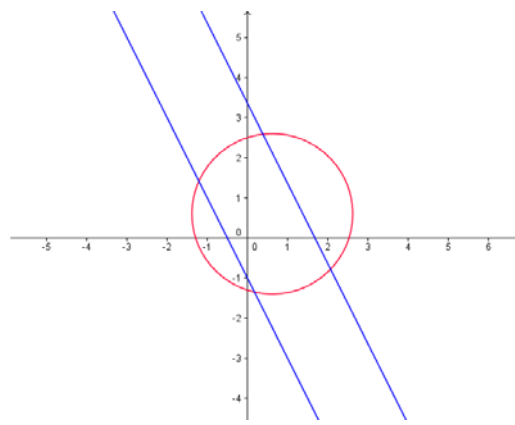
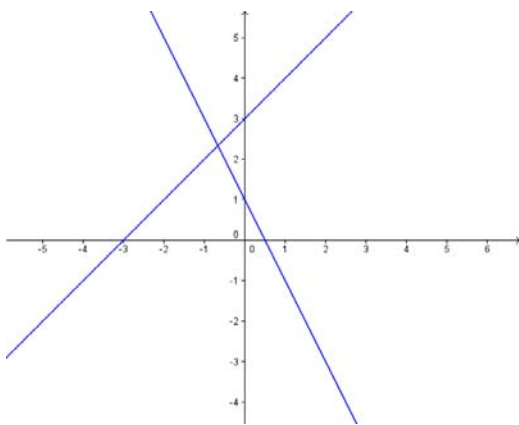


Lines, Circles and Regions

Project Competition – 2010

Instructions: Include a cover sheet with your submission that includes the school name and the names of the students on the project team. Then give solutions to as many problems as possible, create a table of contents, and then bind the cover sheet, the table of contents, and the solutions in a folder. **Projection submissions will be graded based upon the correctness of the solutions, the creativity of the process, and the clarity of the write-up.**

This project involves regions in the xy -plane that are determined by lines and/or circles. A region in the xy -plane is said to be **bounded** if there is a distance $d > 0$ so that all of the points in the region are less than the distance d from the origin. Otherwise a region is said to be **unbounded**. As a point of reference, I have included a two plots below. The first of these plots shows 2 lines separating the plane into 4 regions. All 4 of these regions are unbounded. The second of these plots shows 2 lines and a circle separating the plane into 7 regions. In this case, 3 of these regions are bounded.



The following questions refer to the regions that can be determined by different collections of lines and/or circles in the xy -plane.

1. What are the different numbers of regions that can be determined by 2 distinct lines in the xy -plane? Will any of the regions ever be bounded? Draw representative sketches.
2. What are the different numbers of regions that can be determined by 3 distinct lines in the xy -plane? Will any of the regions ever be bounded? Draw representative sketches.
3. What are the different numbers of regions that can be determined by 2 distinct circles in the xy -plane? Will any of the regions ever be bounded? Draw representative sketches.
4. What are the different numbers of regions that can be determined by 3 distinct circles in the xy -plane? Will any of the regions ever be bounded? Draw representative sketches.

5. For each natural number n , let $m(n)$ be the smallest number of regions that can be determined by n distinct lines in the xy -plane. Give the value of $m(n)$ for each n . If you are able to give a formula in terms of n for $m(n)$, then prove that the formula is correct.
6. For each natural number n , let $M(n)$ be the largest number of regions that can be determined by n distinct lines in the xy -plane. Give the value of $M(n)$ for each n . If you are able to give a formula in terms of n for $M(n)$, then prove that the formula is correct.
7. For each natural number n , let $S(n)$ be the set of possible numbers of regions that can be determined by n distinct lines in the xy -plane. Give $S(1)$, $S(2)$ and $S(3)$, and then give the set $S(n)$ for each natural number n . If you are able to give a representation of the set $S(n)$ in terms of the parameter n , then prove that the representation is correct.
8. A collection of n distinct lines is said to be a **regular collection** if and only if no two lines in the collection are parallel, and each point in the xy -plane lies on at most 2 of the lines in the collection. How many different regions will be determined by a regular collection of 3 distinct lines in the xy -plane? How many different regions will be determined by a regular collection of 4 distinct lines in the xy -plane? In general, if n is a natural number, what is the largest number of regions that can be determined by a regular collection of n distinct lines in the xy -plane. Explain.
9. In general, if n is a natural number, what is the largest number of **bounded regions** that can be determined by a regular collection of n distinct lines in the xy -plane. Explain.
10. In general, if n is a natural number, what is the largest number of **unbounded regions** that can be determined by a regular collection of n distinct lines in the xy -plane. Explain.
11. Repeat exercises 1-7 with the word “lines” replaced by the word “circles”.
12. Give a formulation of the analog of question 8 with the word “lines” replaced by the word “circles”.
13. Repeat exercises 9 and 10 with the word “lines” replaced by the word “circles”.
14. Repeat exercises 1-10 above with the word “lines” replaced by the phrase “lines and/or circles”.