Geometry Exam - University of Houston 2023 Math Contest
January 28, 2023

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1) Given any conditional statement, the contrapositive of the converse of the conditional statement is also known as the _____.

a) converse b) conditional c) inverse d) contrapositive e) biconditional

2) Suppose that $\overline{RU}$ bisects $\overline{BY}$ at point D.

A list of numbered statements is found below. Create a number using the numbers associated with the statements that are necessarily true. For example, if only 3, 4 and 6 are true, then your answer should be 346.

1. $\overline{UD} \cong \overline{DR}$
2. $\overline{RU} \perp \overline{BY}$
3. $BD = DY$
4. $\overline{DY} \cong \overline{BY}$
5. D is the midpoint of $\overline{BY}$.
6. $RU = BY$

a) 56 b) 246 c) 12356 d) 135 e) 1345 f) 35
g) 346 h) 12 i) 1235 j) 123456 k) 1245 l) 125

3) Find the measure of the angle that satisfies the following statement:
Seven times the complement of an angle is 10° less than twice the supplement of the angle.

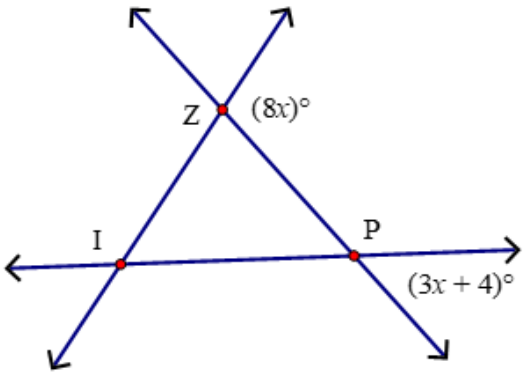
a) 52° b) 24.5° c) $71\frac{1}{9}^\circ$ d) 56° e) 21.8°
f) None of the above.

4) Fill in the blanks to the following statements and then find the sum of your answers.

- Through any _____ distinct point(s) there is exactly one line.
- If two distinct lines intersect, then they intersect in exactly _____ point(s).
- If two distinct planes intersect, then they intersect in exactly _____ line(s).
- Through any _____ distinct noncollinear point(s) there is exactly one plane.
- If there is a line and a point not on the line, then there is/are exactly _____ plane(s) that contain(s) them.

a) 10 b) 9 c) 5 d) 6 e) 7 f) 8

5) $\triangle ZIP$ is isosceles with base $\overline{ZP}$. Find the measure of $\angle ZIP$.



a) 52° b) 128° c) 76° d) 167.2° e) 16°
f) None of the above.

6) The distance between $(-1, 2)$ and $(5, b)$ is $2\sqrt{13}$. Find the product of all possible solutions for b.

a) 144 b) 16 c) -24 d) 36 e) -12

f) None of the above.

7) Consider the following conditional statement:

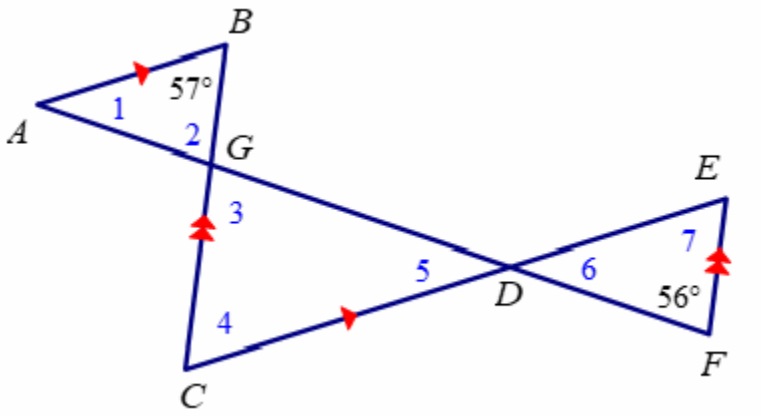
If two lines are not skew, then they are parallel.

