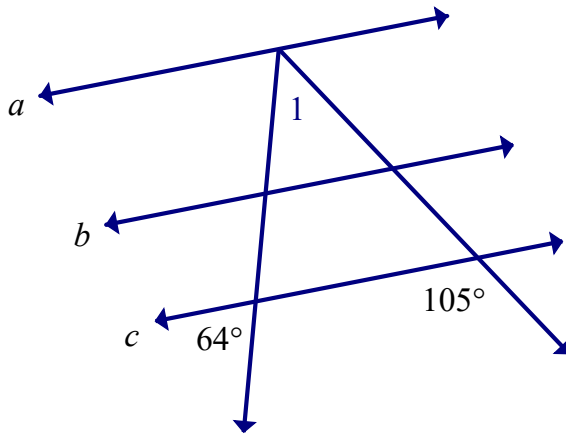


Geometry Exam

University of Houston Math Contest 2024

Answer the following. Note that diagrams may not be drawn to scale.

1. If $a \parallel c$, find $m\angle 1$.



- A. 11°
- B. 75°
- C. 61°
- D. 64°
- E. 41°
- F. Cannot be determined

2. How many distinct lines are determined by five coplanar points, given that exactly three of them are collinear?

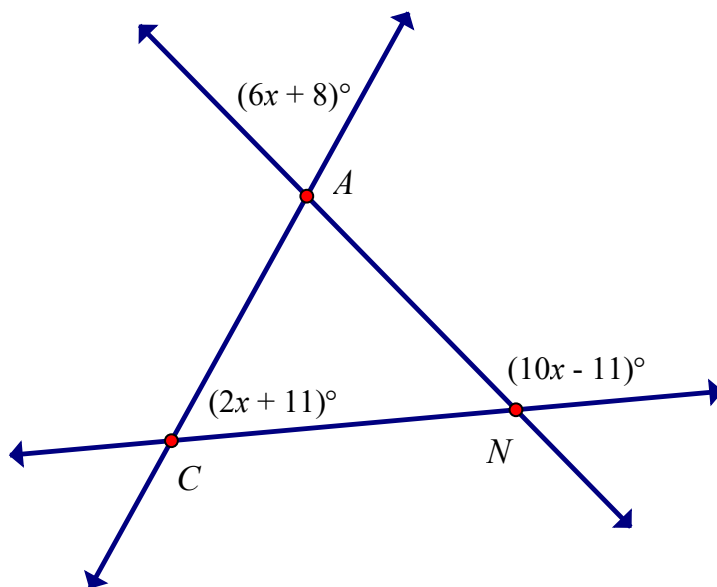
- A. 4
- B. 5
- C. 6
- D. 7
- E. 8
- F. 10

3. R is between C and T , and X is the midpoint of \overline{RC} . If $CT = 27$ and the ratio of CX to RT is $2:5$, find the length of \overline{XT} .

- A. 15
- B. 21
- C. 3
- D. 6
- E. 27
- F. None of the above

4. The slope of the line passing through $(2, -5)$ and $(6, d)$ is $\frac{7}{2}$. Find the value of d .
- A. 19
 - B. 23
 - C. 9
 - D. 14
 - E. $\frac{27}{7}$
 - F. None of the above

5. Classify $\triangle CAN$ based on the information in the diagram.



- A. obtuse, scalene
- B. acute, isosceles
- C. right, scalene
- D. obtuse, isosceles
- E. acute, scalene
- F. None of the above

6. Given the following: $\angle 1$ and $\angle 2$ are supplementary
 $\angle 2$ and $\angle 3$ are supplementary

What conclusion can be drawn about $\angle 1$ and $\angle 3$?

- A. $\angle 1$ and $\angle 3$ are complementary
- B. $\angle 1$ and $\angle 3$ are supplementary
- C. $\angle 1$ and $\angle 3$ are vertical angles
- D. $\angle 1 \cong \angle 3$
- E. $\angle 1$ and $\angle 3$ are right angles
- F. None of the above

7. Suppose that \overline{AB} represents one edge of a cube. If an edge of the cube is randomly chosen from the remaining edges, what is the probability that the randomly chosen edge and \overline{AB} are skew?

