

# Algebra I Exam - Key

1. Ten lines are graphed in the  $xy$ -plane, and they are labeled L1, L2, L3, ..., L10. L1 is perpendicular to L2, L2 is parallel to L3, L3 is perpendicular to L4, L4 is parallel to L5, etc... The slope of L9 is  $1/5$  and L1 passes through the point  $(-2,4)$ . Give an equation for L1 in slope-intercept form.

<b>Answer:</b>	$y = \frac{x}{5} + \frac{22}{5}$
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2. There are infinitely many pairs  $(a,b)$  such that  $x - 2$  is a factor of  $x^2 + ax + b$ . Describe the graph of the points  $(a,b)$ .

<b>Answer:</b>	The graph is the line $2x + y = -4$
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3. A room contains only horses and people. If there are 25 heads in the room and 74 legs in the room, how many horses are in the room?

<b>Answer:</b>	12 horses, 13 people
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4. Give values for  $A$ ,  $B$  and  $C$  so that  $-3x^2 + 7x + 11 = A(x + B)^2 + C$ .

<b>A = -3</b>	<b>B = -7/6</b>	<b>C = 181/12</b>
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5. Mold is growing in a dish, and the amount of mold doubles every 30 minutes. After 4 hours of growth, the dish is covered with mold. What percent of the dish was initially covered with mold?

<b>Answer:</b>	25/64 %
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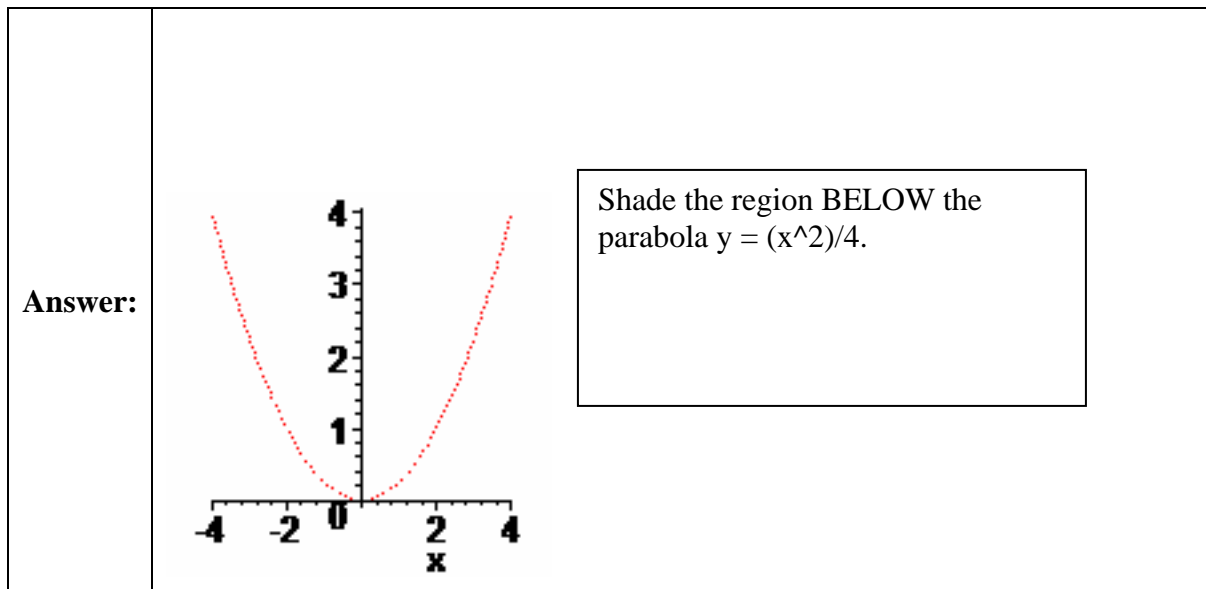
6. An old car was driven for its first 2 years by Tom, then for  $1/6$  of its age by Ann,  $1/5$  if its age by Jill,  $1/4$  of its age by Harry, and  $1/3$  of its age by Julie. How old is the car?

