

Name: _____ School: _____

University of Houston High School Mathematics Contest Geometry Exam – Spring 2014

Note that diagrams may not be drawn to scale.

1. Given the following conditional statement:

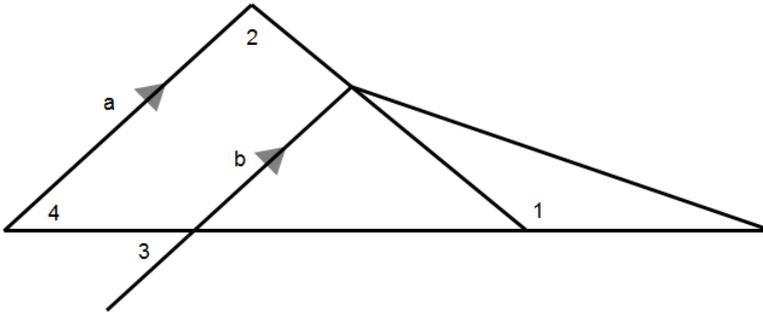
If two lines are parallel, then they do not intersect.

Find the truth value of the contrapositive and converse, respectively.

- (A) False, True (B) False, False (C) True, False (D) True, True

2. In the diagram below, $a \parallel b$.

Suppose that $m\angle 1 = 4x + 23$, $m\angle 2 = 2x - 1$, and $m\angle 3 = 3x$. Find $m\angle 4$.



- (A) 30.5° (B) 24° (C) 61° (D) 72° (E) 47°

3. A rectangular walkway measures $6 \text{ m} \times 800 \text{ cm}$. Find the perimeter of the walkway.

- (A) 14 m (B) 1612 m (C) 28 m (D) 806 m (E) 48 m

4. The sum of the measures of all but one of the interior angles of a convex polygon is 1428° . How many sides does the polygon have?

- (A) 10 (B) 9 (C) 8 (D) 12 (E) Cannot be determined

5. One leg of a right triangle measures 3 cm. If the area of the right triangle is $6\sqrt{2} \text{ cm}^2$, find the length of its hypotenuse.

- (A) $\sqrt{41} \text{ cm}$ (B) $9\sqrt{2} \text{ cm}$ (C) $\sqrt{17} \text{ cm}$ (D) $\sqrt{73} \text{ cm}$ (E) 9 cm

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6. 7 squares of the same size are placed side by side to make a single rectangle. The area of this new rectangle is 63 in^2 . Find its perimeter.

- (A) 48 in. (B) 12 in. (C) 9 in. (D) 84 in. (E) 42 in.

7. **Given:** \overline{KL} bisects \overline{FG} at point M.
What conclusion(s) follow from the given information?

- I. M is the midpoint of \overline{FG} .
- II. M is the midpoint of \overline{KL} .
- III. $\overline{KM} \cong \overline{ML}$
- IV. $\overline{FM} \cong \overline{MG}$
- V. $\overline{KL} \perp \overline{FG}$

- (A) V only (B) I, IV and V only (C) I, II, III, and IV only
(D) II and III only (E) I and IV only

8. How many cubes with an edge of 8 inches can be cut out from a cube with an edge of 48 inches?

- (A) 36 (B) 216 (C) 252 (D) 180 (E) 288

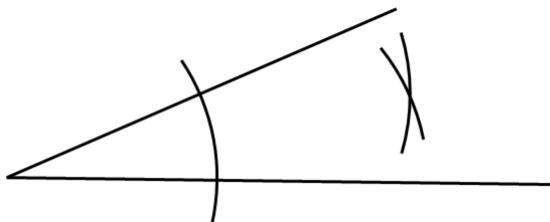
9. Which of the following statements is/are true?

- I. Every rectangle has at least 2 lines of symmetry.
- II. Every parallelogram has at least one line of symmetry.
- III. Every rhombus has exactly 4 lines of symmetry.
- IV. Every square has exactly 4 lines of symmetry.

- (A) I, II, III and IV (B) IV only (C) I only
(D) I, II and IV only (E) I and IV only

10. A construction can be found below with all steps shown except for any final straightedge marking(s). Which of the following constructions is being performed?

- (A) Perpendicular bisector
- (B) Equilateral triangle
- (C) Centroid
- (D) Obtuse triangle
- (E) Angle bisector



11. The tires on a car have a diameter of 24 inches. The car runs over a pebble, which gets stuck in the tread of one of the tires. If the pebble is rotating at a speed of 420 revolutions per minute, find the speed of the car in feet per second.

(A) $2,016\pi$ (B) 28π (C) 84π (D) 14π (E) 840π

12. Jackie creates a solid paperweight in the shape of a hemisphere. She paints the entire bottom of the paperweight (the flat part) and then runs out of paint. If Jackie used 12 grams of paint on the bottom of the paperweight, how many grams of paint would we expect her to need to paint the remainder of the paperweight?

