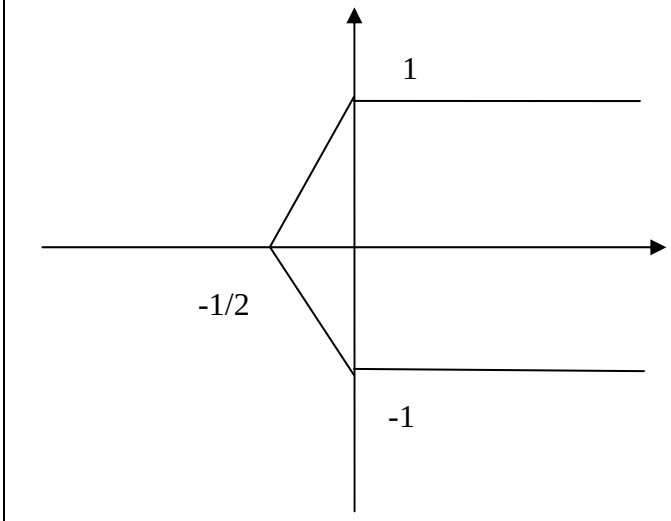
PreCalculus Exam - Key

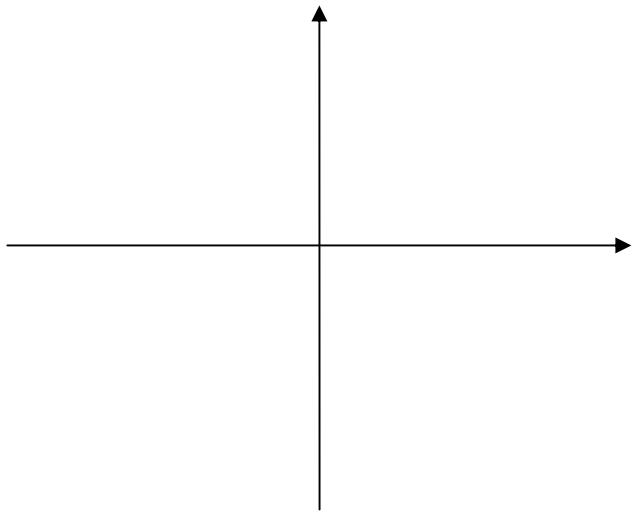
1. Graph the equation $|x|+|y|=1+x$.

Answer:



2. Graph the points (*a*,*b*) for which the function $f(x)=\frac{ax+2}{bx-3}$ is not invertible.

Answer:



The line

-3a -2b = 0

3. Find all positive solutions to the equation $x^{\left(x^x\right)} = \left(x^x\right)^x$.

Answer:	x = 1, 2

4. Simplify $16\frac{\log_4(x)}{\log_2(x)} - 32\frac{\log_2(x)}{\log_4(x)} + 7\frac{\log_5(y)}{\log_{25}(y)}$.

Answer:	-42

5. Simplify the expression $\sin^{-1}(x) + \cos^{-1}(x)$ for $0 < x < 1$.

Answer:	$\pi / 2$

6. The decay equation for radon-222 gas is known to be $y = y_0 e^{-9t/50}$, with *t* given in days. How long will it take for the radon in a sealed sample of air to fall to 90% of its original value?

Answer:	t = (50/9) ln(10/9) days

7. Take any number. Add *c* to the number, double the result, subtract 6, divide by 2 and subtract 2. Give the value of *c* so that the resulting number is the original number.

Answer:	c = 5

8. A wheel with radius 2 feet is positioned in the *xy*-plane with its center at the origin. It is resting on the horizontal line *y* = -2, and it begins to roll forward at with angular velocity of 1 revolution per minute. A point *P* on the wheel is located at (2,0) before the wheel starts to roll. Give functions *x*(*t*) and *y*(*t*) (with *t* representing time in minutes) so that the point *P* is located at (*x*(*t*),*y*(*t*)) for all *t* > 0.

