

## ALGEBRA II EXAM

UNIVERSITY OF HOUSTON  
2008 HIGH SCHOOL MATHEMATICS CONTEST  
FEBRUARY 16, 2008

**Name:**

**School:**

**Score:**

**Circle your answers.**

- (1) Find  $i^{673}$
- (a)  $i$
  - (b)  $-i$
  - (c) 1
  - (d)  $-1$
  - (e) None of the above.
- (2)  $\log_8 x = -\frac{1}{3}$ . Solve for  $x$ .
- (a) 512
  - (b) No solutions.
  - (c) 2
  - (d)  $\frac{8}{3}$
  - (e)  $\frac{1}{2}$
  - (f) None of the above.

- (3) In 3 years John will be half his mother's current age. John's grandfather is twice as old as John's mom. The grandfather is 48. How old is John?
- (a) 18
  - (b) 9
  - (c) 6
  - (d) 12
  - (e) 8
  - (f) None of the above.
- (4) A boat has a speed of 20 miles per hour (mph) in still water. In a stream that has a current of 10 mph it travels a certain distance downstream and returns. Find the ratio of the time the round trip takes to the time it would take in still water.
- (a) 3:2
  - (b) 1:2
  - (c) 3:4
  - (d) 4:3
  - (e) 5:4
  - (f) None of the above.
- (5) Simplify  $\log_2 24 - \log_2 \frac{3}{2}$
- (a) 36
  - (b)  $2\log_2 36$
  - (c) 4
  - (d) 2
  - (e)  $\log_2 22.5$
  - (f) None of the above.

- (6) Find the coordinates of the center of the circle

$$x^2 + y^2 - 8x + 4y + 16 = 0.$$

- (a)  $(4, -2)$
  - (b)  $(4, 2)$
  - (c)  $(-4, -2)$
  - (d)  $(-4, 2)$
  - (e)  $(2, -4)$
  - (f) None of the above.
- (7) At the conclusion of a party, 55 handshakes are exchanged among the guests. Assuming that each guest shook hands with each of the others, find the number of guests  $n$  at the party.
- (a) 11
  - (b) 10
  - (c) 9
  - (d) 8
  - (e) None of the above.
- (8) Each edge of a cube is increased by 40%. What percentage does the surface area of the cube increase?
- (a) 40%
  - (b) 196%
  - (c) 156%
  - (d) 96%
  - (e) 256%
  - (f) None of the above.
- (9) Simplify:  $(2x^2 - 7x - 4)^{-1}(2x^2 + 7x + 3)(x^2 + 3x - 28)(x^2 - 9)^{-1}$
- (a)  $\frac{x+3}{x+4}$
  - (b)  $\frac{x+7}{x-3}$
  - (c)  $\frac{2x+1}{x-4}$
  - (d) 1
  - (e)  $\frac{x+4}{x+3}$
  - (f) None of the above.

(10) How many solutions in non-negative integers does the inequality  $x + y < 50$  have? The solutions  $(x, y)$  and  $(y, x)$  are considered to be different when  $x \neq y$ .

- (a) 1200
- (b) 1225
- (c) 1250
- (d) 1275
- (e) 2300
- (f) None of the above.

(11) Simplify:  $\sqrt[3]{81} - \sqrt[3]{\frac{1}{9}}$ .

- (a)  $\frac{8\sqrt[3]{3}}{3}$
- (b)  $\frac{\sqrt[3]{3}}{24}$
- (c)  $\frac{-8\sqrt[3]{3}}{49}$
- (d)  $\frac{\sqrt[3]{24}}{3}$
- (e)  $\sqrt[3]{9}$
- (f) None of the above.

(12) Simplify:  $\frac{xy-y^2}{xy-x^2} - \frac{x^2-y^2}{xy}$ .

- (a)  $\frac{x}{y}$
- (b)  $\frac{-x}{y}$
- (c)  $\frac{-y}{x}$
- (d)  $\frac{y}{x}$
- (e)  $\frac{xy-x^2}{xy}$
- (f) None of the above.

(13) Which triples can NOT be the lengths of the sides of a right triangle.

