Geometry Exam
2009 University of Houston Math Contest

Name: _____

School: _____

Please read the questions carefully and give a clear indication of your answer on each question.

There is no penalty for guessing.

Judges will use written comments and/or calculations to settle ties.
Good luck.

Name: _____ **School:** _____

University of Houston
High School Mathematics Contest
Geometry Exam – Spring 2009

Directions:
You have 60 minutes to complete this exam. Calculators are not permitted. Choose the correct answer for each question, and write the letter (A, B, C, D or E) corresponding to that answer in the blank to the right of each question. There is no penalty for guessing. In the case of a tie, students' work will be used to determine the winner – so show all work clearly on either your exam or on scrap paper. Write your name and school on top of each page in the blanks provided.
Note: Geometric figures in the problems may not be drawn to scale.

1. Find the center and radius of the circle defined by the equation $(x+2)^2+(y-7)^2=9$.

(A) Center $(2,-7)$, Radius 3
(B) Center $(-2,7)$, Radius 9
(C) Center $(-2,7)$, Radius 3
(D) Center $(2,-7)$, Radius 9
(E) Center $(-2,7)$, Radius 81

1. _____

2. If a circle has a circumference of 16π inches, find its area.

2. _____

(A) 8π in^2 (B) 16π in^2 (C) 32π in^2 (D) 64π in^2 (E) 256π in^2

3. W hat can you conclude about about ΔABC and ΔDEF, given that $\overline{AB}\cong\overline{DE}$, $\overline{BC}\cong\overline{EF}$, and $\angle A\cong\angle D$?

(A) The two triangles are similar.
(B) The two triangles are congruent.
(C) The two triangles are both scalene
(D) The two triangles are both acute.
(E) There is not enough information to draw a conclusion.

3. _____

4. Find the ratio of the edge of a cube to the cube's diagonal.

4. _____

(A) $\sqrt{2}$ (B) $\frac{\sqrt{2}}{2}$ (C) $\frac{1}{3}$ (D) $\sqrt{3}$ (E) $\frac{\sqrt{3}}{3}$

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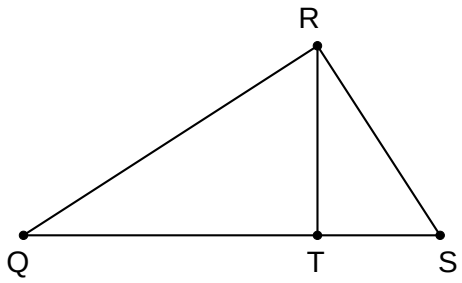
5. Given that the statement "If *p*, then *q*" is false, which of the following statements must always be correct?

I. "If not *p*, then not *q*" is false.
II. "If *p*, then not *q*" is true.
III. "If *q*, then *p*" is true.
IV. "If not *q*, then not *p*" is false.

5. _____

(A) I and II only
(B) III only
(C) I and III only
(D) IV only
(E) II and IV only

6. $\overline{RT}$ is an altitude of ΔQRS, and $m\angle QRS = 90^\circ$. If $ST = 5$ and $QT = 7$, find the length of $\overline{RS}$.



(A) $\sqrt{35}$
(B) $2\sqrt{15}$
(C) $2\sqrt{21}$
(D) $\sqrt{74}$
(E) $2\sqrt{6}$

6. _____

7. A chord of length 24 cm is drawn in a circle of radius 13 cm. Find the distance from the chord to the center of the circle.

7. _____

(A) 25 cm (B) 11 cm (C) 12 cm (D) 5 cm (E) 13 cm

8. Find the measure of an angle satisfying the following:
Six times the complement of an angle is five less than the supplement of the angle.

(A) 25°
(B) 71°
(C) 73°
(D) 35°
(E) $26\ ^3/_7{}^\circ$

8. _____

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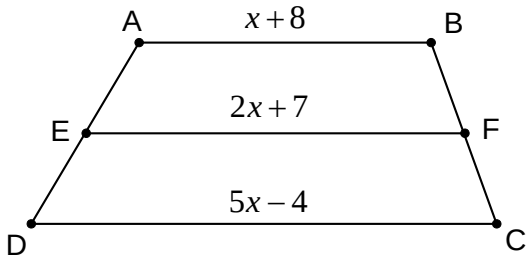
9. Isosceles triangle TRI has base angles T and I and median $\overline{RM}$. If the perimeter of ΔTRI is 54 inches and $TM = 4$ inches, find the length of TR.

