Math Contest – Precalculus Test, 2008

Name: _____

School: _____

Grade: _____

Teacher: _____

Question	Answer Letter
1	A, B, C, D, E
2	A, B, C, D, E
3	A, B, C, D, E
4	A, B, C, D, E
5	A, B, C, D, E
6	A, B, C, D, E
7	A, B, C, D, E
8	A, B, C, D, E
9	A, B, C, D, E
10	A, B, C, D, E
11	A, B, C, D, E
12	A, B, C, D, E
13	A, B, C, D, E
14	A, B, C, D, E
15	A, B, C, D, E
16	A, B, C, D, E
17	A, B, C, D, E
18	A, B, C, D, E
19	A, B, C, D, E
20	A, B, C, D, E
21	A, B, C, D, E
22	A, B, C, D, E

1. Where is the graph of $f(x) = -3\sin(\pi x)$ decreasing in the interval $(0, \pi)$? Use the convention that the turn-around point is NOT included in the answer interval(s).

A. $(0, \frac{\pi}{2})$

B. $(\frac{\pi}{2}, \pi)$

C. $(0, \frac{1}{2})$ and $(\frac{3}{2}, \frac{5}{2})$

D. It is decreasing on the whole interval.

E. It is increasing on the whole interval.

2. A wire of length $(\pi\sqrt{2})x$ is bent into a circle. What is the area of the circle?

A. $2\pi x^2$

B. $\frac{\pi x^2}{2}$

C. $2(\pi x)^2$

D. $\frac{x^2}{2\pi}$

E. none of the above

3. Evaluate: $\left(\dfrac{\sin\frac{19\pi}{3}}{\cos(-\frac{\pi}{6})}\right)\cdot\tan(\dfrac{-101\pi}{4})$

A. 0

B. 1

C. $\dfrac{\sqrt{3}}{2}$

D. $\dfrac{\sqrt{3}}{4}$

E. –1

4. What is the area of the polygon between these two graphs?
$f(x)=|x-3|+1$
$g(x)=-|x-3|+3$

A. 1 sq. unit

B. 2 sq. units

C. $4\sqrt{2}$ sq. units

D. 3 sq. units

4. 4 sq. units

5. What is the range for $f(x)=\dfrac{|x|}{x}$?

A. {–1, 1}

B. all Real numbers

C. [–1, 1]

D. $(-\infty,0)\cup(0,\infty)$

E. $[0,\infty)$

6. What is the domain for $f(x) = \sin^{-1}(x+1)$?

A. [–1, 1]

B. [0, 2]

C. $(0, \infty)$

D. [–2, 0]

E. $(-\infty, \infty)$

7. Where is the asymptote for $f(x) = 2\log_5(1-3x)+10$?

A. y = 3

B. x = 0

C. y = 1

D. x = 3

E. $x = \frac{1}{3}$

8. The following table was found by evaluating a function of x:

x	1/2	1	2	8
y	–1	0	1	3

Which of the following statements are true?

1. y is a logarithmic function of x
2. x is an exponential function of y
3. y is an exponential function of x
4. y is an increasing function
5. y is a decreasing function

A. 1 and 4 only

B. 2 and 4 only

C. 3 and 5 only

D. 1 and 5 only

E. 3 and 4 only

9. Given this fact:

The maximum of two real numbers a and b is
$\max(a, b) = \frac{1}{2}(a + b + |a - b|)$

Which of the following is a formula for the minimum of the same two numbers a and b?

A. $\min(a, b) = |a| + |b|$

B. $\min(a, b) = \frac{1}{2}\sqrt{a^2 + b^2}$

C. $\min(a, b) = \frac{1}{2}(a - b + |a - b|)$

D. $\min(a, b) = \frac{1}{2}(|a + b| - |a - b|)$

E. none of the above

10. A utility company has a fleet of vans. The annual operating cost of each van is estimated to be $C = 0.11\,m + 2300$ where C is cost measured in dollars and m is measured in miles. The company wants the annual operating cost of each van to be less than \$10,000. What is the preferred domain for the formula?

