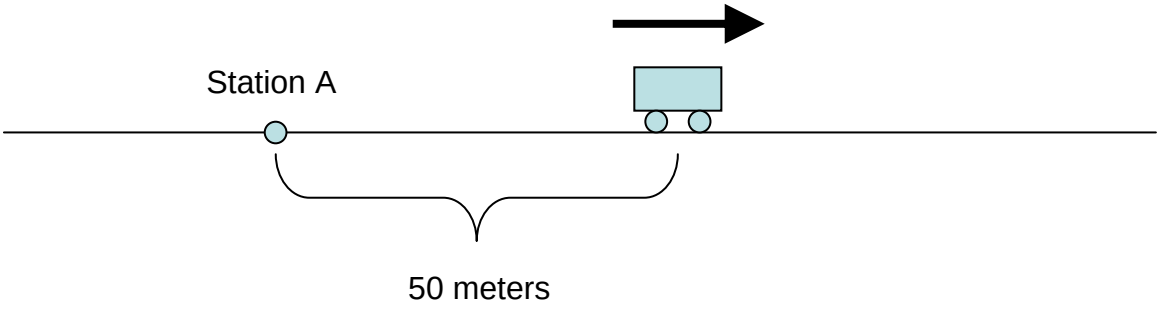
The Cart Problem

Directions: Complete as much work as possible on the 4 parts below. Your write-up is important, and all answers should be justified. Make sure your team's submission includes a cover sheet containing the full name of your school and the full names of each team member. Submissions can be made in person by 9:00 am on the day of the contest (February 16th), or by emailing an electronic version of your solution to jmorgan@math.uh.edu by 5:00 pm on February 15th. Good luck!

Scenario: Suppose a motorized supply cart is automated to bring parts to different stations along a long straight horizontal track. We will assume that the mass of the cart is essentially independent of the amount of supplies being carried by the cart. The cart is equipped with a motor that is capable of accelerating or decelerating the cart at the rate of ½ meter per second^2, or being switched off so that the cart can coast.

Part 1: At some time during the day, the cart is moving away from station A when it receives a call to return. At the time, the cart is 50 meters from the station and traveling at a speed of 5 meters per second. How should the motor be used to bring the cart back to station A as quickly as possible? Note that when the cart arrives at station A it must arrive with velocity 0. (Assume there is no friction and the decision is made by the cart instantaneously upon receipt of the call.)



Part 2: Later in the day, the cart is moving away from station A towards station B when it receives simultaneous calls from both station A and station B to bring supplies. At the time, the cart is 40 meters from station A, 60 meters from station B, and traveling at a speed of 11 meters per second. Assume it will take 10 seconds to unload the cart at each station. How should the motor be used to deliver the materials to both stations A and B as quickly as possible? (Again, assume there is no friction and the decision is made by the cart instantaneously upon receipt of the call.) Give a graph that shows the position of the cart from the time of the call until the time both station have unloaded their supplies.

Part 3: Place an orientation on the track from so that the positive direction corresponds to the direction from station A in the direction of station B, and the negative direction is the opposite direction. Assume the track extends far past each station in each direction. We consider station A to be at position 0 on the track and station B to be at position 100 on the track (with units measured in meters). At time $t = 0$ the cart is at position x along the track and is traveling with velocity v when it receives simultaneous calls from both stations A and B. Give (if possible) a relationship between x and v that can be used to determine whether the cart should proceed first to station A or station B, assuming that the goal is to deliver the parts to both stations in as little time as possible. (Again, assume there is no friction and the decision is made by the cart instantaneously upon receipt of the call.)

Part 4: Repeat Part 1 assuming that friction (opposing motion) is present and that the acceleration due to friction is proportional to the velocity of the cart. Is it possible that the cart could arrive at station A faster with the presence of friction? Explain your answer.