Find the truth values of the conditional, inverse, contrapositive, and converse, respectively.

a) False, True, False, True b) True, False, True, False c) True, True, True, True

d) False, False, False, False e) True, True, False, False f) False, False, True, True

8) In the diagram below, $\overline{AB} \parallel \overline{CE}$, $\overline{BC} \parallel \overline{EF}$, $m\angle B = 57^\circ$, and $m\angle F = 56^\circ$. Find $m\angle 6 - m\angle 2$.



a) Cannot be determined b) 0° c) 1° d) 11° e) 12° f) 10°

9) Draw a diagram such that $\overline{AB}$ and $\overline{CD}$ bisect each other at point E. Then draw segments $\overline{AC}$, $\overline{AD}$, $\overline{BD}$, and $\overline{BC}$. Which of the following statements is true?

a) $\triangle DAE \cong \triangle CBE$ by Side-Side-Angle

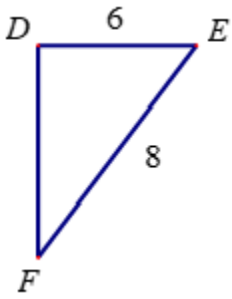
b) $\triangle AEC \cong \triangle DEB$ by Side-Angle-Side

c) $\triangle CEB \cong \triangle DEB$ by Side-Angle-Side

d) $\triangle AEC \cong \triangle AED$ by Side-Side-Side

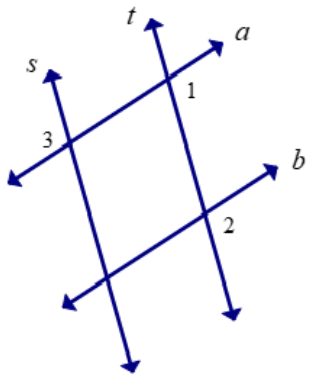
e) None of the above.

10) If $\overline{DE} \perp \overline{DF}$, find $\tan(E) - \sin(F)$ in the diagram below.



a) $\dfrac{4\sqrt{7} - 9}{12}$ b) $\dfrac{11}{12}$ c) $\dfrac{\sqrt{7} - 4}{3}$ d) $\dfrac{\sqrt{7} - 3}{4}$ e) $\dfrac{\sqrt{7}}{12}$ f) $\dfrac{12\sqrt{7} - 21}{8}$

11) In the following diagram, $s \parallel t,\ m\angle 1 = (8x + 9)^\circ$, and $m\angle 2 = (10x - 15)^\circ$. Find $m\angle 3$.



a) 75° b) $88.\overline{3}^\circ$ c) 12° d) 105° e) Cannot be determined f) $91.\overline{6}^\circ$

12) L is between C and X, and E is the midpoint of $\overline{CL}$. If $LX = 20$ and the ratio of EX to CL is 4:3, find the length of $\overline{CE}$.

a) 8 b) 12 c) 1.5 d) 15 e) 16
f) None of the above.

13) State the minimum number of vertices in a hexahedron. (A hexahedron is a polyhedron with 6 faces, where each face is a polygon.)

a) 6 b) 8 c) 5 d) 7 e) 12
f) None of the above.

14) Find the number of diagonals in a convex 43-gon.

a) 903 b) 1680 c) 1806 d) 12,341 e) 860
f) None of the above.

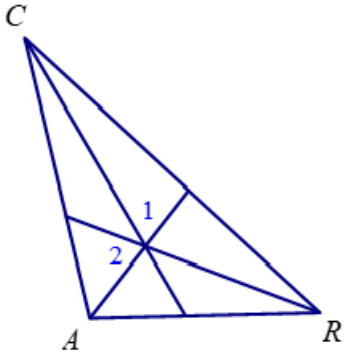
15) One base of an isosceles trapezoid is 10 cm longer than the other. If the height of the trapezoid is 12 cm and its area is 132 cm^2, find the perimeter of the trapezoid, in centimeters.

a) 48 b) 52 c) 60 d) 72 e) 290
f) None of the above.

16) Find the altitude, in centimeters, of an equilateral triangle with area $150\sqrt{3}$ cm^2.

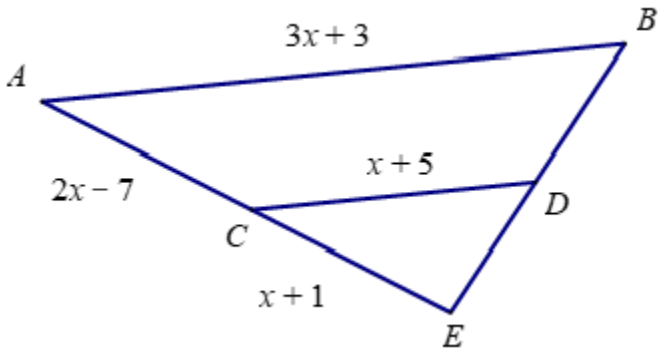
a) 15 b) $15\sqrt{2}$ c) $10\sqrt{6}$ d) $5\sqrt{6}$ e) 45
f) None of the above.