(A) 36π (B) 24 (C) 72π (D) 48 (E) 12

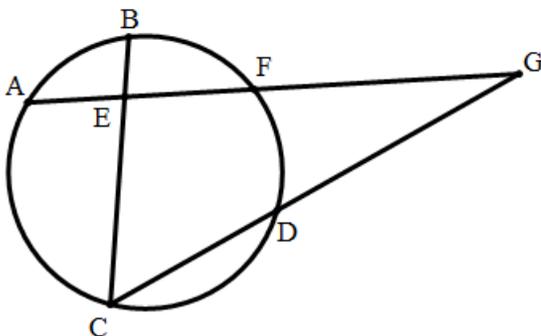
13. A dilation is performed on triangle RBV with a scale factor of 4% , resulting in image $R'B'V'$. If $R'B' = 9$, find the length of RB .

(A) 225 (B) 36 (C) $\frac{9}{25}$ (D) $\frac{224}{25}$ (E) $\frac{9}{4}$

14. Triangle CAT is a right triangle with hypotenuse \overline{CT} , and CAT is similar to triangle DOG . If $DG = 3CT$ and $\cos(C) = \frac{2}{7}$, find $\tan(D)$.

(A) $\frac{2\sqrt{5}}{15}$ (B) $\frac{3\sqrt{5}}{2}$ (C) $\frac{2\sqrt{53}}{53}$ (D) $\frac{3\sqrt{5}}{7}$ (E) $\frac{9\sqrt{5}}{2}$

15. In the circle below, \overline{AG} and \overline{CG} are secant segments intersecting the circle in points F and D , respectively. \overline{AG} intersects chord \overline{BC} at point E . If $m\angle BEF = 75^\circ$, $m\widehat{BF} = 63^\circ$, and $m\angle G = 23^\circ$, find the degree measure of \widehat{FD} .



- (A) 46°
 (B) 64°
 (C) 41°
 (D) 87°
 (E) 17°

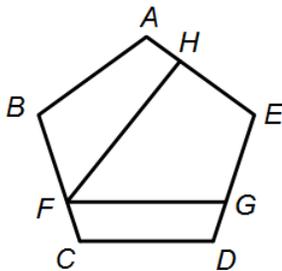
16. A plastic kitchen measuring spoon is in the shape of a hemisphere, connected to a handle, and has a volume of one tablespoon. The diameter of its hemispherical cavity is 38.4 mm. If the manufacturer makes a hemispherically-shaped measuring spoon with a volume of 1 teaspoon, find the diameter of the teaspoon's cavity, in millimeters. *Note that 1 tablespoon = 3 teaspoons.*

(A) $38.4(\sqrt[3]{3})$ (B) 12.8 (C) $12.8(\sqrt[3]{3})$ (D) $12.8(\sqrt[3]{9})$ (E) $12.8\sqrt{3}$

17. Kathy makes a three-layer cake for Michael's birthday. All three layers are cylindrical in shape. The bottom layer is 9 inches in diameter and 2 inches high. The middle layer is 7.5 inches in diameter and 3 inches high. The top layer is 5.5 inches in diameter and 4 inches high. Shortly before the party, Kathy realizes that she did not buy enough frosting, so she decides to first stack the three layers, and then only frost the portions of the cake which can be seen (instead of putting frosting between the layers as well). How many square inches of cake will be frosted?

(A) 82.75π (B) 143.5π (C) 51.5π (D) 104.375π (E) 133.1875π

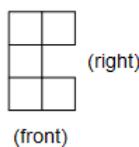
18. Regular pentagon ABCDE is shown below. Points F, G, and H are drawn on the diagram, where $\overline{FG} \parallel \overline{CD}$ and $m\angle FHE = 93^\circ$. Find $m\angle HFG$.



- (A) 81°
 (B) 54°
 (C) 27°
 (D) 87°
 (E) 51°

19. A building is made from stacked cubes, and views from three different perspectives are illustrated below. How many cubes are needed to construct this building?

Top View:



Front View:

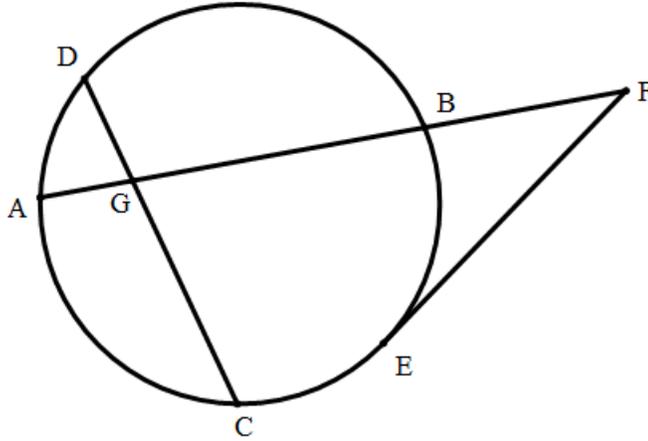


Right View:



- (A) 12
 (B) 9
 (C) 17
 (D) 10
 (E) 7

20. In the circle below, chords \overline{AB} and \overline{CD} intersect at point G. Secant \overline{AF} and tangent \overline{EF} are also shown. If the ratio of DC:DG is 3:1, $AG = 2$, $BF = 5$, and $EF = 5\sqrt{3}$, find the length of \overline{DC} .