I.  $\{12, 5, 13\}$ .

II.  $\{21, 29, 20\}$ .

III.  $\{19, 27, 20\}$ .

IV.  $\{11, 12, 5\}$ .

(a) II,III,IV

(b) II,IV

(c) I,II,III,IV

(d) II,III

(e) III,IV

(f) None of the above.

(14) If  $x$  workers can do a job in  $d$  days, then how many days will it take  $x + y$  workers?

(a)  $\frac{xy}{d+y}$

(b)  $\frac{yd}{x+d}$

(c)  $\frac{xd}{x+y}$

(d)  $\frac{xd}{d+y}$

(e)  $\frac{d}{x+y}$

(f) None of the above.

(15) If  $\frac{1}{a+x} = \frac{1}{a} + \frac{1}{x}$ , and  $a$  is a non-zero constant, how many real number solutions will this equation have?

(a) 0

(b) 1

(c) 2

(d) 4

(e) Infinitely many.

(f) None of the above.

(16) Solve the following system for  $x$  and  $y$ :

$$\frac{7}{x} + \frac{2}{y} = 5$$
$$\frac{1}{x} + \frac{4}{y} = -3$$

- (a) Infinitely many solutions
  - (b)  $\{(\frac{15}{7}, \frac{15}{13})\}$
  - (c)  $\{(\frac{15}{13}, \frac{15}{7})\}$
  - (d) No solutions
  - (e)  $\{(1, -1)\}$
  - (f) None of the above.
- (17) Jim can't remember the last digit of his best friend's phone number. He plans to just dial the number and guess the last digit. What's the probability that he'll dial the correct number in the first 2 tries?
- (a)  $\frac{1}{10}$
  - (b)  $\frac{1}{45}$
  - (c)  $\frac{17}{10}$
  - (d)  $\frac{1}{5}$
  - (e)  $\frac{2}{5}$
  - (f) None of the above.
- (18) Jeanini bought a ring in 1990 for \$200. By 1995 it had lost 3% of its value. In 2000 it was worth 33% more than in 1995. By 2005 it had lost 10% of its value from 5 years earlier. What was the ring worth in 2005 (in nearest whole dollars)?
- (a) \$232
  - (b) \$229
  - (c) \$227
  - (d) \$213
  - (e) \$221
  - (f) None of the above.

- (19) Find the real number solutions:  $\sqrt{3x+1} + \sqrt{x-4} = 3$ .
- (a) 5
  - (b) 8
  - (c) 5,8
  - (d) No real number solutions.
  - (e) 4,5,8
  - (f) None of the above.
- (20) Find the remainder of  $x^8 + 1$  divided by  $x - 1$ .
- (a) 1
  - (b) -1
  - (c) 2
  - (d) -2
  - (e) 0
  - (f) None of the above.
- (21) Find the largest number that always divides the difference of the squares of two consecutive even numbers.
- (a) 1
  - (b) 2
  - (c) 3
  - (d) 4
  - (e) None of the above.
- (22) Jim buys pixie sticks at 4 for 10 cents. He sells them at 4 for 15 cents. How many does he have to sell in order to make a profit of \$1.00?
- (a) 80
  - (b) 40
  - (c) 8
  - (d) 125
  - (e) 25
  - (f) None of the above.

- (23) Instead of walking along the adjacent sides of a rectangular field, Joe took a shortcut along the diagonal and saved a distance equal to half the length of the longer side. Find the ratio of the longer side to the shorter side of the field.
- (a) 5:3
  - (b) 3:2
  - (c) 2:3
  - (d) 5:2
  - (e) 4:3
  - (f) None of the above.
- (24) Write 25.25 in base 2.
- (a) 1011.01
  - (b) 1101.01
  - (c) 11001.001
  - (d) 1101.001
  - (e) 11001.01
  - (f) None of the above.
- (25) There are 500 red and blue balls in a jar. 1% of the balls are blue. Some red balls, but no blue balls, are withdrawn. Then 2% of the balls are blue. How many balls are still in the jar?
- (a) 490
  - (b) 495
  - (c) 485
  - (d) 250
  - (e) 245
  - (f) None of the above.