(A) 8 inches
(B) 38 inches
(C) 23 inches
(D) 25 inches
(E) 46 inches

9. _____

10. $\overline{EF}$ is the median of trapezoid $ABCD$, with measures listed as below. Find the length of $\overline{AB}$.

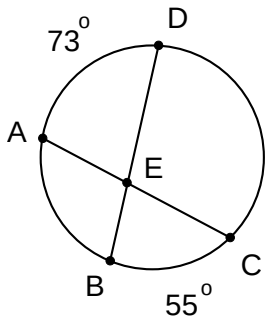
(A) 5
(B) 13
(C) $\frac{21}{4}$
(D) 17
(E) 21



10. _____

11. In the circle below, chords $\overline{AC}$ and $\overline{DB}$ intersect at point E. If $m\widehat{AD} = 73^\circ$ and $m\widehat{BC} = 55^\circ$, find $m\angle AED$.

(A) 36.5°
(B) 55°
(C) 64°
(D) 73°
(E) 128°



11. _____

12. Let ℓ represent the line which passes through the point $(-8, -9)$ and is perpendicular to the line $y = -\frac{2}{5}x + 6$. Find the x-intercept of ℓ.

12. _____

(A) $-\frac{58}{5}$ (B) $\frac{29}{2}$ (C) 11 (D) 15 (E) $-\frac{22}{5}$

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13. A spherical balloon has volume 240π in^3. If 20% more air is blown into the balloon (and it still retains its spherical shape), find the radius of the enlarged balloon.

13. _____

(A) $2\sqrt[3]{9}$ in. (B) 6 in. (C) $6\sqrt{2}$ in. (D) $\sqrt[3]{36}$ in. (E) $\frac{6\sqrt[3]{180}}{5}$ in.

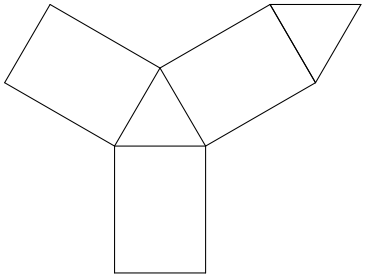
14. Find the area of a hexagon with apothem 6 inches.

14. _____

(A) $36\sqrt{3}$ in^2 (B) 72 in^2 (C) $72\sqrt{3}$ in^2 (D) $216\sqrt{3}$ in^2 (E) 144 in^2

15. Below is a net that can be used to create a solid, formed by three congruent rectangles and two congruent equilateral triangles. How many vertices and edges does the solid have?

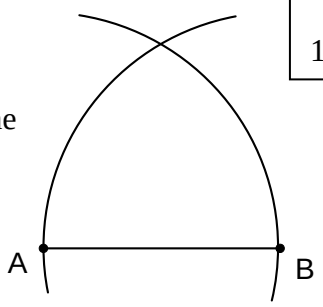
(A) 10 vertices and 14 edges
(B) 18 vertices and 18 edges
(C) 6 vertices and 14 edges
(D) 9 vertices and 9 edges
(E) 6 vertices and 9 edges



15. _____

16. A construction can be found below with all steps shown except for any final straightedge marking(s). Which of the following constructions is being performed?

(A) Angle bisector
(B) Equilateral triangle
(C) Segment bisector
(D) Line through a point parallel to a given line
(E) Construct a circle given 3 points



16. _____

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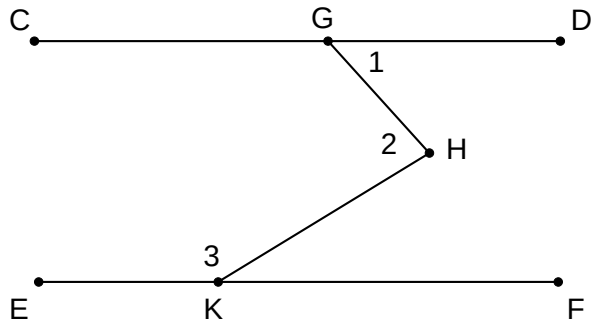
17. In spherical geometry, a line is defined to be a great circle of the surface of a sphere. Two lines are parallel in spherical geometry if they never intersect.

Given line ℓ on a sphere along with the above definitions, how many lines on the sphere are parallel to line ℓ?