- A. $\frac{3}{11}$ B. $\frac{1}{4}$ C. $\frac{4}{11}$ D. $\frac{2}{11}$ E. $\frac{1}{3}$

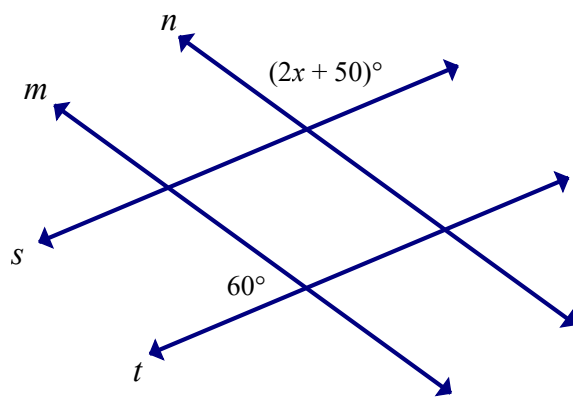
F. None of the above

8. On the planet Elcric, the following conditional statement is true: "If Serauqs are purple, then they are not tall." Which statement(s) below must also be true on the planet Elcric?

- I. If Serauqs are not purple, then they are tall.
 II. If Serauqs are tall, then they are not purple.
 III. If Serauqs are not tall, then they are purple.

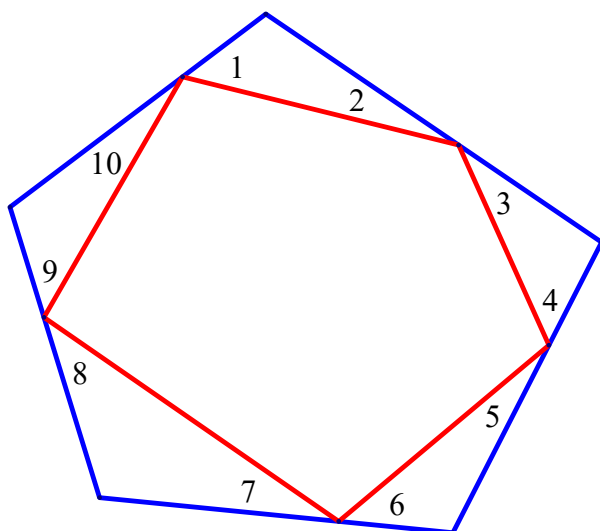
- A. I only
 B. II only
 C. III only
 D. I, II, and III
 E. I and III
 F. None of the above

9. What value of x will guarantee that $s \parallel t$?



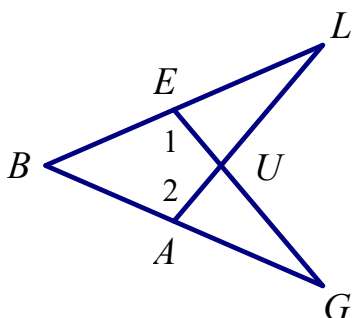
- A. 85
 B. 5
 C. 65
 D. 35
 E. 20
 F. Cannot be determined

10. Find the sum of the numbered angles in the diagram below.



- A. 540°
- B. 270°
- C. 360°
- D. 720°
- E. 180°
- F. None of the above

11. In the diagram below, $\overline{EG} \cong \overline{AL}$ and $\angle 1 \cong \angle 2$. What theorem or postulate can be used to prove that $\triangle EBG \cong \triangle ABL$?