Answer:	$x(t) = 4\pi t + 2\cos(2\pi t),\ y(t) = -2\sin(2\pi t)$

9. Find the cosine of the acute angle of intersection of the lines $2x+3y=7$ and $x-4y=-2$.

Answer:	$\frac{10}{\sqrt{13}\sqrt{17}}$ *or* $\frac{10}{\sqrt{221}}$ *or* $\frac{10\sqrt{221}}{221}$

10. The point $P=\left(\frac{1}{2}, 2+\sqrt{3}\right)$ lies on the ellipse $4x^2+y^2=4y$. The line tangent to the ellipse at this point has slope $\frac{-2}{\sqrt{3}}$. Suppose it is possible to rotate and slide the ellipse in the *xy*-plane so that the point *P* is located at the origin and all other points on the ellipse are in quadrants I and II. What is the new location of the center of the ellipse?

Answer:	$\left(-\frac{2\sqrt{7}}{7}, \frac{5\sqrt{21}}{14}\right)$

11. Let $A=\begin{pmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{pmatrix}$. Find A^{50}.

Answer:	$A^{50}=\begin{pmatrix} -1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & -1/2 \end{pmatrix}$

12. Let $f(x) = x^4$. Simplify $\dfrac{f(x+h) - f(x-h)}{2h}$.

Answer:	$4x^3 + 4xh^2$

13. Let $g(x) = a\dfrac{f(x+h) - f(x-h)}{2h} + (1-a)\dfrac{f\left(x+h/2\right) - f\left(x-h/2\right)}{h}$ where *f* is the function given in the problem above. There is a number *a* so that $g(x)$ is independent of both *a* and *h*. Give both *a* and the resulting value of $g(x)$.

Answer:	$a = -1/3,\ g(x) = 4x^3$

14. The graphs of the equations $y = |x+2| - 5$ and $y = 5 - |x-2|$ enclose a rectangular region in the coordinate plane. What is the area of this rectangle?

Answer:	42

15. Simplify the expression $\left(1-\dfrac{1}{2^2}\right)\left(1-\dfrac{1}{3^2}\right)\cdots\left(1-\dfrac{1}{n^2}\right)$ where *n* is an integer that is larger than 3.

Answer:	$\dfrac{n+1}{2n}$

16. Determine the positive integers *n* for which $3^{2n+1}+2^{n+2}$ is divisible by 7.

Answer:	All positive integers.

17. Suppose $f(x)=x^3$ and $F(x)=\left(1-\frac{1}{x^4}\right)^2$. Give a function g so that $f\circ g=F$.

Answer:	$g(x)=\left(1-\frac{1}{x^4}\right)^{2/3}$

18. Suppose ℓ_1 and ℓ_2 are non-vertical lines in the plane with slopes m_1 and m_2 respectively. Assume $m_1m_2\neq -1$ and let α be the acute angle between ℓ_1 and ℓ_2. Find a formula for $\tan(\alpha)$ in terms of m_1 and m_2.

Answer:	$\tan(\alpha)=\left\|\frac{m_1-m_2}{1+m_1m_2}\right\|$

19. Give the value of 1 + 2 + 4 + 8 + 16 + 32 + 64 + 128 + 256 + 512 + … + 1048576.

Answer:	2(1048576) – 1 = 2097151

20. *b* is a fixed positive real number. A point (0,*c*) lies on the positive *y*-axis, and an arbitrary point (*a*,4*b*) is chosen so that -2 < *a* < 2. A vertical line segment is drawn downward (parallel to the *y*-axis) from this point until it contacts the parabola given by $y=bx^2$ at a new point. Then a second line segment is drawn to connect this new point to the point (0,*c*). How should *c* be chosen so that the sum of the lengths of these two line segments is independent of the value of *a*?

Answer:	*c* = 1/(4*b*)