Algebra I Exam – University of Houston Math Contest 2024

1. Solve the equation
$$2[(3-2x)-6(3+2x)-7]=4\big(3x+6(2-x)\big).$$
   a. -6.5
   b. -6.25
   c. -6
   d. -5.75
   e. -5.5
   f. None of these.

2. Give the slope of the line passing through the points $(-2,1)$ and $(3,2)$.
   a. $2/5$
   b. $1/5$
   c. $3/5$
   d. $4/5$
   e. $6/5$
   f. None of these.

3. Give the sum of the solutions to the equation
$$2x^2-3x=5.$$
   a. 3
   b. $3/2$
   c. 1
   d. $1/2$
   e. 2
   f. None of these.

4. What is the average of the smallest possible slope and largest possible slope of a line that passes through two of the following three points?
$$(2,3),(-1,6),(3,-2)$$
   a. -5
   b. -6
   c. -4
   d. -7
   e. -3
   f. None of these.

5. Give the equation of the line that passes through the point $(1,2)$ and is perpendicular to the line $3x-2y=5$.
   a. $2x-y=0$
   b. $3x+2y=7$
   c. $2x+3y=8$
   d. $x-2y=-3$
   e. $4x+6y=10$
   f. None of these.

6. Find the solution x to the equation

$$\frac{1}{10} = \frac{1}{18x} + \frac{1}{45x}.$$

   a. $7/9$
   b. $2/3$
   c. $8/9$
   d. $5/9$
   e. $4/3$
   f. None of these.

7. Find the average of the solutions to

$$3x^2 = 7 - 5x.$$

   a. $-2/3$
   b. $2/3$
   c. $5/6$
   d. $-5/6$
   e. $5/7$
   f. None of these.

8. Solve the equation

$$8\sqrt{x^3} = \sqrt{\left(\frac{26}{9} + 3x\right)^3}.$$

   a. $19/9$
   b. $13/9$
   c. $17/9$
   d. $16/9$
   e. $26/9$
   f. None of these.

9. Give the average of the solutions to the equation

$$\sqrt{1-x} - x = 5.$$

   a. $11/2$
   b. $9/2$
   c. $13/2$
   d. $-7/2$
   e. $-5/2$
   f. None of these.

10. Use interval notation to give the solution to

$$-\frac{4(x+7)}{12} < \frac{4}{5}.$$

   a. $(47/5, \infty)$
   b. $(-47/5, \infty)$
   c. $(27/5, \infty)$
   d. $(-\infty, 47/5)$
   e. $(-\infty, -27/5)$
   f. None of these.

11. $g(x) = x^2 + 7$. Simplify

$$g(x+2) + x + g(x).$$

a. $2x^2 + 5x + 14$
b. $2x^2 + 7x + 18$
c. $2x^2 + 5x + 18$
d. $2x^2 + 7x + 19$
e. $2x^2 + 7x + 14$
f. None of these.

12. Give the sum of the solutions to

$$(x-5)(x+2)(x^2-9)(3x^2+6x+3) = 0.$$

a. 5
b. 4
c. 3
d. 2
e. 1
f. None of these.

13. Give the 8^{th} term in the sequence

$$4, -12, 36, -108, \cdots$$

a. -7642
b. -5698
c. -8748
d. -6844
e. -9844
f. None of these.

14. Two sequences are shown below (without simplification)

$$3 - \frac{20}{2}, 3 - \frac{20}{3}, 3 - \frac{20}{4}, 3 - \frac{20}{5}, \cdots$$
$$-2 + 10, -2 + \frac{10}{2}, -2 + \frac{10}{3}, -2 + \frac{10}{4}, \cdots$$

Give the average of the 8^{th} term in the first sequence and the 8^{th} term in the second sequence.

a. 1/36
b. 3/72
c. 1/72
d. 1/18
e. 5/72
f. None of these.

15. The first few terms of two sequences are shown below (without simplification)

$$3-\frac{40}{2}, 3-\frac{40}{3}, 3-\frac{40}{4}, 3-\frac{40}{5}, \cdots$$
$$-2+10, -2+\frac{10}{2}, -2+\frac{10}{3}, -2+\frac{10}{4}, \cdots$$

Give the smallest positive integer n so that the n^{th} term of the first sequence is larger than the n^{th} term of the second sequence.

a. 9
b. 10
c. 7
d. 8
e. 11
f. None of these.