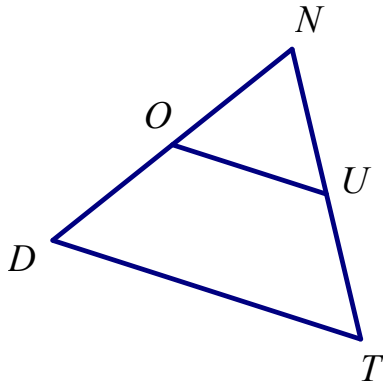


- A. Angle-Angle-Side
- B. Angle-Side-Angle
- C. Side-Angle-Side
- D. Side-Side-Side
- E. Side-Side-Angle
- F. The triangles are not necessarily congruent

12. In $\triangle BLT$, $m\angle T < m\angle B < m\angle L$, list the side lengths from largest to smallest.

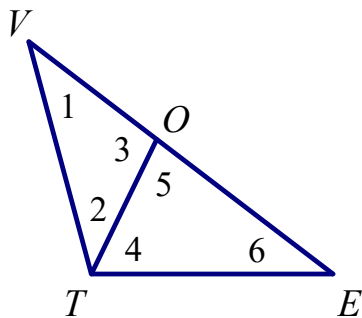
- A. LT, BL, BT
- B. BT, BL, LT
- C. BT, LT, BL
- D. BL, LT, BT
- E. BL, BT, LT
- F. None of the above

13. \overline{OU} is a midsegment of $\triangle DNT$, the ratio of $ON:NT:DT$ is 3:7:8, and the perimeter of $\triangle DNT$ is 126. Find the length of \overline{OU} .



- A. 28
 B. 24
 C. 48
 D. 6
 E. 12
 F. None of the above

14. If $VT = TE$ and $VO > OE$, which of the following statements is true?



- A. $m\angle 1 > m\angle 6$
 B. $m\angle 4 > m\angle 2$
 C. $m\angle 1 < m\angle 6$
 D. $m\angle 4 = m\angle 2$
 E. $m\angle 4 < m\angle 2$
 F. None of the above

15. Andrew has four straws of the following lengths, in centimeters: 4, 7, 2, and 5. If he randomly chooses three of the straws, what is the probability that he can form a triangle with those three straws?

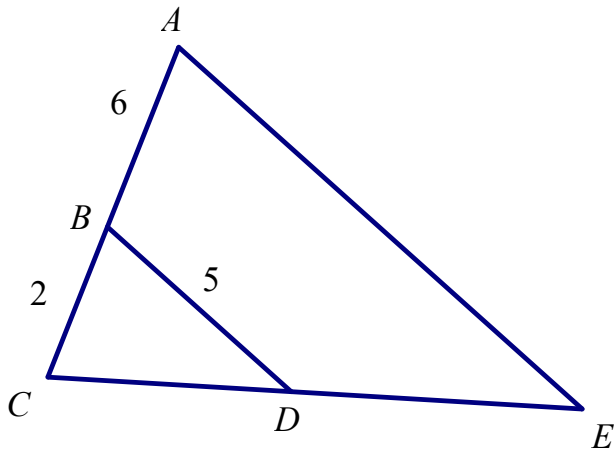
- A. $\frac{3}{4}$ B. 1 C. $\frac{2}{3}$ D. $\frac{1}{2}$ E. $\frac{1}{4}$
 F. None of the above

16. Draw parallelogram $MATH$ with diagonals \overline{MT} and \overline{AH} . If $\angle MHT$ measures 42° and $\angle ATM$ measures 91° , find the measure of $\angle AMT$.

- A. 138°
 B. 89°
 C. 47°
 D. 91°
 E. 57°
 F. Cannot be determined

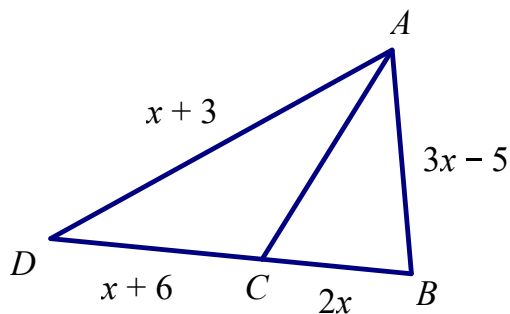
17. If \overline{AD} is a median of $\triangle ABC$ and $\overline{AD} \cong \overline{CD}$, classify $\triangle ABC$.
- A. scalene
 - B. isosceles
 - C. equilateral
 - D. obtuse
 - E. right
 - F. None of the above
18. Jenna is hosting a party with five guests. She hugs each of the five guests once, and each guest hugs each other guest exactly once. How many hugs occur?
- A. 10
 - B. 15
 - C. 9
 - D. 12
 - E. 14
 - F. None of the above

