p so that $g(p) = p$. Find the positive fixed point of $g(x)$.
a) 0.7101 b) 0.7146 c) 0.7132 d) 0.7097 e) 0.7113 f) None of these.

11) Give the distance from the solution to the system

$$47x + 21y = -1200$$

$$213x - 18y = 1401$$

to the solution to the system

$$-35x + 16y = -1300$$

$$136x - 8y = 2701.$$

a) 23.2672 b) 23.4283 c) 23.2472 d) 23.3116 e) 23.3127 f) None of these.

12) Assume the earth is a perfect sphere with circumference $24,859.73$ miles. Based upon this assumption, give the radius of the earth in miles.
a) 3956.5363 b) 3956.5489 c) 3956.5237 d) 3956.5132 e) 3956.5499 f) None of these.

13) The points given by $(\cos(3t), \sin(2t))$ for $0 \leq t \leq 2\pi$ enclose a region in the xy plane, and it also divides this region into several pieces. Give the number of pieces.
a) 9 b) 7 c) 6 d) 8 e) 10 f) None of these.

14) Give the area of the triangle determined by the vertices of the parabolas $y = 3x^2 - 7x + 2$, $y = 6x^2 - 2$ and $y = 3x^2 - 4x + 3$.
a) 2.3333 b) 2.4333 c) 2.4666 d) 2.1666 e) 2.1333 f) None of these.

15) Give $1 + 1/2 + 1/3 + 1/4 + 1/5 + \cdots + 1/999$.
a) 5.1773 b) 5.2248 c) 5.4136 d) 5.1137 e) 5.1562 f) None of these.

16) A game is played where a player gets 20 turns flipping a fair coin. After each coin flip, the player moves depending on the outcome. If the result of the coin flip is heads, then the player moves 2 meters forward. If the result of the coin flip is tails, then the player moves 1 meter back. Consequently, at the end of 20 turns, a player ends up between -20 meters behind their original position or 40 meters in front of their original position. How many meter marks between -20 and 40 are not possible as a final position for a player in this game?
a) 19 b) 22 c) 18 d) 24 e) 21 f) None of these.

17) Find the slope of the line that passes through the origin and the midpoint of the line segment from $(-23, 17)$ and $(14, 36)$.
a) -5.4488 b) -5.8844 c) -5.8888 d) -5.8866 e) -5.4444 f) None of these.

18) A sandwich shop serves over 1000 customers each day, seven days per week, and the management decides to boost interest in its sandwiches by giving a free sandwich to a select number of customers each day. The number of customers who will receive a free sandwich on a given day is determined by a drawing that the manager and assistant manager have each morning from a sack that has 5 tokens with the numbers 5, 12, 15, 17 and 20. The number on the token that is drawn determines the number of customers who will each receive one free sandwich that day. Suppose there is an equal likelihood of selecting each token. How many positive integers from 1 to 140 represent numbers of sandwiches that will never be the total number of sandwiches given away free in a week?
a) 63 b) 46 c) 67 d) 58 e) 51 f) None of these.

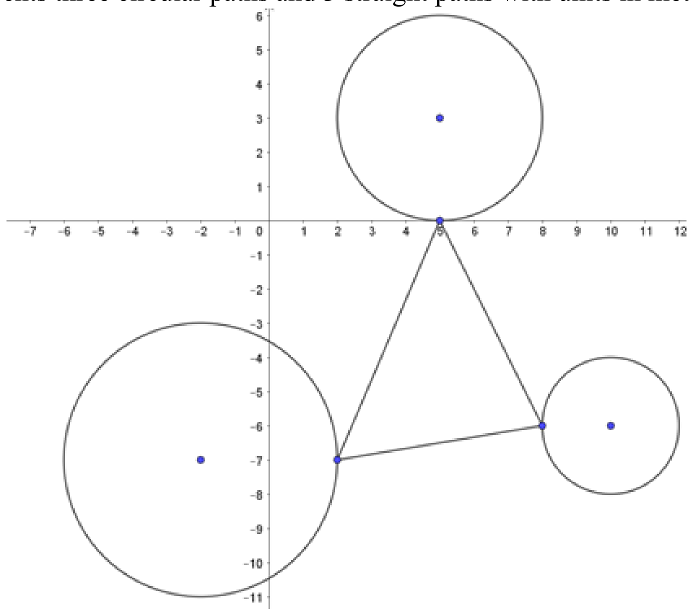
19) Give the number of points on the graph of $f(x) = 3\cos(5x) + 6\sin(7x)$ that intersect the circle of radius 4 centered at $(1, 0)$.
a) 30 b) 26 c) 28 d) 27 e) 29 f) None of these.

20) A number is written in base 7 as 33645611231. Give this number in base 10.
a) 1,456,231,112
b) 1,163,142,117
c) 1,007,061,497
d) 1,271,164,389
e) 1,311,2666,443
f) None of these.

21) A parallelogram is drawn in the xy plane. The vertices all have integer coordinates. A set M contains the numbers of the form $n/4$ where $n \in \{1, \cdots, 100\}$. Give the number of values in M that cannot be the area of this parallelogram.

a) 75 b) 45 c) 55 d) 25 e) 65 f) None of these.

22) The figure below represents three circular paths and 3 straight paths with units in meters.



What is the total distance travelled in meters by a particle that traverses all of the paths?

a) 77.2131 b) 76.8365 c) 76.7333 d) 76.9554 e) 77.1132 f) None of these.

23) What number is represented by the curve given parametrically by $x = -\cos^2(2t), y = \sin^3(t)$ for $0 \le t \le 2\pi$?

a) 0 b) 3 c) ∞ d) 1 e) 8 f) None of these.

24) The pattern in a sequence of numbers is indicated below.

$(1 + 1/2 + 1/3), (2 + 3/2 + 4/3), (3 + 5/2 + 7/3), \cdots$ Give the sum of the first 1000 numbers is this sequence.

a) 1500334.333 b) 1500333.666 c) 1500332.666

d) 1500332.333 e) 1500333.333 f) None of these.

25) Let C be the circle of radius 10 centered at the origin. $P_1, P_2, \cdots, P_{24}$ are placed counterclockwise around the circle, with the distance from P_i to P_{i+1} the same as the distance from P_{24} to P_1 for each $i = 1, \cdots, 23$. Give the distance from P_1 to P_9.

a) 17.3205 b) 17.3464 c) 17.2957 d) 17.2761 e) 17.3357 f) None of these.

26) The 5 tuple $(x_1, x_2, x_3, x_4, x_5)$ is said to be a nonnegative integer solution to the equation $x_1 + 2x_2 + 3x_3 + 2x_4 + x_5 = 15$ provided each of x_1, x_2, x_3, x_4, x_5 are nonnegative integers. Give the number of nonnegative integer solutions.

a) 483 b) 482 c) 489 d) 481 e) 486 f) None of these.

27) The 5 tuple $(x_1, x_2, x_3, x_4, x_5)$ is said to be a nonnegative integer solution to the equation $x_1 + 2x_2 + 3x_3 + 2x_4 + x_5 = 15$ provided each of x_1, x_2, x_3, x_4, x_5 are nonnegative integers. Give the largest value of $3x_1 + 2x_2 + x_3 + 2x_4 + 3x_5$ for a nonnegative integer solution.

a) 51 b) 46 c) 48 d) 42 e) 45 f) None of these.

28) Give the sum of the solutions to $\frac{1}{5}x^3 - 3x^2 + 8 = 2x - 20\cos(25x)$.

a) 105.4811 b) 104.7963 c) 108.2312 d) 107.7601 e) 106.2356

f) None of these.