

1. Which of the following defines a function  $f$  for which  $f(-x) = -f(x)$ ?
  - a.  $f(x) = 3x^2 + 4x$
  - b.  $f(x) = \sin(2x)$
  - c.  $f(x) = \cos(2x)$
  - d.  $f(x) = xe^x$
  - e.  $f(x) = \log x$
  
2.  $\ln(4 - 2x) < 0$  if and only if
  - a.  $x < 2$
  - b.  $1 < x < 2$
  - c.  $\frac{3}{2} < x < 2$
  - d.  $x > 2$
  - e.  $x > 1$
  
3. If  $f(x) = (x - 2)(x - k)$  and if the remainder is 15 when  $f(x)$  is divided by  $(x + 1)$ , then  $k =$ 
  - a. 4
  - b. 3
  - c. 2
  - d. 0
  - e. -2
  
4. If  $f(x) = \frac{2}{x-1}$  and  $g(x) = 3x$ , then the solution set of  $f(g(x)) = g(f(x))$  is
  - a.  $\left\{\frac{1}{4}\right\}$
  - b.  $\{4\}$
  - c.  $\left\{\frac{1}{3}\right\}$
  - d.  $\{-1, 4\}$
  - e.  $\left\{\frac{1}{3}, 1\right\}$

5. If  $a, b, c, d,$  and  $e$  are real numbers and  $a \neq 0$ , then the polynomial equation  $ax^7 + bx^5 + cx^3 + dx + e = 0$  has
- only one real root.
  - at least one real root.
  - an odd number of non-real roots.
  - no real roots.
  - no positive real roots.
6. If  $f(x) = e^{3x}$ , which of the following lines is an asymptote to the graph of  $f$ ?
- $y = 0$
  - $x = 0$
  - $y = 3$
  - $y = \ln(3x)$
  - $3x = \frac{1}{y}$
7. If  $f(x) = 2x^3 + Ax^2 + Bx - 3$  and if  $f(2) = -5$  and  $f(-3) = -15$ , what is the value of  $A - B$ ?
- 10
  - 10
  - 11
  - 12
  - It cannot be determined from the information given
8. Let  $f(x) = \sin(\arctan x)$ . What is the range of  $f$ ?
- $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
  - $(0, 1]$
  - $[0, 1]$
  - $(-1, 1)$
  - $[-1, 1]$

9. If  $\log_a(2^a) = \frac{a}{3}$ , then  $a =$

- a. 2
- b. 4
- c. 8
- d. 16
- e. 32

10. The fundamental period of the function defined by  $f(x) = 2 - 3 \cos^2\left(\frac{\pi x}{3}\right)$  is

- a.  $2\pi$
- b.  $\frac{\pi}{3}$
- c. 3
- d. 6
- e.  $6\pi$

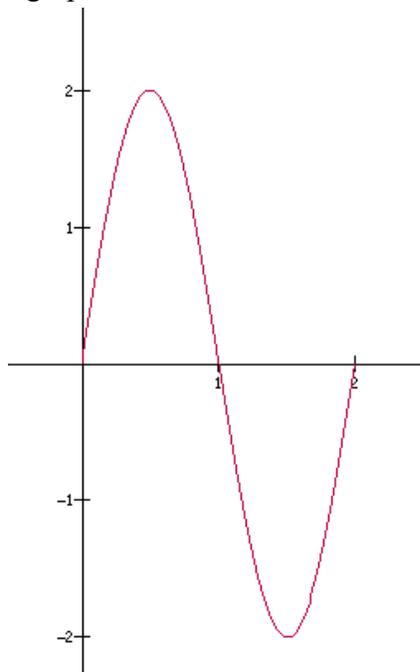
11. If  $f(x_1) + f(x_2) = f(x_1 + x_2)$  for all real numbers  $x_1$  and  $x_2$ , which of the following could define  $f$ ?

- a.  $f(x) = x + 1$
- b.  $f(x) = 2x$
- c.  $f(x) = \frac{1}{x}$
- d.  $f(x) = e^x$
- e.  $f(x) = x^2$

12. The graph of  $y^2 = x^2 + 4$  is symmetric to which of the following?

- I. The  $x$ -axis
  - II. The  $y$ -axis
  - III. The origin
- a. I only
  - b. II only
  - c. III only
  - d. I and II only
  - e. I, II and III

13. The figure below shows the graph of a cosine function for one period. Which of the following is an equation for the graph?