A. $m = 70{,}000$

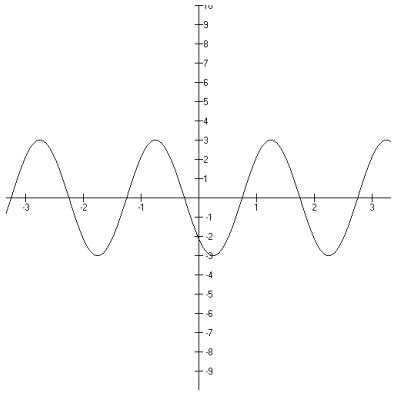
B. $[0, 70{,}000)$

C. $[7{,}000, \infty)$

D. $(-\infty, 700)$

E. $[0, 10{,}000)$

11. Find a, b, and c so that the function: $y = a \sin(bt + c)$ matches the following graph. You are given the following 3 points on the graph: $(0, \frac{-3\sqrt{2}}{2})$, $(\frac{3}{4}, 0)$, and $(\frac{5}{4}, 3)$.



A. $-3 \sin(x + \frac{1}{4})$

B. $3 \sin(2x + \frac{1}{2})$

C. $-3 \sin(\pi x + \pi/4)$

D. $3 \sin(2\pi x + \pi/8)$

E. $-3 \sin(x + \pi/4)$

12. Which of the following is the interval described by $\frac{1}{|x|} < 1$?

A. $(-\infty, -1) \cup (1, \infty)$

B. $(0, \infty)$

C. $(-\infty, 0) \cup (0, \infty)$

D. $(-1, 1)$

E. $(-\infty, \infty)$

13. Which of the following is the formula for the area of an equilateral triangle expressed as a function of a side length x?

A. $A = \frac{x^2}{2}$

B. $A = \frac{x^2}{4}$

C. $A = \frac{x^2\sqrt{3}}{4}$

D. $A = \frac{x\sqrt{2}}{3}$

E. none of the above

14. What is the behavior of the graph as x approaches 2 through values slightly less than 2 for the graph: $f(x) = -\frac{x+3}{x-2}$?

A. y approaches 0

B. y approaches 3/2

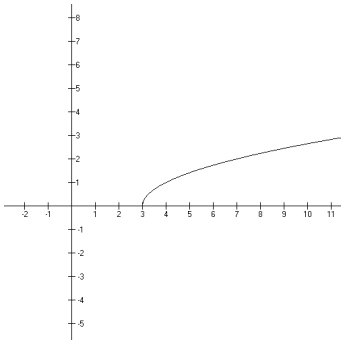
C. y approaches –3

D. y approaches ∞

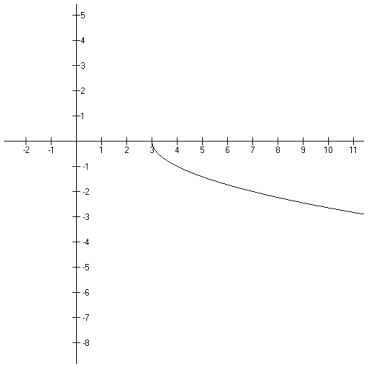
E. y approaches $-\infty$

15. Which of the following is the graph of $f(x) = -\sqrt{3-x}$?

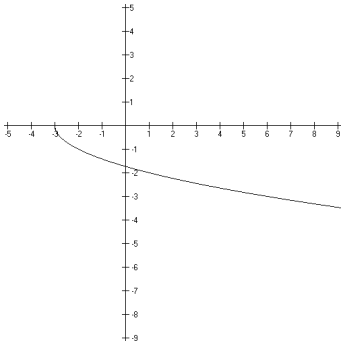
A.



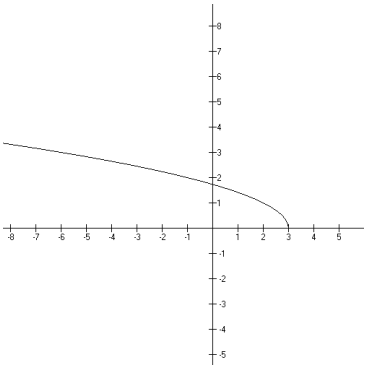
B.