17) The angle bisectors are drawn for $\triangle CAR$. If $m\angle CAR = 128^\circ$, find $m\angle 1 + m\angle 2$.



a) Cannot be determined b) 52° c) 128° d) 104° e) 116° f) 154°

18) In the diagram below, $\overline{AB} \parallel \overline{CD}$, $AB = 3x + 3$, $AC = 2x - 7$, $CD = x + 5$, and $CE = x + 1$. Find the value of x.



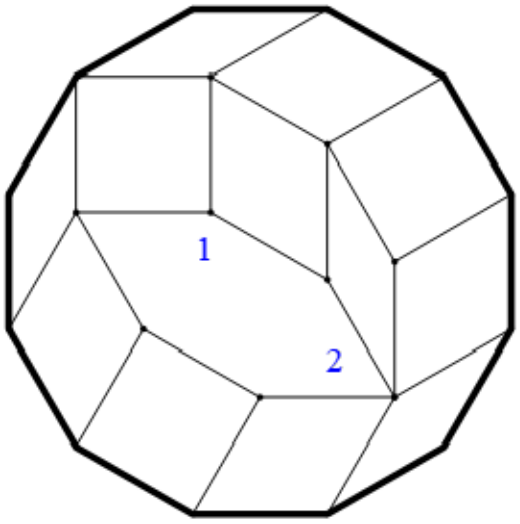
a) $-\frac{3}{2} + \frac{\sqrt{161}}{2}$ b) 8 c) $-\frac{3}{2} + \frac{\sqrt{143}}{2}$ d) 7 e) 11
f) None of the above.

19) Quadrilateral SOUP is inscribed in circle C. If $m\angle U = 54^\circ$, $m\angle O = 75^\circ$, and $m\widehat{SP} = 41^\circ$, find $m\widehat{OU}$.
a) 126° b) 54° c) 75° d) 105° e) 143°
f) None of the above

20) Find the area of the triangle with vertices $(2, 4)$, $(3, 9)$, and $(6, 7)$.
a) 10 b) 8.5 c) 11.5 d) 9.5 e) 10.5
f) None of the above

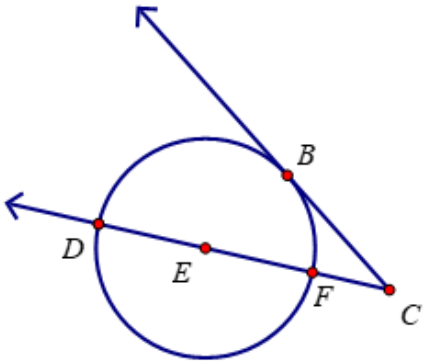
21) A cube with bases $ABCD$ and $EFGH$ has lateral edges $\overline{AE}$, $\overline{BF}$, $\overline{CG}$, and $\overline{DH}$. Let M and N represent the midpoints of lateral edges $\overline{AE}$ and $\overline{CG}$, respectively. If the volume of the cube is $8x^3$, find the area of quadrilateral $MBNH$.
a) $x^2\sqrt{30}$ b) $5\,x^2$ c) $2\,x^2\sqrt{6}$ d) $2\,x^2\sqrt{3}$ e) $3\,x^2$
f) None of the above

22) The outside of the diagram below is a regular dodecagon (a 12-sided polygon). Inside this dodecagon are 12 parallelograms and a hexagon. Find $7m\angle 2 - 2m\angle 1$.



a) 600° b) 330° c) 930° d) 120° e) 15°
f) None of the above.

23) In the diagram below, $\overrightarrow{CB}$ is tangent to circle E at point B. If $DF = 42$ and $m\overset{\frown}{DB} = 150^\circ$, find BC.

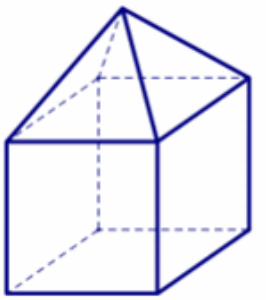


a) $7\sqrt{3}$ b) $21\sqrt{3}$ c) $21\sqrt{2}$ d) 21 e) 42
f) None of the above.

24) John's birthday cake is in the shape of a right rectangular pyramid. The height of the cake is 20 cm, and the cake's volume is 4000 cm^3. John slices the cake parallel to the base to remove the first slice. If the height of John's slice is 6 cm, what is the volume of the remaining portion of the cake (the portion that does not contain John's first slice), in cubic centimeters?

