

## Precalculus Exam - University of Houston Math Contest 2024

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- Given:  $P(3x - 2) = 3x^2 - 5x + 7$  find  $P(7)$ .
  - 21
  - 25
  - 27
  - 23
  - 19
  - none of the other answer choices provided
  
- If  $\cos(40^\circ) = b$ , what is  $\sin(50^\circ)$  in terms of  $b$ ?
  - $1/b$
  - $b\sqrt{2}$
  - $b$
  - $90 - b$
  - $-b$
  - none of the other answer choices provided
  
- If  $\tan(y) = k$ , what is  $\sin(y)$  in terms of  $k$ ?
  - $1/\sqrt{k^2 + 1}$
  - $k/\sqrt{k^2 - 1}$
  - $1/\sqrt{1 - k^2}$
  - $k/\sqrt{k^2 + 1}$
  - $2k$
  - none of the other answer choices provided

4. Which of the following is equivalent to the expression

$$\cos(x) + \cos(90^\circ - x) + \sin(x) + \sin(90^\circ - x)$$

for any angle  $x$ ?

- (a) 0
- (b)  $2\sin(x)$
- (c)  $2(\sin(x) + \cos(x))$
- (d)  $2\cos(x)$
- (e) 1
- (f) none of the other answer choices provided

5. If the graph of  $f(x) = (m + 3)x^4 + 2x^{n+6} - 1$  is a line then what is the value of  $m + n$ ?

- (a)  $-3$
- (b)  $-8$
- (c) 0
- (d) 3
- (e) 2
- (f) none of the other answer choices provided

6. What is the positive value of

$$x + \frac{1}{x}$$

given that

$$\left(x + \frac{1}{x}\right)^2 - x - \frac{1}{x} = 6?$$

- (a) 4
- (b) 6
- (c) 1
- (d) 8
- (e) none of the other answer choices provided
- (f) 3

7. If  $f(x) = \sin(x)$  and  $g(x) = \cos(x)$  what is  $(f \circ g)(\pi/2) + (g \circ f)(\pi)$  ?

- (a) 0
- (b)  $-1$
- (c) 3
- (d) 1
- (e) 2
- (f) none of the other answer choices provided

8. If  $y = 6x - 1$  and  $y = bx + 3$  are the equations of perpendicular lines, then what is the value of  $b$ ?

- (a)  $-6$
- (b)  $1/6$
- (c)  $-1/6$
- (d) 6
- (e) 8
- (f) none of the other answer choices provided

9. Find the value of the expression

$$\cos(\tan^{-1}(-\frac{3}{2})).$$

- (a)  $2\sqrt{13}/13$
- (b)  $-3\sqrt{13}/13$
- (c)  $-2\sqrt{13}/13$
- (d)  $\sqrt{13}/13$
- (e)  $-3/13$
- (f) none of the other answer choices provided

10. Find the value of the expression  $\cot(\cos^{-1}(12/13))$ .

- (a)  $9/5$
- (b)  $2$
- (c)  $1$
- (d)  $12/5$
- (e)  $11/5$
- (f) none of the other answer choices provided

11. Evaluate the following trigonometric expression.

$$\frac{\cos^2(\pi/7) + \sin^2(15\pi/7) - 1}{\sin(\pi/8) \sin(\pi/6) + \cos(\pi/4)}$$

- (a)  $1$
- (b)  $2$
- (c)  $0$
- (d)  $3$
- (e)  $4$
- (f) none of the other answer choices provided

12. Find the value of  $\sqrt{3} \cos(15^\circ) - \sin(15^\circ)$ .

- (a)  $1$
- (b)  $\sqrt{3}$
- (c)  $\sqrt{2}$
- (d)  $3\sqrt{2}$
- (e)  $-1$
- (f) none of the other answer choices provided

13. Find the value of

$$\frac{\sqrt{1 - \cos(40^\circ)} + \sqrt{1 + \cos(20^\circ)}}{\cos(115^\circ) \cos(25^\circ)}.$$

- (a)  $-2\sqrt{6}$
- (b) 2
- (c)  $\sqrt{6}$
- (d)  $-\sqrt{6}$
- (e) 0
- (f) none of the other answer choices provided