17. _____

(A) 0 (B) 1 (C) 2 (D) 4 (E) An infinite number

18. In the diagram below, $\overline{CD} \parallel \overline{EF}$, $m\angle 1 = (x-7)^\circ$, $m\angle 2 = (3x)^\circ$, and $m\angle 3 = (9x+19)^\circ$. Find the value of x.



18. _____

(A) $\frac{161}{12}$ (B) 14 (C) $\frac{84}{5}$ (D) $-\frac{3}{2}$ (E) $\frac{168}{13}$

19. Given regular octagon ABCDEFGH, remove vertex G and use the remaining vertices to form heptagon ABCDEFH. Find the measure of $\angle F$.

19. _____

(A) 45°

(B) 157.5°

(C) $128\frac{4}{7}^\circ$

(D) 135°

(E) 112.5°

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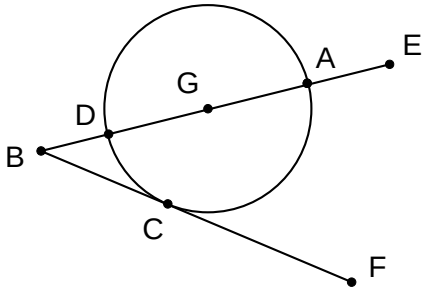
20. $\overline{BE}$ is a secant of circle *G* and $\overline{BF}$ is tangent to circle *G* at point *C*. If $m\overset{\frown}{AC} = 120^\circ$ and the diameter of circle *G* is 10 cm, find the length of $\overline{BC}$.

(A) $5\sqrt{3}$ cm
(B) $5\sqrt{2}$ cm
(C) 10 cm
(D) 5 cm
(E) 30 cm



20. _____

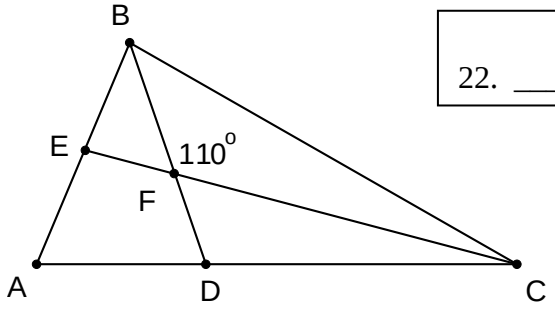
21. Quadrilateral ABCD is inscribed in a circle, with $m\angle C = 70^\circ$. Find the measure of $\angle A$.

21. _____

(A) 70° (B) 110° (C) 145° (D) 140° (E) Can not be determined

22. $\overline{CE}$ and $\overline{BD}$ are angle bisectors of ΔABC which intersect at point *F*. If $m\angle BFC = 110^\circ$, find the measure of $\angle A$.

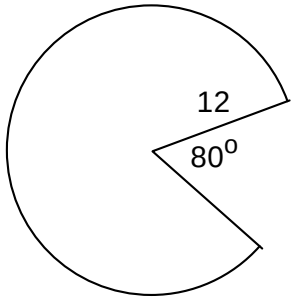
(A) 40°
(B) 55°
(C) 70°
(D) 110°
(E) Cannot be determined



22. _____

23. A sector with central angle 80° and radius 12 has been removed from a figure, as shown below. Find the area of the portion of the circle which remains.

(A) $\frac{56\pi}{3}$
(B) $\frac{56\pi + 72}{3}$
(C) 32π
(D) 128π
(E) 112π



23. _____

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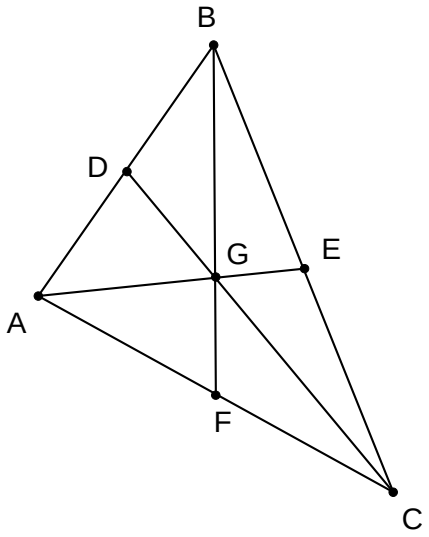
24. Triangle ABC has medians $\overline{AE}$, $\overline{BF}$, and $\overline{CD}$ which intersect in point G as shown below. Given that $AF = DB$, $BC = 10$, and $FG = \sqrt{11}$, find the length of $\overline{AE}$.