<b>Answer:</b>	40 years
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7. A small dog slips down the side of a steep ravine whose side is 18 feet long. The dog becomes tired after struggling for one minute to climb three feet up the slippery ravine, and rests for a minute, during which time it slides two feet down the ravine. This process continues. How long does it take for the dog to climb out of the ravine?

<b>Answer:</b>	31 minutes (assuming the dog it out when it reaches the edge at the top). We will take 31 or 32.666666 or 33.
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8. Graph the points  $(a,b)$  for which the quadratic  $x^2 + ax + b$  can be written as the product of two distinct linear factors.



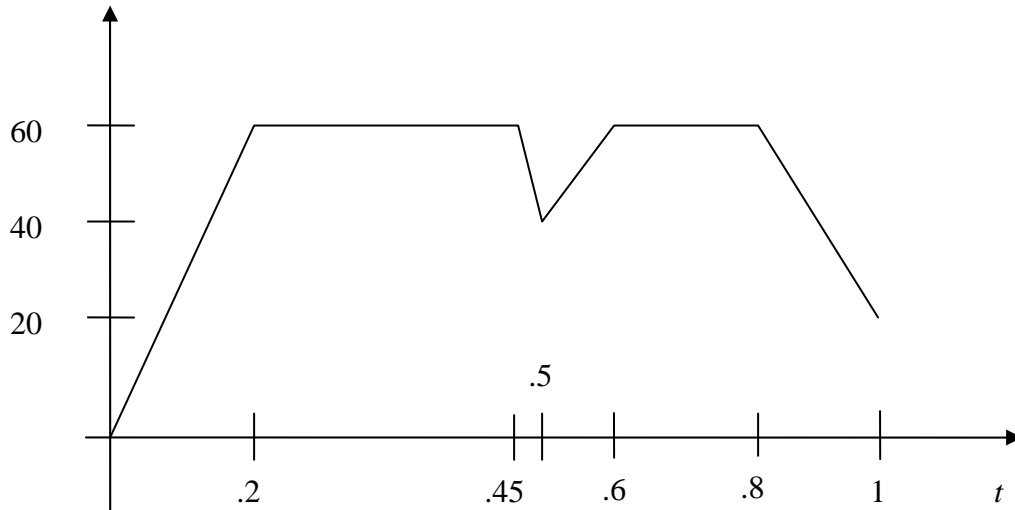
9. Describe the set of points  $(a,b)$  so that the system  $\begin{cases} ax + by = 6 \\ -x + 2y = 3 \end{cases}$  has a unique solution.

<b>Answer:</b>	We need $2a + b \neq 0$ . So, we get all of the points off of the line $2x + y = 0$ .
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10. Give the area of the polygon in the  $xy$ -plane with vertices  $(1,2)$ ,  $(2,5)$ ,  $(5,1)$  and  $(3,-2)$ .

<b>Answer:</b>	$27/2$
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11. A car is driving north on I-45 from Houston towards Dallas. The speed of the car (in miles per hour) for the first hour of travel is given in the graph below. How far did the car travel during this hour?