14. Evaluate  $\cos(8^\circ) \sin(38^\circ) - \cos(82^\circ) \sin(52^\circ)$ .

- (a)  $-1/2$
- (b)  $\sqrt{2}/2$
- (c)  $\sqrt{3}/2$
- (d)  $1/2$
- (e)  $-\sqrt{3}/2$
- (f) none of the other answer choices provided

15. Given  $x + y + z = 90^\circ$ . Find  $\cos(x) \cos(y) - \sin(x) \sin(y)$ .

- (a)  $-\sin(z)$
- (b)  $\cos(z)$
- (c)  $\sin(z)$
- (d)  $-\cos(z)$
- (e)  $\tan(z)$
- (f) none of the other answer choices provided

16. What is the largest possible value of  $3 \sin(x) + 4 \cos(x)$  ?

- (a) 5
- (b) 7
- (c) 6
- (d) 8
- (e) 9
- (f) none of the other answer choices provided

17. The parabola  $y = px^2 + x - 2$  has a vertex on the  $x$ -axis. Find  $p$ .

- (a)  $-1/16$
- (b) 0
- (c) 1
- (d)  $-1/8$
- (e)  $1/2$
- (f) none of the other answer choices provided

18. Find the exact value of

$$\cos \left[ \tan^{-1} \left( \frac{12}{5} \right) + \cot^{-1} \left( \frac{5}{12} \right) \right].$$

- (a)  $-119/169$
- (b)  $5\sqrt{13}/169$
- (c)  $50/169$
- (d)  $-5\sqrt{13}/169$
- (e)  $119/169$
- (f) none of the other answer choices provided

19. If

$$\frac{-3}{\sin(x)} = \frac{4}{\cos(x)}$$

and  $0 < x < \pi$ , find the exact value of  $\cos(x) - \tan(x)$ .

- (a)  $1/20$
- (b)  $-1/20$
- (c)  $9/20$
- (d)  $23/20$
- (e)  $-3/20$
- (f) none of the other answer choices provided

20. Find the number of solutions of the equation  $\cos^2(x) - 8\sin(x) - 8 = 0$  on  $0 < x < 2\pi$ .

- (a) 1
- (b) 2
- (c) 3
- (d) 4
- (e) 5
- (f) none of the other answer choices provided

21. Simplify the following expression

$$\frac{(\tan(x) + 1)(\tan(x) + 1) - \sec^2(x)}{\tan(x)}$$

- (a)  $2\sin(x)$
- (b) 1
- (c) 2
- (d)  $\tan(x)$
- (e)  $-1$
- (f) none of the other answer choices provided

22. Simplify

$$2 + \frac{1}{\cos^2(x)} - \tan^2(x)$$

- (a) 3
- (b)  $2 \sin(x)$
- (c)  $2 \cos(x)$
- (d)  $2/5$
- (e) 1
- (f) none of the other answer choices provided

23. Which of the following is the period (in radians) of the function

$$f(x) = 3 \cot^4(2x - 15^\circ) + 1?$$

- (a)  $\pi/2$
- (b)  $3\pi/2$
- (c)  $\pi$
- (d)  $15\pi/2$
- (e)  $6\pi$
- (f) none of the other answer choices provided

24. Consider the equation

$$\frac{\cot(x) + \frac{\cos^3(x) + \sin^2(x) \cos^3(x)}{1 - \sin^4(x)} - \cos(x)}{\sin(x)} = 0.$$

Find the number of the solutions of the equation.