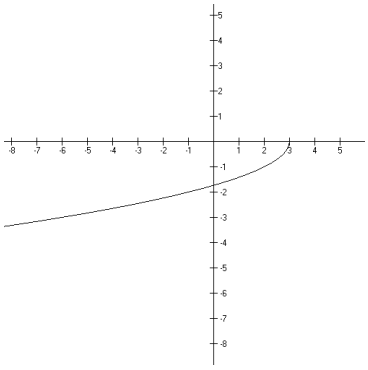
C.



D.



E.



16. Given: $f(x) = \begin{cases} \sqrt{x} & x > \frac{3\pi}{2} \\ \sin x & -\frac{\pi}{2} < x \le \frac{3\pi}{2} \\ x^2 & x \le -\frac{\pi}{2} \end{cases}$

Calculate: $\dfrac{f(12) + f(\frac{\pi}{3})}{f(-2)}$

A. $\dfrac{\sin(12) + \sqrt{3}}{8}$

B. $\dfrac{5\sqrt{3}}{8}$

C. -1

D. $\dfrac{2\sqrt{3} + 1}{-2\sin(2)}$

E. none of the above

17. What is the y-intercept for the inverse function to $f(x) = \dfrac{x+3}{x-1}$?

A. (0, –3)

B. (0, 3)

C. (–3, 0)

D. (3, 0)

E. none of the above

18. If the point (a, b) is reflected about the line y = 3x, what are the point's new coordinates?

A. (b + 3, a + 3)

B. $(\frac{3b-4a}{5}, \frac{3a+4b}{5})$

C. (a, b)

D. $(\frac{b}{3}, \frac{a}{3})$

E. $(\frac{a+b}{3}, \frac{a-b}{3})$

19. Which of the following are true statements?

1. Given two natural numbers a and b: $\sqrt{ab} \le \frac{a+b}{2} \le \frac{\sqrt{a^2+b^2}}{2}$

2. The following is symmetric with respect to the x-axis:
$y = 2^x - 2^{-x}$

3. Let $S(x) = 0.5(3^x - 3^{-x})$, $S(3x) = S(x)+4S(x)^3$.

4. $\log_3(8\sqrt{2}) = \frac{3}{2}$.

A. 2 and 3 only

B. 1, 2, and 3 only

C. none of them

D. 1 and 3 only

E. all of them

20. Give the equation of the circle that is tangent to the x-axis, tangent to the y-axis and tangent to the line 3x + 4y =12.

A. $(x-1)^2 + (y-1)^2 = 4$

B. $(x-1)^2 + (y-1)^2 = 1$

C. $(x+1)^2 + (y+1)^2 = 1$

D. $(x+1)^2 + (y+1)^2 = 4$

21. Given f(x) = sin (x) and g(x) = $\frac{x-2}{2}$, h(x) = $\pi x + \frac{\pi}{6}$, and k(x)= cos x

Calculate: $(k \circ h \circ g^{-1} \circ f)(x)$ for x = $\frac{\pi}{6}$.

A. ½

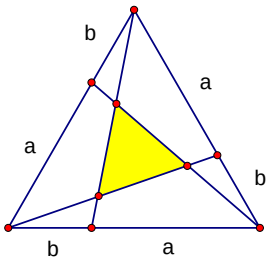
B. 1

C. $-\frac{\sqrt{3}}{2}$

D. –½

E. –1

22. Find the length of one side of the shaded triangle given that the largest triangle is an equilateral triangle with side length 3 inches. Each side of the largest triangle is divided into two pieces; one of length 2 inches (a) and the other of length 1 inch (b). Use the Laws of Sines and Cosines. Hint: the shaded triangle is equilateral.



A. $\frac{3\sqrt{7}}{7}$

B. $\frac{2\sqrt{5}}{6}$

C. $\frac{5\sqrt{3}}{3}$

D. $\frac{\sqrt{3}}{5}$

E. $\frac{\sqrt{2}}{3}$