19. Find AE , given that $\overline{BD} \parallel \overline{AE}$.



- A. 25
- B. 20
- C. 11
- D. 15
- E. 9
- F. None of the above

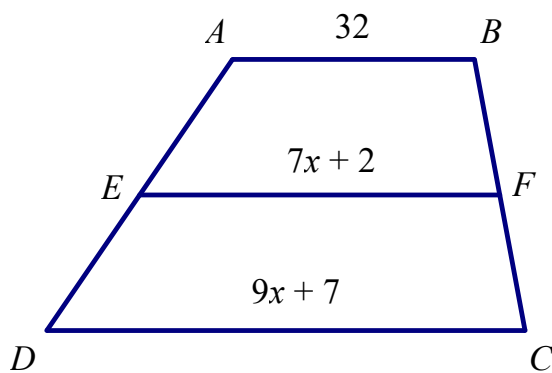
20. The sum of the measures of the interior angles of a regular convex polygon is 1800° . Find the measure of one of its exterior angles.
- A. 30°
 B. 144°
 C. 150°
 D. 36°
 E. 15°
 F. None of the above
21. A rhombus can have at most _____ line(s) of symmetry.
- A. 0
 B. 3
 C. 2
 D. 4
 E. 1
 F. None of the above
22. $\triangle ABC$ has vertices $A(2, 5)$, $B(8, -1)$ and $C(-3, -4)$. Find the length of median \overline{CD} .
- A. $\sqrt{130}$
 B. $2\sqrt{17}$
 C. $\sqrt{85}$
 D. 10
 E. $\sqrt{106}$
 F. None of the above
23. Find the value of x so that \overline{AC} is an angle bisector of $\triangle ABD$.



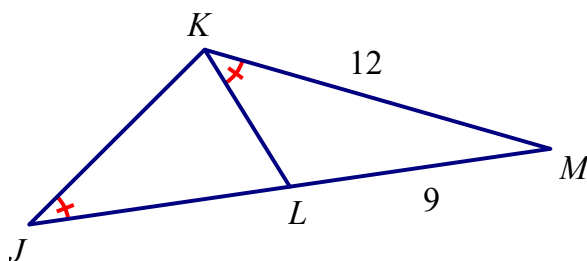
- A. $x = 15$
 B. $x = 3$
 C. $x = 5$
 D. $x = 10$
 E. $x = 6$
 F. No such triangle exists

24. A triangle with side lengths 5, $2\sqrt{10}$, and 9 is what type of triangle?
- A. obtuse
 - B. equiangular
 - C. acute
 - D. isosceles
 - E. right
 - F. None of the above

25. \overline{EF} is a median of trapezoid $ABCD$. Find the length of \overline{CD} .

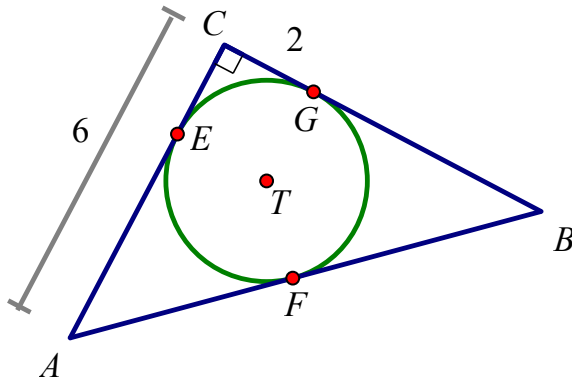


- A. 173.5
 - B. 48
 - C. 70
 - D. 64
 - E. 51
 - F. None of the above
26. In the diagram below, $\angle JKM$ is obtuse. Find JL .



