

Algebra II Exam - Key

1. Find the distance from the line $3x + 4y = 6$ to the origin.

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| Answer: | $6/5$ |
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2. Write the equation in slope-intercept form of the line that passes through $(-3, 2)$ and divides the circle $x^2 - 4x + y^2 + 8y = 1$ in half.

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| Answer: | $y = -6x/5 - 8/5$ |
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3. The Pnut company sells three different boxes of nuts; small, medium and large. Last week they shipped three crates containing different numbers of the different sizes of boxes of nuts. The first crate contained seven boxes, with one of them large, three of them medium, and three of them small. The second crate contained eight boxes, with two of them large and six of them small. The third crate contained ten boxes, with four of them medium and six of them small. Each crate was weighed, and it was determined that the total weight of the boxes of nuts in each crate was 18 pounds. Give the weight of each of the boxes of nuts.

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| Answer: | 6 pounds, 3 pounds and 1 pound. |
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4. Suppose $f(x) = \sqrt{x-3} - 6$. Find $f^{-1}(x)$ and give the domain of the inverse function.

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| Answer: | $f^{-1}(x) = x^2 + 12x + 39, x \geq -6$ |
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5. Give the area enclosed by the graph of $\frac{|x|}{3} + 2|y| = 4$.

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| Answer: | 48 square units |
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6. Give the solution to the inequality $x^3 - 1 < 2x - 2$.

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| Answer: | $x < -\frac{1}{2} - \frac{\sqrt{5}}{2}$ or $-\frac{1}{2} + \frac{\sqrt{5}}{2} < x < 1$ |
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7. Give the remainder of $\frac{x^5}{x^3 + x + 1}$.

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| Answer: | $-x^2 + x + 1$ |
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8. Give the 20,000th term in the sequence 1,2,2,3,3,3,4,4,4,4,5,5,5,5,6,6,6,6,6,7,...

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| Answer: | 199 |
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9. Let $f(x) = 3x^2 - 6x + 7$. Simplify the expression $\frac{f(x+h) - f(x-h)}{2h}$ for $h \neq 0$.

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| Answer: | $6x - 6$ |
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10. Determine the values of a and b so that $\frac{x^4 - 2x^3 + ax^2 - bx + 1}{x^2 - x - 2}$ is a polynomial.

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| Answer: | $a = -3/2, b = -5/2$ |
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11. Give an equation (in slope-intercept form) for the line that passes through the intersection of the lines $2x + 3y = 7$ and $x - 4y = -2$, and is perpendicular to the vector $3i - 5j$.

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| Answer: | $y = 3x/5 - 1/5$ |
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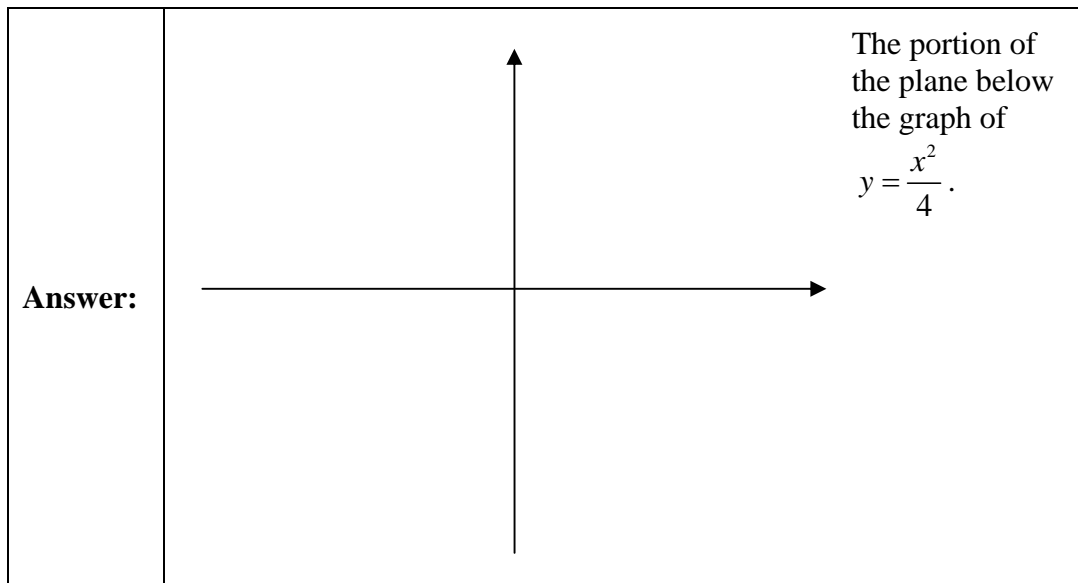
12. Give all of the solutions to the equation $|x + 2|^2 = 3|x + 2| + 4$.

