Calculator Exam
2009 University of Houston Math Contest

Name: _____

School: _____

Please read the questions carefully. Unless otherwise requested, round your answers to 8 decimal places.

There is no penalty for guessing.

Judges will use written comments and/or calculations to settle ties.
Good luck.

Name: _____

School: _____

Calculator Exam - 2009

1. Give the largest prime number that is less than 950,774.

Answer:	

2. Write 96,942 in base 7.

Answer:	

3. Find the smallest natural number k so that $1+\dfrac{1}{2}+\dfrac{1}{3}+\dfrac{1}{4}+\cdots+\dfrac{1}{k}>7.2$.

Answer:	

4. Give the smallest positive root of $f(x)=\cos\left(\dfrac{900}{1000x+1}\right)$.

Answer:	

5. Give the sum of the x coordinates for the points of intersection of the graphs of $f(x)=\sin(x)+2x$ and $g(x)=5-x^2$.

Answer:	

6. Approximate the largest value of $f(x)=-x^8+787x^4+673x^3+521x^2+840x+12$.

Answer:	

7. A set of points (i,j) is plotted in the plane, where i and j are both positive integers and are factors of 7,351,344. How many points are plotted?

Answer:	

Name: _____

School: _____

8. Give a decimal approximation to $\left(1+\dfrac{1}{3}\right)^2\left(1+\dfrac{1}{9}\right)^3\cdots\left(1+\dfrac{1}{4,782,969}\right)^{15}$.

Answer:	

9. Two perpendicular lines intersect at the point (7,4). One of these lines passes through the point (-2,1). Give the decimal approximation for the radian measure of the acute angle of intersection of the other line with the *y* axis.

Answer:	

10. Give the *y* coordinate of the solution to the system $\begin{pmatrix} 31x - 29y = 43 \\ -51x + 19y = 16 \end{pmatrix}$, in reduced

fraction form.

Answer:	

11. Suppose the government is close to approving $1.55 trillion dollars in stimulus money that will hopefully create 5 million new jobs, when suddenly someone asks, "How about if we forget about creating the jobs, and we just divide this money evenly between the 5 million people?" If the government took the latter approach, how much money would each person get, rounded down to the nearest penny?
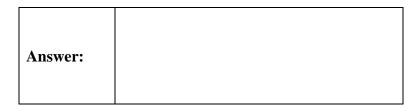
Answer:	

12. A family has met each year on September 1st to visit since 1961. At that time, there were 71 family members, and each year the family grew as a net result of marriages, divorces, adoptions, births and deaths by an amount equal to the largest integer less than 6% of the total from the previous year. There were 39 family members who missed the 2008 meeting. How many attended the 2008 meeting?

Answer:	

Name: _____

School: _____

13. According to Google, Dallas is 239 miles from Houston. A young woman leaves Houston at 4:00 pm to make the drive. She stops for 20 minutes to get gas at 6:00 pm, spends 8 minutes driving through Taco Bell at 7:00 pm, and wastes 25 minutes of driving time after leaving Taco Bell because she misses a turn and doesn't realize her mistake until later. She averages 67 mph before stopping for gas, and 64 mph for the first 20 minutes after making her way back to the correct turn (after missing it). If she never drove more than 75 mph, then what was her earliest time of arrival (rounded to the nearest minute)?

Answer:

14. Add the sum of the first 3 positive integer multiples of 2, to the sum of the first 5 positive integer multiples of 5, to the sum of the first 7 positive integer multiples of 11, to the sum of the first 9 positive integers multiples of 17, and so forth, up to the sum of the appropriate number of positive integer multiples of 47. Give the result.

Answer:

15. $s_1 = \dfrac{1}{2}$, $s_2 = 3s_1(1 - s_1)$, $s_3 = 3s_2(1 - s_2)$, ... , $s_{50} = 3s_{49}(1 - s_{49})$. Assuming this pattern continues, give the values s_{51}, s_{52} and s_{53}.

Answer:

16. There are 46 prime numbers less than 200. Denote these as $p_1, p_2, p_3, \cdots, p_{46}$.

Approximate the value of $\dfrac{1}{p_1^{\,2}} + \dfrac{1}{p_2^{\,2}} + \dfrac{1}{p_3^{\,2}} + \cdots + \dfrac{1}{p_{46}^{\,2}}$.

Answer:

Name: _____

School: _____

17. An IT group designs a software solution to be used to help grade *fill in the blank* mathematics questions submitted by students in online quizzes. Most of the questions will require students to type a formula in a box, and then the computer will have to determine whether the formula is correct. Unfortunately, many expressions can be expressed in multiple ways, so there will be problems for which there is no single expression representing the correct answer. As a potential solution, the programmers devise a strategy that does not require students to simplify their answers. It just requires the problems to be written so that the answers are mathematical expressions of a single variable. When a problem is designed for use in an online quiz, the author is required to supply a formula for the correct answer, along with the name of the variable and an interval on which the formula can be evaluated. Then, when a student submits their answer, the formula for the correct answer will be evaluated at 10 randomly chosen values in the interval, and the formula submitted by the student will be evaluated at the same 10 values. If for each of the 10 values, the result of evaluating correct formula at the value is within 10^{-9} of the result of evaluating the student supplied formula at the value, the answer is assumed to be correct. Suppose a problem is written and the author gives e^{-x^2} as the correct solution, a statement that the variable is x and the interval is $[-9,9]$. What is the probability that a student who submits 0 as their answer will get credit for the problem?

Answer:	

18. Many years ago, Cindy had some bills that were due every other month, and as a result, she had some extra money on the months when she didn't have to pay these bills. She decided to put money in savings at the end of every month, alternating the amounts between $600 and $900. She deposited $600 at the end of the first month and continued her plan for 21 years without ever increasing the amount. One day, she mentioned to her daughter that she did this, and although she didn't tell her how much money was currently in the account, she said that the interest rate was typically something like 4% per year, compounded monthly. Assuming the daughter knows how to use a calculator and understands compound interest, how much money does she determine was in the account after the last deposit was made?

Answer:	

19. Give the number of positive integers that are no larger than 1,100 and are not divisible by any of the first 25 prime numbers?

Answer:	

20. Thirteen men and nineteen women are placed in line so that 3 women are at the front of the line, 4 women are at the end of the line, and men and women alternate in the line from the fourth position to the 29th position, with a man standing in the 4th position. Each of the first 31 people choose a random number in the set $\{2, 3, \ldots , 100\}$. You don't know their choices, but you know that the 32nd person knows their choices, and you hear her say that all of the men have chosen prime numbers in increasing order, and all of the women have chosen composite numbers in increasing order. What is the probability that she can choose a composite number in the set $\{2, 3, \ldots , 100\}$ that is larger than any of the numbers chosen by the women and forces the sum of all chosen numbers to be an integer multiple of 103?

Answer:	