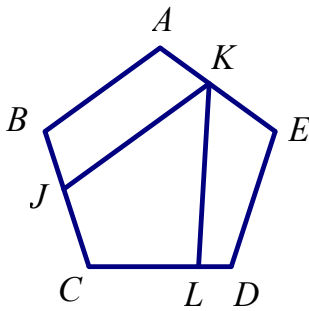
- A. 7
- B. 6
- C. $6\sqrt{3}$
- D. 9
- E. $3\sqrt{7}$
- F. Cannot be determined

27. Circle T is inscribed in right triangle ABC . Find the area of $\triangle ABC$, given that $CG = 2$ and $AC = 6$.



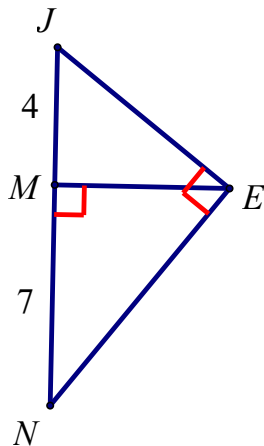
- A. 48
- B. 18
- C. 36
- D. 24
- E. 30
- F. None of the above

28. Regular pentagon $ABCDE$ is shown below. If $\overline{BA} \parallel \overline{JK}$ and $m\angle KLD = 85^\circ$, find $m\angle JKL$.



- A. 36°
- B. 54°
- C. 59°
- D. 23°
- E. 49°
- F. None of the above

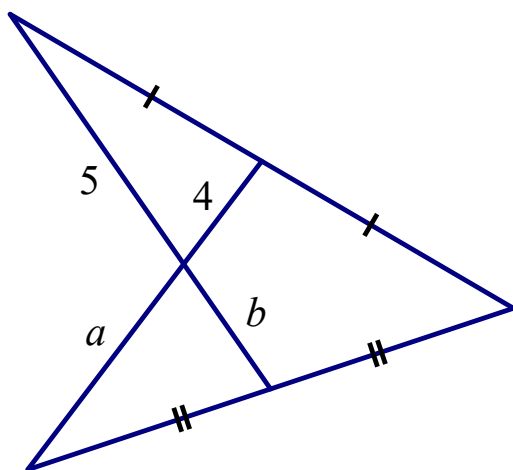
29. Find JE , given that $JM = 4$ and $MN = 7$.



- A. $2\sqrt{7}$
- B. 22
- C. 14
- D. $\sqrt{77}$
- E. $2\sqrt{11}$
- F. None of the above

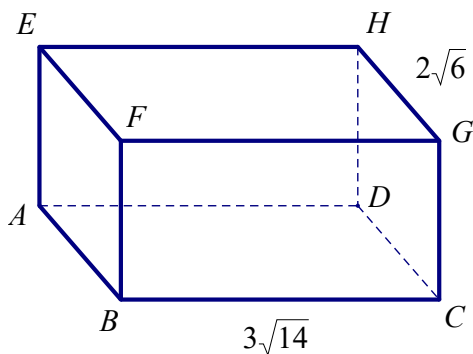
30. Quadrilateral DEFG is inscribed in a circle and $m\angle G = 71^\circ$. Find $m\angle E$.
- A. 142°
 - B. 71°
 - C. 19°
 - D. 35.5°
 - E. 109°
 - F. Cannot be determined

31. Find the value of $a - b$.



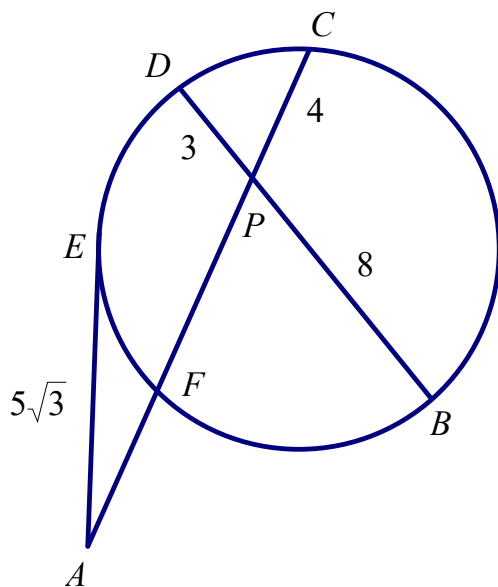
- A. -0.5
- B. 2
- C. 6
- D. 1
- E. 5.5
- F. None of the above

32. Find the height of the following right rectangular prism, given that $BC = 3\sqrt{14}$, $GH = 2\sqrt{6}$, and $CE = 10\sqrt{2}$.