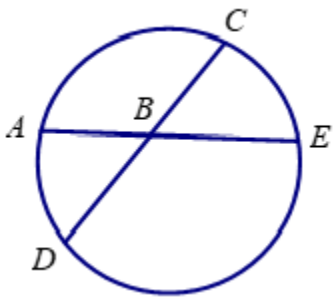
a) 2800 b) 3964 c) 400
d) Cannot be determined e) 3640 f) 3892

25) A three-dimensional solid is comprised of a cube which is topped with a right square pyramid. The edge of the cube measures 10 cm and the height of the pyramid is 12 cm. Find the total surface area of the solid, in square centimeters.



a) $1433.\overline{3}$ b) 840 c) 740 d) 760 e) 860 f) 1400

26) In the circle below, chords $\overline{CD}$ and $\overline{AE}$ intersect at point B. If $m\angle CBE = (3x + 11)^\circ$, $m\angle DBE = (6x - 11)^\circ$, and $m\overset{\frown}{AD} = (4x + 12)^\circ$, find $m\overset{\frown}{CE}$.

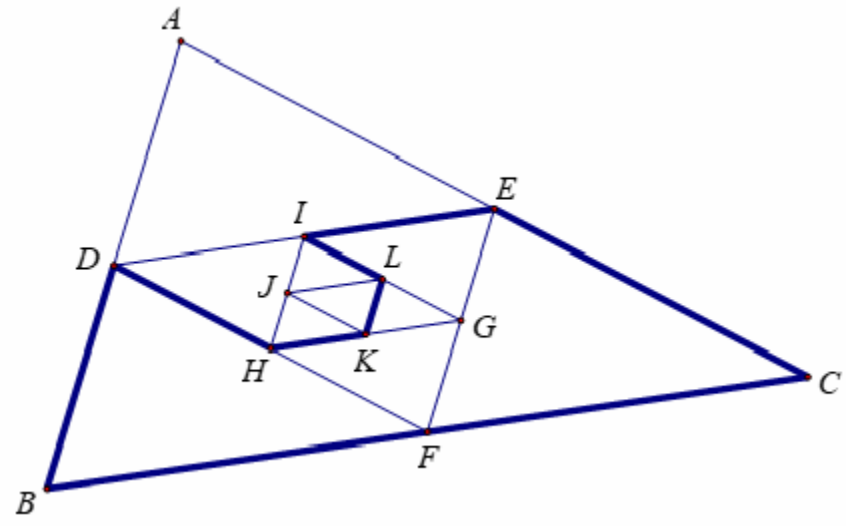


a) 109° b) 50° c) 71° d) 33° e) 92° f) $24\frac{2}{3}^\circ$

27) A right circular cone has a height of 5 cm and a volume of 40π cm^3. Find the radius of the cone, in centimeters.
a) $2\sqrt{10}$ b) 4 c) $2\sqrt{2}$ d) $2\sqrt{3}$ e) $2\sqrt{6}$
f) None of the above.

28) In the diagram below:
Points D, E, and F are the midpoints of the sides of $\triangle ABC$.
Points G, H, and I are the midpoints of the sides of $\triangle DEF$.
Points J, K, and L are the midpoints of the sides of $\triangle GHI$.

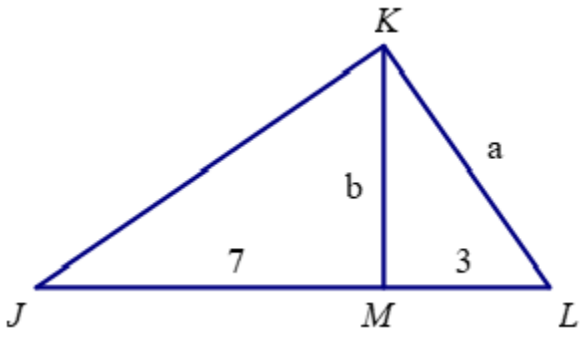
If $JK = 5$, $FG = 6$, and if AC is 16 less than BC, find the perimeter of $DHKLIECB$.



a) 143 b) 130 c) 120 d) 127 e) 99
f) None of the above.

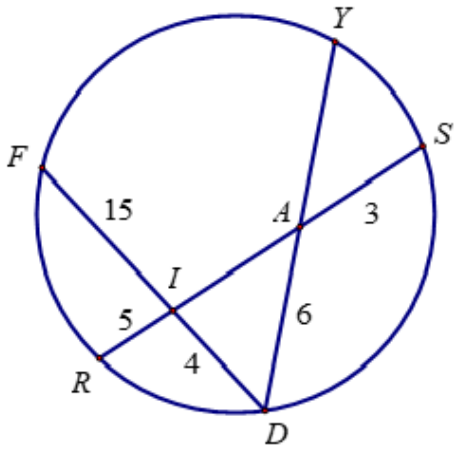
29) Circle P with radius 12 cm is inscribed in $\triangle ABC$. If $PA = 13$ cm, $PC = 20$ cm, and $PB = 15$ cm, find the perimeter of $\triangle ABC$ in centimeters.
a) 45 b) 44 c) 84 d) 60 e) 48
f) None of the above.

