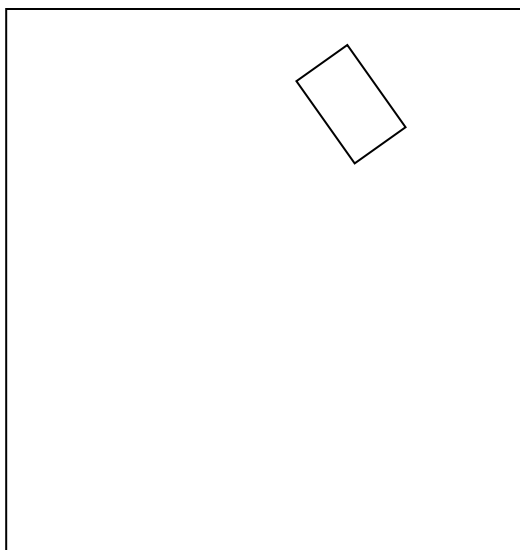


What Is The Point?

Aman and Hisson enjoy digital photography, and they spend many hours enhancing and resizing photographs. One afternoon, Aman and Hisson were working together on a particular digital photo. They printed the image on their photo printer and laid it on the desk. Afterwards, Aman left the room and Hisson decided it would be fun to print a slightly different version of the picture. He first scaled it horizontally to make the picture much thinner. Then he scaled it vertically to make it much shorter. Hisson purposely chose scalings which were different in the horizontal and vertical directions, and he laughed aloud at the resulting picture. Then, just for fun, he printed the new picture and laid it on top of the original picture in a manner similar to the illustration below, where the details of the photos are omitted.



When Aman returned to the room, Hisson expected him to see the smaller picture and laugh at the distorted view. Instead, to his surprise, Aman exclaimed “This is incredible! There is exactly one point in the original picture which is directly beneath the corresponding point in the smaller picture.” He showed Hisson the location of the point, and Hisson agreed with his statement. Then Aman picked up the smaller picture and placed it on top of the larger picture again (in the manner shown above, but in a different location), and he made the same observation. This time Hisson immediately saw the common point. Then just for fun, Aman slid the smaller picture along a line segment on top of the original picture and observed how the common point changed as the smaller picture moved, and he followed this action by sliding the smaller picture along other curves while he twisted it slowly in a counter clockwise direction.

Problem: Use algebra, geometry and/or trigonometry to show that Aman’s observation was correct. As a follow-up activity, explore how the common point changed when Aman slid the smaller photo along a line segment, and discuss how the common point changed as Aman slid and rotated the smaller photo along other curves (in this latter case, use specific examples).