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| Answer: | $x = -6$ or $x = 2$ |
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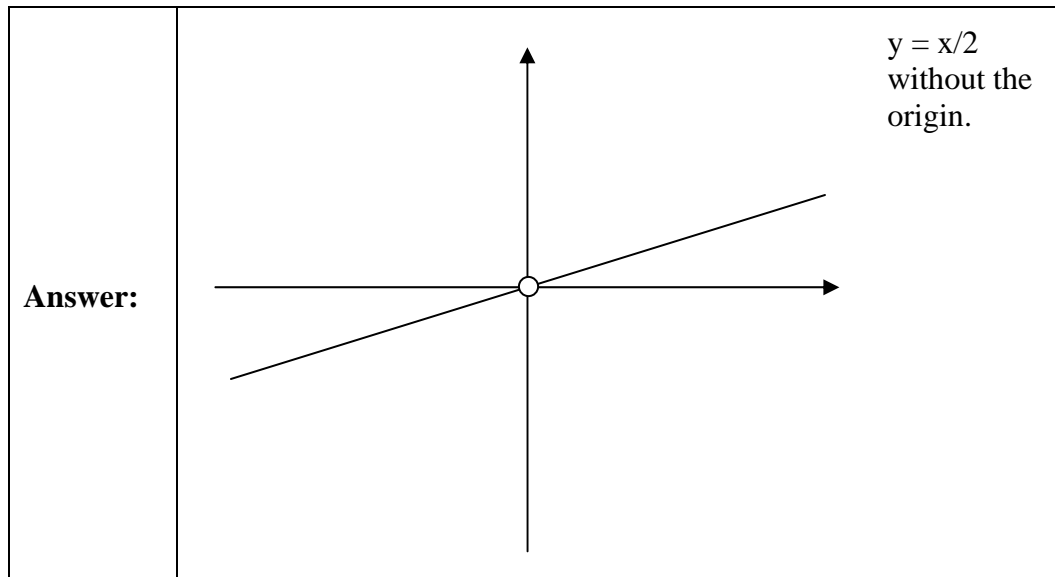
13. a, b and c are positive integers between 1 and 9. What prime numbers must always divide the sum of the three 3-digit numbers abc, bca and cab ?

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| Answer: | 3 and 37 |
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14. Graph the points (a,b) for which the quadratic $x^2 + ax + b$ can be written as the product of two distinct linear factors.



15. Graph the equation $(2x + 6y)/(x + 3y) = x/y$.



16. Write the number 897 in base 6.

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| Answer: | 4053 |
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17. Which of the following are true statements?

- a. The sum of two rational numbers is always a rational number.
- b. The sum of two irrational numbers is always an irrational number.
- c. The product of two irrational numbers can be an irrational number.
- d. The product of an integer and a rational number is a rational number.
- e. The sum of an irrational number and a rational number is an irrational number.
- f. The product of an irrational number and a rational number is an irrational number.

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| Answer: | <i>a, c, d and e</i> |
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18. An integer n lies between 901 and 923. What is the least number of divisions that must be performed to determine whether n is prime, regardless of the value of n (assuming, of course, that you do not know the primes between 901 and 923)?

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| Answer: | The number of primes less than 30. i.e. 10 |
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19. Give the value of $1 + 2 + 4 + 8 + 16 + 32 + 64 + 128 + 256 + 512 + \dots + 1048576$.

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| Answer: | $2(1048576) - 1 = 2097151$ |
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20. b is a fixed positive real number. A point $(0,c)$ lies on the positive y -axis, and an arbitrary point $(a,4b)$ is chosen so that $-2 < a < 2$. A vertical line segment is drawn downward (parallel to the y -axis) from this point until it contacts the parabola given by $y = bx^2$ at a new point. Then a second line segment is drawn to connect this new point to the point $(0,c)$. How should c be chosen so that the sum of the lengths of these two line segments is independent of the value of a ?

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| Answer: | $c = 1/(4b)$ |
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