- (a) 1
- (b) 2
- (c) 3
- (d) 4
- (e) 0
- (f) none of the other answer choices provided



25. Consider  $\triangle ABC$  with  $\angle CBA = 135^\circ$ ,  $|BC| = 2\sqrt{2}$  in,  $|AB| = 3$  in. Find the area of the triangle.
- (a)  $3$  in<sup>2</sup>
  - (b)  $9$  in<sup>2</sup>
  - (c)  $6$  in<sup>2</sup>
  - (d)  $2$  in<sup>2</sup>
  - (e)  $12$  in<sup>2</sup>
  - (f) none of the other answer choices provided
26. Consider  $\triangle ABC$  with  $\angle A = 75^\circ$ ,  $\angle B = 60^\circ$  and  $|AC| = 8$  in. Find  $|AB|$ .
- (a)  $4\sqrt{6}/3$  in
  - (b)  $8\sqrt{3}/6$  in
  - (c)  $\sqrt{6}$  in
  - (d)  $8$  in<sup>2</sup>
  - (e)  $8\sqrt{6}/3$  in
  - (f) none of the other answer choices provided
27. Consider  $\triangle ABC$  with  $\angle B = \angle C + 90^\circ$ ,  $|AC| = 4$  in and  $|AB| = 3$  in. What is  $\cot(C)$ ?
- (a)  $4/3$
  - (b)  $5/3$
  - (c)  $1$
  - (d)  $2$
  - (e)  $5$
  - (f) none of the other answer choices provided

28. Find the smallest solution for the equation  $\sin\left(2x - \frac{\pi}{3}\right) + \sin(x) = 0$  on  $[0, 2\pi)$ .

- (a)  $\pi/9$
- (b) 0
- (c)  $\pi/8$
- (d)  $\pi/6$
- (e)  $2\pi/9$
- (f)  $8/\pi$
- (g) none of the other answer choices provided

29.  $x = \ln(2)$  and  $y = \ln(3)$ . What is  $\log_6(12)$  in terms of  $x$  and  $y$ ?

- (a)  $x/(2y)$
- (b)  $(2 + y)/y$
- (c)  $(2x + y)/(x + y)$
- (d)  $2x/y$
- (e)  $(x + 2)/(2y)$
- (f) none of the other answer choices provided

30. Find the radius of the circle  $x^2 + y^2 + 12x - 14y + 84 = 0$ .

- (a) 6
- (b) 3
- (c) 1
- (d) 8
- (e) 5
- (f) none of the other answer choices provided

31. A rectangular board has perimeter of  $72 \text{ ft}$  and an area of  $288 \text{ ft}^2$ . Find the width of the rectangle.
- (a)  $24 \text{ ft}$
  - (b)  $10 \text{ ft}$
  - (c)  $12 \text{ ft}$
  - (d)  $14 \text{ ft}$
  - (e)  $11 \text{ ft}$
  - (f) none of the other answer choices provided

32. If  $\vec{a} = \vec{i} + 2\vec{j}$  and  $\vec{b} = 3\vec{i} + 3\vec{j}$ , what is the the magnitude of the vector

$$\frac{1}{3}\vec{b} - \vec{a}?$$

- (a) 1
  - (b) 2
  - (c) 3
  - (d) 8
  - (e) 5
  - (f) none of the other answer choices provided
33. Find the number of points where the circle  $x^2 + y^2 = 100$  and the parabola  $x^2 + 2y = 1$  intersect.
- (a) 0
  - (b) 3
  - (c) 1
  - (d) 2
  - (e) 6
  - (f) none of the other answer choices provided.

34. The graph of the equation  $y = -|x| - 6$  is symmetric with respect to

- (a) the  $x$ -axis
- (b) the  $y$ -axis
- (c) the origin
- (d) the  $x$ -axis and the  $y$ -axis
- (e) the  $x$ -axis and the origin
- (f) none of the other answer choices provided.

35. If

$$ax^3 + bx^2 + cx + d = (x^2 - 1)(mx + n) + 3x + 4,$$

what is  $b + d$ ?

- (a) 4
- (b) 1
- (c) 2
- (d) 5
- (e) 3
- (f) none of the other answer choices provided.