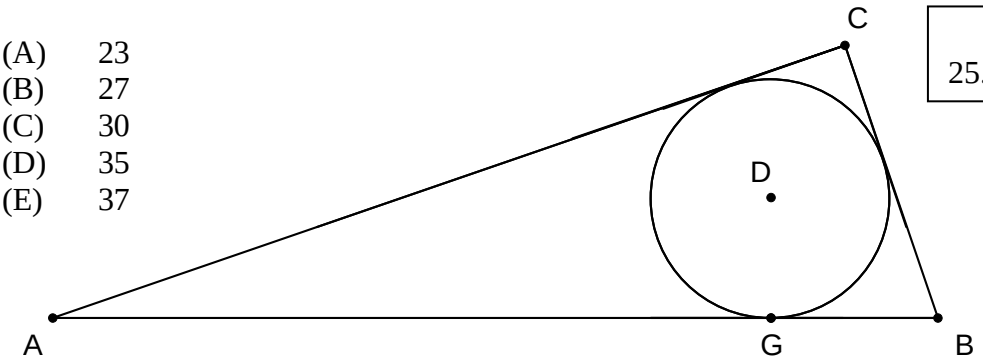
(A) $4\sqrt{6}$
(B) $5\sqrt{3}$
(C) $3\sqrt{11}$
(D) $\sqrt{21}$
(E) $3\sqrt{19}$



24. _____

25. Circle D is inscribed in right triangle ABC with right angle C, and $\overline{AB}$ is tangent to the circle at G. If $CB = 12$ and $GB = 7$, find AC.

(A) 23
(B) 27
(C) 30
(D) 35
(E) 37



25. _____

26. Two pyramids are similar. The lateral area of the first pyramid is 240 cm^2, and the lateral area of the second pyramid is 540 cm^2. If the larger pyramid has a volume of 810 cm^3, find the volume of the smaller pyramid.

(A) $\frac{640}{9}$ cm^3
(B) 540 cm^3
(C) 360 cm^3
(D) 720 cm^3
(E) 240 cm^3

26. _____

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27. A company manufactures dice in large quantities and ships them to other game companies. The manufacturer gets a cube-shaped cardboard box ready to send to a customer, and fills it with 512 dice. If the dice are stacked neatly in rows in such a way that they fill the entire volume of the box, how many dice are touching at least one face of the cardboard box?

(A) 296
(B) 216
(C) 256
(D) 384
(E) 512

27. _____

28. A cube is shown in Figure 1 below. In Figure 2, vertices A, B, C, and D of the same cube are shown as vertices of a right triangular pyramid. If the volume of the cube is 54 cm^3, find the volume of the right triangular pyramid.

(A) 18 cm^3
(B) 12 cm^3
(C) 36 cm^3
(D) 9 cm^3
(E) 27 cm^3

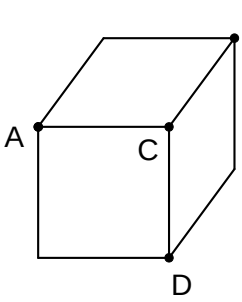


Figure 1

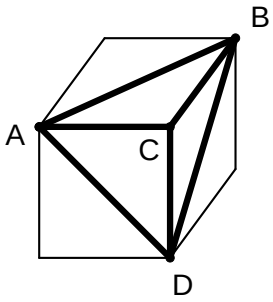
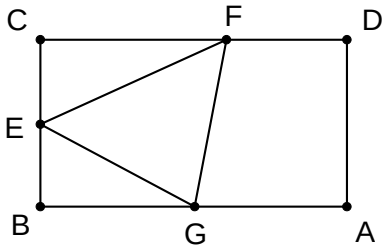


Figure 2

28. _____

29. Rectangle ABCD is shown below with $CD = 10$ and $AD = 8$. If E is the midpoint of $\overline{BC}$, G is the midpoint of $\overline{AB}$, F is a point on $\overline{CD}$, and the area of ΔEFG is 18, find the length of $\overline{DF}$.