- A. $4\sqrt{11}$
- B. $5\sqrt{2}$
- C. $\sqrt{74}$
- D. $5\sqrt{6}$
- E. $5\sqrt{10}$
- F. None of the above

33. \overline{EA} is tangent to the circle below. Find AF .



- A. 5
 B. 6
 C. 7.5
 D. 15
 E. $2 + \sqrt{3}$
 F. None of the above

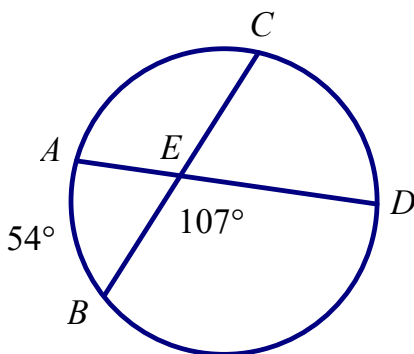
34. A car travels due west for $7\sqrt{2}$ miles, then travels northwest for 18 miles, then travels southwest for 8 miles, and then travels south for $15\sqrt{2}$ miles. How far is the car from its starting point, in miles?

- A. $30\sqrt{2}$
 B. $\sqrt{1042}$
 C. 40
 D. $10\sqrt{10}$
 E. $26 + 22\sqrt{2}$
 F. None of the above

35. Two identically-sized balls fit snugly into a cylindrical container of the same radius so that the combined height of the balls is the same as the height of the container. If each ball has a radius of 5 cm, find the volume of the air in the container which is surrounding the balls, in cubic cm.

- A. 300π B. $\frac{500\pi}{3}$ C. $\frac{2000\pi}{3}$ D. 50π E. $\frac{1000\pi}{3}$
 F. None of the above

36. In the circle below, the degree measure of arc AB is 54° and $m\angle BED = 107^\circ$. Find the degree measure of arc CD .



- A. 92°
 B. 170°
 C. 63.5°
 D. 80.5°
 E. 73°
 F. None of the above

37. Regular hexagon $ABCDEF$ has perimeter $12\sqrt{3}$ cm. Find the length of \overline{BD} , in centimeters.

- A. 24
 B. $12\sqrt{3}$
 C. 6
 D. $4\sqrt{3}$
 E. 8
 F. None of the above

38. In right triangle ABC with hypotenuse \overline{AB} , $\cos(B) = \frac{2}{7}$. Find $\tan(A)$.

- A. $\frac{3\sqrt{5}}{2}$ B. $\frac{2\sqrt{5}}{15}$ C. $\frac{5}{2}$ D. $\frac{7\sqrt{53}}{53}$ E. $\frac{3\sqrt{5}}{7}$
 F. None of the above

39. Regular pentagon $ABCDE$ is dilated to create $A'B'C'D'E'$. The perimeter of $A'B'C'D'E'$ is 20 cm greater than that of $ABCDE$, and the area of $A'B'C'D'E'$ is 9 times that of $ABCDE$. Find the perimeter of $A'B'C'D'E'$, in cm.

- A. 30
 B. 60
 C. 10
 D. 180
 E. 90
 F. Cannot be determined

40. The height of a right square pyramid is equal to the length of a base edge. If the volume of the pyramid is 72 cm^3 , find the lateral area, in square centimeters.
- A. $72\sqrt{2}$
 - B. $24\sqrt{5}$
 - C. $36\sqrt{2}$
 - D. 72
 - E. $36\sqrt{5}$
 - F. None of the above

END OF EXAM ☺