- a.  $y = 2 \cos\left(\pi\left(x - \frac{1}{2}\right)\right)$
- b.  $y = 2 \cos\left(\frac{\pi}{2}(x+1)\right)$
- c.  $y = 2 \cos(2x+1)$
- d.  $y = 2 \cos(\pi(x+1))$
- e.  $y = \cos\left(2\left(x + \frac{\pi}{2}\right)\right)$
14. Find the length of an arc intercepted by a central angle of measure  $\frac{2\pi}{3}$  if the radius of the circle is 12 in.
- a.  $4\pi$  in.
- b.  $16\pi$  in.
- c.  $8\pi$  in.
- d.  $\frac{4\pi}{3}$  in.
- e.  $\frac{8\pi}{3}$  in.

15. The terminal side of an angle  $\theta$ , in standard position passes through the point  $(7, -24)$ . What is  $\csc \theta$ ?

- a.  $-\frac{24}{25}$
- b.  $-\frac{25}{24}$
- c.  $\frac{24}{25}$
- d.  $-\frac{25}{7}$
- e.  $-\frac{24}{7}$

16. The graph of the polar equation  $r = 2 \cos \theta + 1$  is a

- a. cardioid
- b. rose
- c. limacon
- d. spiral
- e. circle

17. In the expression  $\frac{u}{\sqrt{u^2 - 1}}$ , let  $u = \csc x$ . Which of these is equivalent to the resulting expression?

- a.  $\sec x(\sec x + \tan x)$
- b.  $\csc x$
- c.  $\sec x$
- d.  $\csc x(\csc x + \cot x)$
- e.  $\frac{\cos x(\sin x - 1)}{\sin x}$

18. Suppose  $0 < t < \frac{\pi}{2}$ ,  $f(t) = \sin t \cos t + \tan t \csc t$  and  $\sin t = \frac{1}{2}$ . Which of this is the value of  $f(-t)$ ?

a.  $\frac{11\sqrt{3}}{12}$

b.  $-\frac{11\sqrt{3}}{12}$

c.  $\frac{7\sqrt{3}}{12}$

d.  $\frac{5\sqrt{3}}{12}$

e.  $\frac{-5\sqrt{3}}{12}$

19. Suppose  $f(x) = 3 \sin\left(2x - \frac{\pi}{3}\right) + 1$ . For which of the following values of  $x$  does the function reach its maximum value?

a.  $\frac{35\pi}{12}$

b.  $\frac{17\pi}{12}$

c.  $\frac{7\pi}{6}$

d.  $\frac{8\pi}{3}$

e. More than one of the answers given in A,B,C and D generates a maximum value

20. Evaluate  $\sin(2142^\circ + 303^\circ) * \sin(2142^\circ - 1947^\circ)$

a.  $\frac{1}{4}$

b.  $\frac{1}{2}$

c.  $-\frac{1}{4}$

d.  $-\frac{1}{2}$

e. 1

21. Suppose  $A = \frac{2^x - 2^{-x}}{2}$  and  $B = \frac{2^x + 2^{-x}}{2}$ . Compute  $(A + B)^2$ .

a.  $2^x$

b.  $4^x$

c.  $2^{x-1}$

d.  $4^{x-1}$

e.  $4^x + 2$

22. Simplify:  $\log_2 x + \log_{\frac{1}{2}} x$

a. 0

b. 1

c.  $\frac{1}{4}$

d.  $\log x$

e. -1

23. If  $\ln 2 = .693$  and  $\ln 3 = 1.098$ , find  $\ln 36$ .

- a. 1.6859
- b. 1.791
- c. 3.582
- d. 1.522
- e. 10.728

24. The graph of  $f(x) = e^x$  passes through the key point  $(0, 1)$ . Which of these gives the transformation of the key point for  $g(x) = -2e^{4-x} + 1$ ?

- a.  $(-4, -1)$
- b.  $(4, 0)$
- c.  $(-4, 1)$
- d.  $(4, -1)$
- e.  $(-4, 0)$

25. State the domain of the function:  $f(x) = \frac{e^x + 1}{e^x - 1}$

- a.  $(-\infty, 1) \cup (1, \infty)$
- b.  $(-\infty, e) \cup (e, \infty)$
- c.  $(-\infty, 0) \cup (0, \infty)$
- d.  $(-\infty, \infty)$
- e.  $[0, \infty)$

26. Suppose a triangle has vertices  $(0, 0)$ ,  $(0, 4)$  and  $(2, 3)$ . For each vertex, write the equation of the line that contains that vertex and is perpendicular to the opposite side of the triangle and then find the point of intersection of the three lines.