30) In the diagram below, $\overline{KM} \perp \overline{JL}$, $\overline{JK} \perp \overline{KL}$, $JM = 7$, and $ML = 3$. Find the value of $a^2 + b^2$.



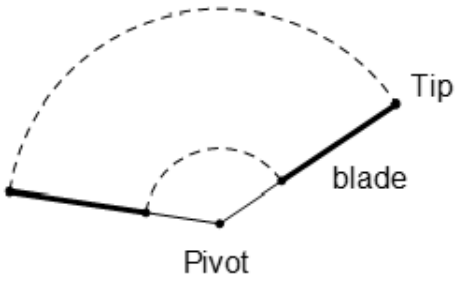
a) 42 b) 88 c) 51 d) 91 e) 9
f) Cannot be determined

31) Chords $\overline{FD}$, $\overline{DY}$, and $\overline{RS}$ are shown in the circle below. Find the sum $IA + AY$, given that $FI = 15$, $RI = 5$, $ID = 4$, $AD = 6$, and $AS = 3$



a) 22 b) 25 c) 27.5 d) 16 e) 19
f) None of the above.

32) The following diagram shows a windshield wiper which is installed on the window of a car. The wiper is 20 inches long, measured from the pivot point to the tip. The rubber blade (the portion that cleans the window) is 15 inches long. If the wiper traces out an angle of 144°, find the area of the windshield which is cleaned with each full sweep, in square inches.



a) 70π b) 300π c) 12π d) 80π e) 140π f) 150π

33) Figure 1 represents a regular hexagon. We will call it Hexagon 1. Figure 2 contains a smaller red hexagon formed by joining the midpoints of the sides of Hexagon 1; we name this Hexagon 2. This process is repeated 3 more times to obtain Hexagon 3, Hexagon 4, and Hexagon 5 (shown in green, blue, and purple, respectively).

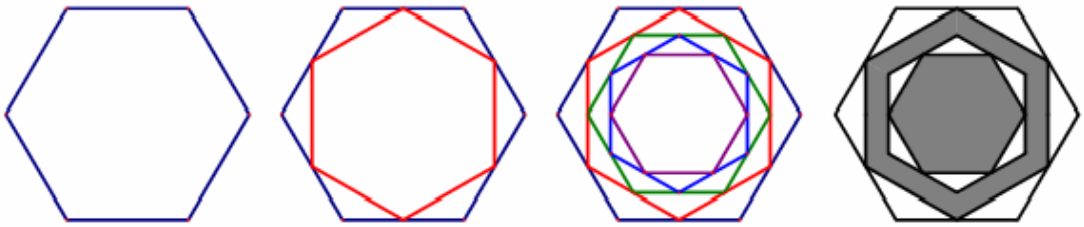


Figure 1 Figure 2 Figure 3 Figure 4

Figure 4 is a dartboard created from the hexagons in Figure 3. If a dart is thrown at the dartboard and is equally likely to land anywhere within the outer hexagon, what is the probability that the dart will land within a shaded region?

a) $\frac{13}{16}$ b) $\frac{39}{64}$ c) $\frac{165}{256}$ d) $\frac{129}{256}$ e) $\frac{2\sqrt{3}+9}{16}$
f) None of the above.

34) Suppose that $\triangle ABC$ is drawn on the coordinate plane with vertices $A\,(-3,-4)$, $B\,(x,y)$, and $C\,(11,-2)$. The centroid of $\triangle ABC$ is the point $P\left(\frac{13}{3},0\right)$. Find the coordinates of point B.

a) $(8,2)$ b) $(5,6)$ c) $(27,18)$ d) $(19,8)$ e) $\left(\frac{37}{3},10\right)$ f) $\left(\frac{35}{3},4\right)$

35) **TIEBREAKER QUESTION:** One side of a triangle measures 3 cm, another side of the triangle measures 5 cm, and the third side is a whole number. Consider the set of all triangles that exist with these conditions (where no two of those triangles are congruent).

Let v represent the number of scalene triangles, w represent the number of isosceles triangles, x represent the number of right triangles, y represent the number of acute triangles, and z represent the number of obtuse triangles. Find $v-w+x-y+z$.

a) 6 b) 3 c) 1 d) 0 e) 5 f) 2 g) 4

h) None of the above.