16. Give the number of integers for which $n^2 + 32$ is a perfect square given that $|n| < 15$.

a. 1
b. 4
c. 3
d. 2
e. 5
f. None of these.

17. Give the smallest value of b that solves

$$b^2 + \frac{1}{b^2} = 6.$$

a. $\sqrt{2} - 1$
b. $2\sqrt{2} + 1$
c. $-2\sqrt{2} + 1$
d. $-\sqrt{2} - 1$
e. $-2\sqrt{2} - 1$
f. None of these.

18. There are two points (x, y) that satisfy the equations $xy = 4$ and $x - 2y = 3$. Give the absolute value of the sum of the x-coordinates of these points.

a. 4
b. 5
c. 6
d. 3
e. 7
f. None of these.

19. List the following lines in order from smallest slope to largest slope

$$L_1:\ x + 2y = 3$$
$$L_2:\ x - 3y = 2$$
$$L_3:\ 2x - 5y = 7$$
$$L_4:\ 2x + 5y = 1$$

   a. L_1, L_4, L_2, L_3
   b. L_1, L_4, L_3, L_2
   c. L_4, L_1, L_2, L_3
   d. L_4, L_1, L_3, L_2
   e. L_1, L_3, L_4, L_2
   f. None of these.

20. Find numbers A and B so that

$$2A - 3B = 1$$

and

$$3A - 2B = 1.$$

Then give the value of $3A - B$.
   a. $6/5$
   b. $3/5$
   c. $2/5$
   d. $4/5$
   e. $1/5$
   f. None of these.

21. Describe the relationship between the point $P = (2,1)$, the line L_1 given by $2x + 4y = 9$, and the line L_2 given by $4x + 2y = 9$.
   a. P lies above L_1 and below L_2
   b. P lies above L_2 and below L_1
   c. P lies above both L_1 and L_2
   d. P lies below both L_1 and L_2
   e. P is the intersection point of L_1 and L_2
   f. None of these.

22. Find the point (a, b) on the line $2x + y = 3$ that is closest to the point $(2,1)$. Then give the value of b/a.
   a. $1/4$
   b. $2/5$
   c. $1/3$
   d. $1/2$
   e. $3/5$
   f. None of these.

23. A rectangle has two vertices on the x-axis and two vertices in quadrants I and II on the portions of the graphs of $y = 5 - x$ and $y = 5 + x$ with $y \leq 5$. Give the height of the rectangle if it has the largest possible area.
    a. 3
    b. $7/2$
    c. $5/2$
    d. $4/3$
    e. $7/3$
    f. None of these.

24. Select the number below that is closest to the area of the region in the first quadrant of the xy-plane determined by the inequalities

$$x + 2y \leq 10$$

and

$$2x + y \leq 6.$$

    a. 8.5
    b. 8.25
    c. 9
    d. 8.75
    e. 9.25
    f. None of these.

25. There are 100 points in the xy-plane with (x, y) coordinates taken from the set

$$\{1,2,3,4,5,6,7,8,9,10\}.$$

A line $y = mx$ is created. Give the smallest value c so that the line does not contain any of these points whenever $c < m < 1$.
    a. 0.99
    b. 0.98
    c. 0.97
    d. 0.96
    e. 0.95
    f. None of these.

26. Bob, Sue and Jen weigh themselves separately and collectively on an accurate industrial scale. Bob weighs 16kg more than Sue, and Sue weighs 12kg less than the average weight of Bob and Jen. The total weight of Bob, Sue and Jen is 230kg. Give the number below that is closest to the average weight of Bob, Sue and Jen.
    a. 77kg
    b. 75kg
    c. 78kg
    d. 76kg
    e. 79kg
    f. None of these.

27. Bob, Sue and Jen weigh themselves separately and collectively on an accurate industrial scale. Bob weighs 16kg more than Sue, and Sue weighs 12kg less than the average weight of Bob and Jen. The total weight of Bob, Sue and Jen is 230kg. Give the number below that is closest to the sum of the weights of Bob and Jen.
    a. 161
    b. 162
    c. 163
    d. 164
    e. 165
    f. None of these.

28. Solve the equation $\frac{2}{x} - \frac{3}{3x-1} = 0$ for x. Then give the value of
$$\frac{2}{x} + \frac{4}{3}.$$
    a. 3
    b. 5
    c. 1
    d. 4
    e. 2
    f. None of these.