(A) 8
(B) 7
(C) 4
(D) 5
(E) 6



29. _____

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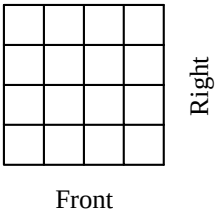
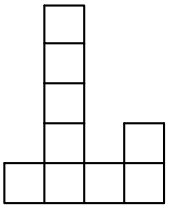
30. A building is made from stacked cubes, and views from three different perspectives are illustrated below. ***There are exactly ten stacks that are one unit high.***

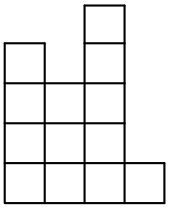
Top View:



Front View:



Right View:



How many cubes are needed to construct this building?

30. _____

(A) 22 (B) 24 (C) 25 (D) 28 (E) 33

31. Which of the following polygons will tessellate the Euclidean plane?

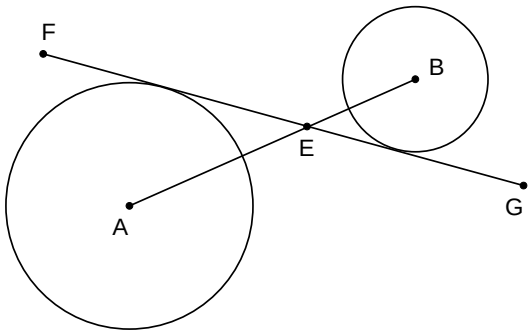
I. Every pentagon
II. Every triangle
III. Every hexagon
IV. Every quadrilateral

31. _____

(A) II only
(B) III only
(C) IV only
(D) II and IV only
(E) None of the above

32. Circle A and Circle B have common internal tangent $\overline{FG}$ which intersects $\overline{AB}$ at E. If circle A has radius 10, circle B has radius 8, and $AB = 27$, find the length of $\overline{BE}$.

(A) 4
(B) 8
(C) 9
(D) 10
(E) 12



32. _____

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33. A square pyramid is made of foam and has base edge 12 cm and height 8 cm. A knife slices through the apex perpendicular to the base and through two opposite vertices of the base (cutting along one of the diagonals of the base), forming two distinct congruent polyhedra. Find the total surface area of one of the resulting polyhedra.

(A) $240\ \text{cm}^2$

(B) $\left(312+96\sqrt{2}\right)\ \text{cm}^2$

(C) $\left(192+24\sqrt{17}\right)\ \text{cm}^2$

(D) $\left(264+60\sqrt{2}\right)\ \text{cm}^2$

(E) $\left(192+48\sqrt{2}\right)\ \text{cm}^2$

33. _____

34. Triangle *ABC* exists on a coordinate plane, and the midpoints of its sides are located at the points $(2,1)$, $(3,7)$, and $(5,4)$. Find the area of triangle *ABC*.

(A) 10 (B) 15 (C) 20 (D) 30 (E) 36

34. _____

35. In the diagram below, an equilateral triangle with side length 4 inches is shown with both its inscribed and circumscribed circles. Find the area of the gray shaded region.

(A) $\frac{4\pi}{3}\ \text{in}^2$

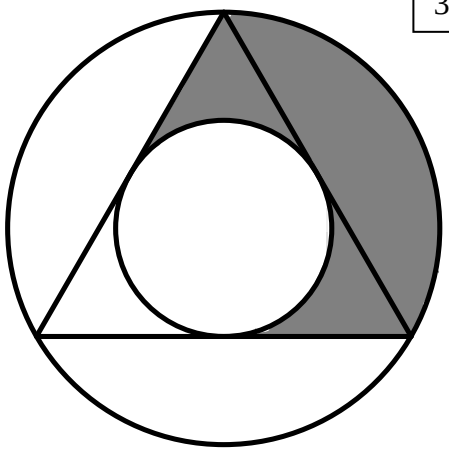
(B) $\left(\frac{8\pi}{9}+\frac{4\sqrt{3}}{3}\right)\ \text{in}^2$

(C) $\left(\frac{16\pi}{9}+\frac{2\sqrt{3}}{3}\right)\ \text{in}^2$

(D) $\left(\frac{8\sqrt{3}}{3}-\frac{8\pi}{9}\right)\ \text{in}^2$

(E) $\frac{8\pi\sqrt{3}}{3}\ \text{in}^2$

35. _____



END OF EXAM ☺

(Make sure all answer blanks are filled in with the letter of the correct answer.)