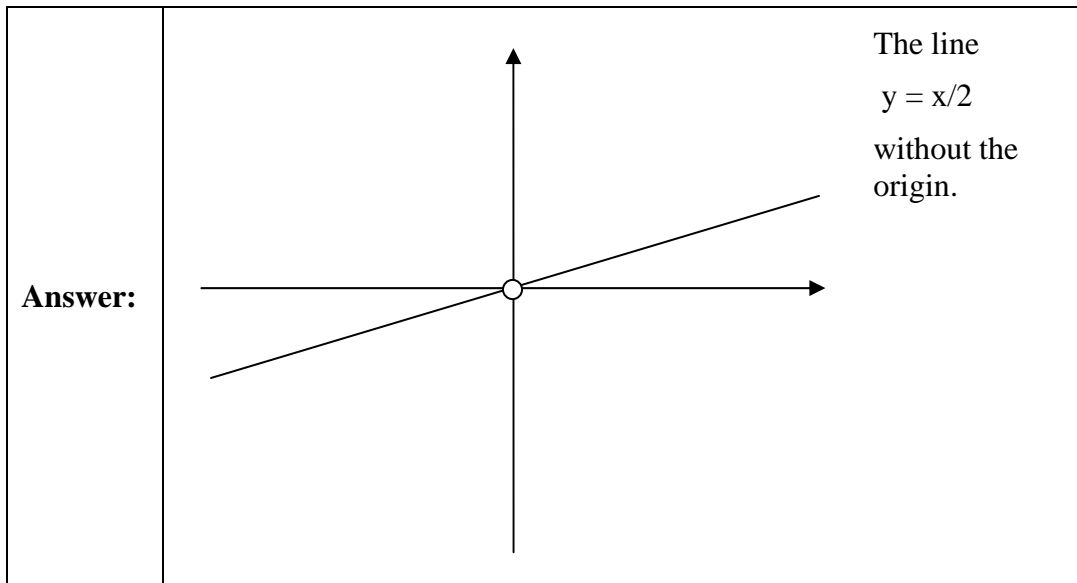


<b>Answer:</b>	48.5 miles
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12. How much pure sugar should be added to 20 ounces of a 30% sugar solution to increase its sugar concentration to 50%?

<b>Answer:</b>	8 ounces
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13. Graph the equation  $(2x + 6y)/(x + 3y) = x/y$ .



14. Write the number 897 in base 6.

<b>Answer:</b>	4053
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15.  $a$ ,  $b$  and  $c$  are positive integers between 1 and 9. What prime numbers must always divide the sum of the three 3-digit numbers  $abc$ ,  $bca$  and  $cab$ ?

<b>Answer:</b>	3 and 37
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16. Which of the following are true statements?

- The sum of two rational numbers is always a rational number.
- The sum of two irrational numbers is always an irrational number.
- The product of two irrational numbers can be an irrational number.
- The product of an integer and a rational number is a rational number.
- The sum of an irrational number and a rational number is an irrational number.
- The product of an irrational number and a rational number is an irrational number.

<b>Answer:</b>	a, c, d and e. (note: in f the rational number could be 0.)
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17. An integer  $n$  lies between 901 and 923. What is the least number of divisions that must be performed to determine whether  $n$  is prime, regardless of the value of  $n$  (assuming, of course, that you do not know the primes between 901 and 923)?

<b>Answer:</b>	The number of primes less than 30. i.e. 10
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18. Give the value of  $1 + 2 + 4 + 8 + 16 + 32 + 64 + 128 + 256 + 512 + \dots + 1048576$ .

<b>Answer:</b>	$2(1048576) - 1 = 2097151$
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19. A Grecian gentleman has the opportunity to win a prize. He is given two urns and four marbles, two red and two white marbles. He may arrange the marbles in the two urns any way he chooses. Then another person will randomly draw one marble out of one of the urns. If the marble is white, the Grecian gentleman will win the prize. If the marble is red, he will win nothing. The Grecian gentleman wants your help. How could he best arrange the four marbles in the two urns to improve his chances of winning the prize?

<b>Answer:</b>	Put all of the marbles in one of the urns.
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20.  $b$  is a fixed positive real number. A point  $(0,c)$  lies on the positive  $y$ -axis, and an arbitrary point  $(a,4b)$  is chosen so that  $-2 < a < 2$ . A vertical line segment is drawn downward (parallel to the  $y$ -axis) from this point until it contacts the parabola given by  $y = bx^2$  at a new point. Then a second line segment is drawn to connect this new point to the point  $(0,c)$ . How should  $c$  be chosen so that the sum of the lengths of these two line segments is independent of the value of  $a$ ?

<b>Answer:</b>	$c = 1/(4b)$
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