36. How many solutions does  $4 \cos^2(\theta) - 3 = 0$  have on  $0 \leq \theta < 2\pi$ ?

- (a) 3
- (b) 2
- (c) 0
- (d) 4
- (e) 1
- (f) none of the other answer choices provided

37. Identify the graph of the equation  $4x^2 - 4xy + 5y^2 - 20 = 0$ .

- (a) ellipse
- (b) parabola
- (c) hyperbola
- (d) circle
- (e) line
- (f) none of the other answer choices provided

38. Identify the type of conic given by

$$4x^2 + 8xy + 4y^2 - 2x - 5y - 2 = 0.$$

- (a) parabola
- (b) ellipse
- (c) hyperbola
- (d) circle
- (e) line
- (f) none of the other answer choices provided

39. Find the  $x$ -coordinate of the center of the circle  $x^2 + y^2 - 6x + 14y + 33 = 0$ .

- (a)  $-7$
- (b)  $5$
- (c)  $-3$
- (d)  $3$
- (e)  $1$
- (f) none of the other answer choices provided

40. Find the  $y$ -coordinate of the vertex of the parabola  $f(x) = 2(x - 4)^2 - 5$ .

- (a)  $-5$
- (b)  $5$
- (c)  $4$
- (d)  $2$
- (e)  $1$
- (f) none of the other answer choices provided

41. Find the radius of the circle

$$(x + 3)^2 + \left(y - \frac{1}{3}\right)^2 = 11.$$

- (a)  $11$
- (b)  $121$
- (c)  $\sqrt{121}$
- (d)  $12$
- (e)  $\sqrt{11}$
- (f) none of the other answer choices provided

42. If

$$\frac{\tan(4x) - \tan(3x)}{1 + \tan(4x)\tan(3x)} = \frac{1}{2},$$

what is the value of  $\tan(2x)$ ?

- (a)  $4/3$
- (b)  $3/4$
- (c)  $-3/4$
- (d)  $-1$
- (e)  $1$
- (f) none of the other answer choices provided

43. Which of the following is **not** a solution of the equation  $\tan(2\theta) + 2\cos(\theta) = 0$  on the interval  $0 \leq \theta < 2\pi$ ?
- (a)  $\pi/2$
  - (b)  $7\pi/6$
  - (c)  $\pi/6$
  - (d)  $3\pi/2$
  - (e)  $3$
  - (f) none of the other answer choices provided
44. The vector  $\vec{v} = 4\vec{i} - 4\vec{j}$ . Find the direction angle of  $\vec{v}$ .
- (a)  $215^\circ$
  - (b)  $45^\circ$
  - (c)  $60^\circ$
  - (d)  $30^\circ$
  - (e)  $315^\circ$
  - (f) none of the other answer choices provided
45. Find the solution set of the equation  $\tan^{-1}(x^2 + x - 12) = -\pi$ .
- (a)  $\{-4, 3\}$
  - (b)  $\{-5, 6\}$
  - (c)  $\{-5, 3, 6\}$
  - (d)  $\{-5, 2, 6\}$
  - (e)  $\{-5, 3, 6\}$
  - (f) none of the other answer choices provided.

46. Find the solution of the equation  $\tan^{-1}(2x + 1) = \cot^{-1}(5)$ .

- (a)  $-1/5$
- (b)  $-1/8$
- (c)  $1/5$
- (d)  $-2/5$
- (e)  $2/5$
- (f) none of the other answer choices provided.

47. One of the vertical asymptotes of

$$y = \frac{2x^2}{2x^2 - x - 3}$$

is

- (a)  $x = 1$
- (b)  $x = -1$
- (c)  $x = -3/2$
- (d)  $x = 3$
- (e)  $x = 4$
- (f)  $x = 2/3$
- (g) none of the other answer choices provided.

48.  $y = 6x^4 + 7|x|$  is

- (a) an even function
- (b) an odd function
- (c) neither an odd nor an even function
- (d) a trigonometric function
- (e) a polynomial function
- (f) a linear function
- (g) none of the other answer choices provided.



49. Simplify  $\sin(x + 2\pi) \cot(x + \pi)$ .

- (a)  $\sin(x)$
- (b)  $\cot(x)$
- (c)  $\cos(x)$
- (d)  $\tan(x)$
- (e)  $\cos(2x)$
- (f) none of the other answer choices provided.

50. Simplify

$$\frac{\cos^2(a) - \cos^2(b)}{\sin(a + b) \sin(a - b)}$$

- (a)  $-1$
- (b)  $\tan(a)$
- (c)  $\tan(2a)$
- (d)  $1$
- (e)  $\tan(3a)$
- (f) none of the other answer choices provided.