- a.  $\left(\frac{2}{3}, 3\right)$
- b.  $(2, 2)$
- c.  $\left(2, \frac{2}{3}\right)$
- d.  $\left(3, \frac{3}{2}\right)$
- e.  $\left(\frac{3}{2}, 3\right)$ .

27. Form the difference quotient for  $f(x) = \frac{1}{5x+7}$  and evaluate it at  $x = -3$ .

- a.  $\frac{-1}{8}$
- b.  $\frac{-5h}{64-40h}$
- c.  $\frac{-5}{40h-64}$
- d.  $\frac{5}{40h-64}$
- e.  $\frac{-5h}{40h-64}$

28. Evaluate  $\sin^2(6A) - \sin^2(3A) + \cos^2(6A) + \cos^2(3A)$

- a.  $1 + \cos^2(6A)$
- b.  $1 + \cos(6A)$
- c.  $1 + \sin(6A)$
- d. 0
- e. 2

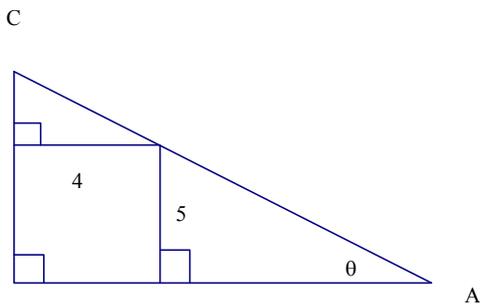
29. Evaluate  $\frac{\sin\left(\frac{11\pi}{6}\right)\sec\left(\frac{4\pi}{3}\right)}{1 + \tan\left(\frac{5\pi}{4}\right)}$

- a. Undefined
- b. 2
- c.  $\frac{1}{2}$
- d.  $-\frac{1}{2}$
- e. 0

30. Suppose  $f(x) = 4x - 2$ . The linear function that is perpendicular to  $f$  and passes through the origin intersects  $f$  at

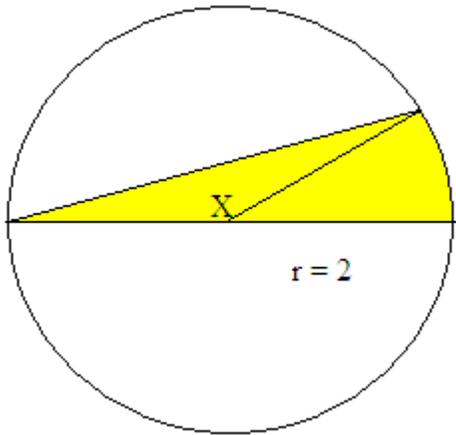
- a.  $\left(\frac{-8}{15}, \frac{-2}{15}\right)$
- b.  $\left(\frac{-8}{15}, \frac{-32}{15}\right)$
- c.  $\left(\frac{8}{17}, \frac{2}{17}\right)$
- d.  $\left(\frac{8}{17}, \frac{-2}{17}\right)$
- e.  $\left(\frac{-8}{17}, \frac{-62}{17}\right)$

31. Express AC as a function of  $\theta$ .



- a.  $9 \sec \theta$
- b.  $4 \sec \theta + 5 \csc \theta$
- c.  $5 \sec \theta + 4 \csc \theta$
- d.  $9 \csc \theta$
- e.  $3 + 5 \csc \theta$

32. Find the area of the shaded region. The radius of the circle is 2, and the measure of angle  $X$  in the obtuse triangle is given in radians.



- a.  $\pi - X + \sin X$
- b.  $2(\pi - X + \sin X)$
- c.  $2x + 2 \sin(\pi - X)$
- d.  $2x + 4 \sin(\pi - X)$
- e.  $2(\pi - X + 2 \sin X)$

33. Write the equation of the conic section with eccentricity 2 which has foci at  $(\pm\sqrt{7}, 0)$ .

- a.  $2x^2 - 7y^2 = 14$
- b.  $4x^2 - 12y^2 = 21$
- c.  $12x^2 + 4y^2 = 21$
- d.  $12x^2 - 4y^2 = 21$
- e.  $4x^2 + 12y^2 = 21$

Name: \_\_\_\_\_ School: \_\_\_\_\_

Tie-breaker:

Show all steps that you use to answer this question:

A rectangular piece of paper that measures  $a$  inches by  $b$  inches ( $a > b$ ) is red on one side and white on the other. The paper is on a table with the red side up. Suppose you fold the paper along the diagonal without lifting the paper off the table. What is the area of the red triangle that you now see?