1. Graph the equation $|x| + |y| = 1 + x$.

Answer:

2. Graph the points $(a, b)$ for which the function $f(x) = \frac{ax + 2}{bx - 3}$ is not invertible.

Answer:

The line

$-3a - 2b = 0$
3. Find all positive solutions to the equation $x^{x^x} = \left(x^x\right)^x$.

Answer: $x = 1, 2$

4. Simplify $16 \frac{\log_4(x)}{\log_2(x)} - 32 \frac{\log_2(x)}{\log_4(x)} + 7 \frac{\log_5(y)}{\log_{25}(y)}$.

Answer: $-42$

5. Simplify the expression $\sin^{-1}(x) + \cos^{-1}(x)$ for $0 < x < 1$.

Answer: $\pi / 2$

6. The decay equation for radon-222 gas is known to be $y = y_0 e^{-9t/50}$, with $t$ given in days. How long will it take for the radon in a sealed sample of air to fall to 90% of its original value?

Answer: $t = (50/9) \ln(10/9)$ days

7. Take any number. Add $c$ to the number, double the result, subtract 6, divide by 2 and subtract 2. Give the value of $c$ so that the resulting number is the original number.

Answer: $c = 5$

8. A wheel with radius 2 feet is positioned in the $xy$-plane with its center at the origin. It is resting on the horizontal line $y = -2$, and it begins to roll forward at with angular velocity of 1 revolution per minute. A point $P$ on the wheel is located at $(2,0)$ before the wheel starts to roll. Give functions $x(t)$ and $y(t)$ (with $t$ representing time in minutes) so that the point $P$ is located at $(x(t),y(t))$ for all $t > 0$.

Answer: $x(t) = 4\pi t + 2\cos(2\pi t), \; y(t) = -2\sin(2\pi t)$
9. Find the cosine of the acute angle of intersection of the lines \(2x + 3y = 7\) and \(x - 4y = -2\).

Answer: \(\frac{10}{\sqrt{13}}, \frac{10}{\sqrt{221}}, \text{ or } \frac{10\sqrt{221}}{221}\)

10. The point \(P = \left(\frac{1}{2}, 2 + \sqrt{3}\right)\) lies on the ellipse \(4x^2 + y^2 = 4y\). The line tangent to the ellipse at this point has slope \(-\frac{2}{\sqrt{3}}\). Suppose it is possible to rotate and slide the ellipse in the \(xy\)-plane so that the point \(P\) is located at the origin and all other points on the ellipse are in quadrants I and II. What is the new location of the center of the ellipse?

Answer: \(\left(-\frac{2\sqrt{7}}{7}, \frac{5\sqrt{21}}{14}\right)\)

11. Let \(A = \begin{pmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{pmatrix}\). Find \(A^{50}\).

Answer: \(A^{50} = \begin{pmatrix} -1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & -1/2 \end{pmatrix}\)
12. Let $f(x) = x^4$. Simplify \( \frac{f(x+h) - f(x-h)}{2h} \).

| Answer: | $4x^3 + 4xh^2$ |

13. Let $g(x) = a \frac{f(x+h) - f(x-h)}{2h} + (1-a) \frac{f(x+h/2) - f(x-h/2)}{h}$ where $f$ is the function given in the problem above. There is a number $a$ so that $g(x)$ is independent of both $a$ and $h$. Give both $a$ and the resulting value of $g(x)$.

| Answer: | $a = -1/3, \ g(x) = 4x^3$ |

14. The graphs of the equations $y = |x+2| - 5$ and $y = 5 - |x-2|$ enclose a rectangular region in the coordinate plane. What is the area of this rectangle?

| Answer: | 42 |

15. Simplify the expression $\left( 1 - \frac{1}{2^2} \right) \left( 1 - \frac{1}{3^2} \right) \cdots \left( 1 - \frac{1}{n^2} \right)$ where $n$ is an integer that is larger than 3.

| Answer: | $\frac{n+1}{2n}$ |
16. Determine the positive integers $n$ for which $3^{2n+1} + 2^{n+2}$ is divisible by 7.

**Answer:** All positive integers.

17. Suppose $f(x) = x^3$ and $F(x) = \left(1 - \frac{1}{x^4}\right)^2$. Give a function $g$ so that $f \circ g = F$.

**Answer:** 
$$g(x) = \left(1 - \frac{1}{x^4}\right)^{2/3}$$

18. Suppose $\ell_1$ and $\ell_2$ are non-vertical lines in the plane with slopes $m_1$ and $m_2$ respectively. Assume $m_1m_2 \neq -1$ and let $\alpha$ be the acute angle between $\ell_1$ and $\ell_2$. Find a formula for $\tan(\alpha)$ in terms of $m_1$ and $m_2$.

**Answer:** 
$$\tan(\alpha) = \frac{m_1 - m_2}{1 + m_1m_2}$$

19. Give the value of $1 + 2 + 4 + 8 + 16 + 32 + 64 + 128 + 256 + 512 + \ldots + 1048576$.

**Answer:** 
$$2(1048576) - 1 = 2097151$$

20. $b$ is a fixed positive real number. A point $(0,c)$ lies on the positive $y$-axis, and an arbitrary point $(a,4b)$ is chosen so that $-2 < a < 2$. A vertical line segment is drawn downward (parallel to the $y$-axis) from this point until it contacts the parabola given by $y = bx^2$ at a new point. Then a second line segment is drawn to connect this new point to the point $(0,c)$. How should $c$ be chosen so that the sum of the lengths of these two line segments is independent of the value of $a$?

**Answer:** 
$$c = 1/(4b)$$