29. An object is launched into the air and wind resistance is assumed negligible. The motion is modelled in the xy-plane with x representing horizontal distance in feet and y representing height in feet (until the object strikes the ground). At time $t \geq 0$ seconds the object is located at the position $\left(x(t), y(t)\right)$ where
$$x(t) = 32t \text{ feet}$$
and
$$y(t) = -16t^2 + 32t + 48 \text{ feet.}$$
The line segment from $\left(x(0), y(0)\right)$ to the point $(x(t), y(t))$ at which $y(t)$ is its largest is the hypotenuse of a right triangle. Give the area of the triangle.
    a. 128
    b. 324
    c. 256
    d. 512
    e. 384
    f. None of these.

30. The number $c > 0$. A parabola passes through the points $(2,0)$, $(5,0)$ and $(3, c)$. Give the slope of the line from passing through the vertex of the parabola and the point $\left(\frac{7}{2}c, \frac{9}{8}\right)$.
    a. $-9/28$
    b. $-7/26$
    c. $-4/13$
    d. $-11/32$
    e. $-13/36$
    f. None of these.

31. Determine the smallest number $C > 0$ so that the point $P = (2,1)$ lines below the line L_1 given by $2x + 4y = c$, and above the line L_2 given by $4x + 2y = c$ whenever $C < c < 10$.
    a. 9
    b. 8.5
    c. 7.5
    d. 8
    e. 7
    f. None of these.

32. Let $f(x) = 2x - 1$, $g(x) = 3 - 2x$ and $h(x) = x^2 - 2x$. Give the absolute value of the difference of the solutions to

$$h\Big(g\big(f(x)\big)\Big) - g(f\big(h(x)\big) = 0.$$

    a. $5\sqrt{2}/2$
    b. $2\sqrt{2}$
    c. $3\sqrt{2}/2$
    d. $\sqrt{2}/2$
    e. $\sqrt{2}$
    f. None of these.

33. a and b are numbers and $(x - 1)$ is a factor of the polynomial $x^2 + ax + b$. Give the value of $2a + 2b + 13$.
    a. 15
    b. 12
    c. 13
    d. 14
    e. 11
    f. None of these.

34. If $x \geq 100$ then there is a natural number n and a number $0 \leq c < n + 1$ so that

$$1 + 2 + \cdots + n \leq x < 1 + 2 + \cdots + n + (n + 1)$$

and

$$1 + 2 + \cdots + n + c = x.$$

What is the smallest possible value of x if $c = 3/5$?
    a. 100.6
    b. 101.6
    c. 102.6
    d. 103.6
    e. 104.6
    f. None of these.

35. Find the smallest value of c so that there is at last one value of x solving

$$-13 \leq 2(3c - x) + 1 \leq 1 + c.$$

a. -14
b. -12
c. -13
d. -10
e. -15
f. None of these.

36. c is a number and $h(x) = x^2 + c$. Determine the largest value of c for which

$$h(x) + h(x + 1) = 0$$

has at least one solution.

a. $-\frac{1}{2}$
b. $-\frac{1}{4}$
c. $-\frac{5}{4}$
d. $-\frac{3}{4}$
e. 0
f. None of these.

37. 100 points (a, b) are in the first quadrant of the xy-plane with the values a and b chosen from the set

$$\{1,2,3,4,5,6,7,8,9,10\}.$$

Find the largest value c so that the line $y = 11 + m(x - 11)$ does not contain any of these points whenever $1 < m < c$.

a. 1.1
b. 1.01
c. 1.11
d. 1.21
e. 1.05
f. None of these.

38. c is a number. Solve the following equation

$$xc = \frac{2y - x}{y + x}$$

for y. Then determine the value of c for which the graph of

$$y = \frac{2}{cx - 2}$$

is a line with a hole removed.

a. 3
b. -2
c. 0
d. 2
e. -3
f. None of these.

39. There are two numbers c so that the lines

$$2cx + y = 1$$

and

$$x + 4cy = 2$$

do not intersect. Give the reciprocal of the sum of the absolute values of these numbers.

a. $2\sqrt{2}$
b. $3\sqrt{2}/2$
c. $3\sqrt{2}$
d. $5\sqrt{2}/2$
e. $\sqrt{2}$
f. None of these.

40. Find the smallest number A so that

$$\sqrt{2c + x} - \sqrt{2c - x} = x$$

has at least one nonzero solution for x whenever $A < c \leq 1$. Then give the value of $A + 1$.

a. $5/2$
b. $1/2$
c. 2
d. 1
e. $3/2$
f. None of these.