- (A) $4\sqrt{3}$
 (B) $\frac{20}{3}$
 (C) $4\sqrt{2}$
 (D) $\frac{8\sqrt{3}}{3}$
 (E) $6\sqrt{2}$

21. A right pyramid has seven faces and its base is a regular polygon with side length 12 cm. If the height of the pyramid is $3\sqrt{13}$ cm, find the total surface area of the pyramid.

- (A) $216\sqrt{39}$ cm² (B) $(216\sqrt{3} + 90)$ cm² (C) $(532\sqrt{3} + 1080)$ cm²
 (D) $108\sqrt{13}$ cm² (E) $(540 + 216\sqrt{3})$ cm²

22. Shawn selects a spherically-shaped orange which has a diameter of 9 cm. He uses his cutting knife to slice the orange into six slices of equal thickness. This results in two large slices identical in size, two medium slices identical in size, and two small end-piece slices identical in size – where all six slices have the same thickness. Find the diameter of the flat surface of one of the small end pieces.

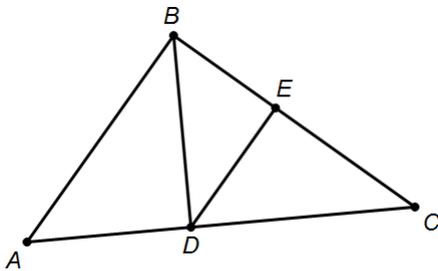
- (A) $5\sqrt{3}$ cm (B) $6\sqrt{5}$ cm (C) $6\sqrt{2}$ cm (D) $3\sqrt{5}$ cm (E) 3 cm

23. In the time period of 225 seconds, the minute hand of a clock sweeps out an area of 5π square centimeters. Find the length of the minute hand.

- (A) $4\sqrt{5}$ cm (B) $20\sqrt{2}$ cm (C) 40 cm (D) $\frac{2\sqrt{3}}{3}$ cm (E) $2\sqrt{2}$ cm

24. A regular convex octagon has an area of $(500 + 500\sqrt{2}) \text{ cm}^2$. Find the length of one side.
- (A) $5\sqrt{10}$ cm
 (B) 50 cm
 (C) 25 cm
 (D) $10\sqrt{2}$ cm
 (E) $10\sqrt{3}$ cm

25. Right triangle ABC is shown below with hypotenuse \overline{AC} and altitude \overline{BD} . Point E is chosen between B and C so that $\overline{AB} \parallel \overline{DE}$. If $BE = 8$ and $ED = 10$, find the length of \overline{AB} .



- (A) 20 (B) $\frac{82}{5}$ (C) 16 (D) $2\sqrt{41}$ (E) $\frac{32}{5}$
26. Zach wants to find the volume of the metal mailbox which is mounted on a post at his curb. The shape of the front of the mailbox (or any other cross-section of the mailbox which is parallel to the mailbox door) is that of a semicircle placed curved-side-up on top of a rectangle of the same width. The front of the mailbox is 10 inches wide at its base, and 12 inches tall at its highest point. Find the volume of the mailbox if the depth of the mailbox (the dimension parallel to the ground) is 20 inches. Disregard the thickness of the metal in your calculations.
- (A) 2400 in^3
 (B) $(1400 + 250\pi) \text{ in}^3$
 (C) $(400 + 50\pi) \text{ in}^3$
 (D) $(500\pi + 1400) \text{ in}^3$
 (E) $(250\pi + 2400) \text{ in}^3$

27. A flower garden is built in the center of a park. Figure 1 shows a view of the garden from above. The garden is the area which looks like a block-shaped “X”, and the dots in the diagram represent people on paths who are viewing the garden. Figure 2 details the geometric shape of the garden, which can be represented by five congruent overlapping squares. The area of each overlapping shaded region represents $\frac{3}{20}$ of the area of any one of the five squares. Flowers are planted with equal distribution (of flowers per square unit) throughout the garden area, and their colors correspond to the chart shown in Figure 3. (Note that blue flowers are planted in the entire cross-shaped area within the center square, and no flowers are planted on any of the paths.) If a gardener randomly picks one of the flowers from the garden, what is the probability that the flower is red?

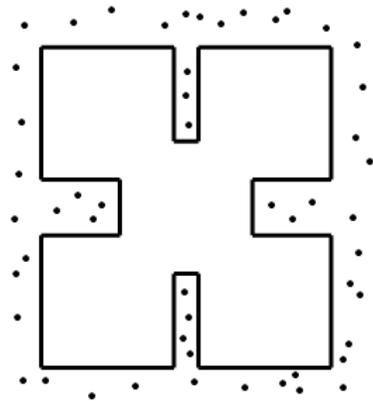


Figure 1

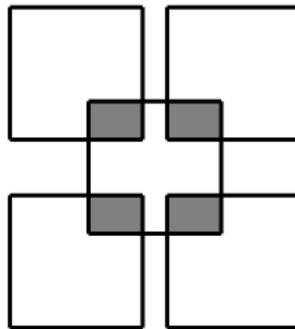


Figure 2

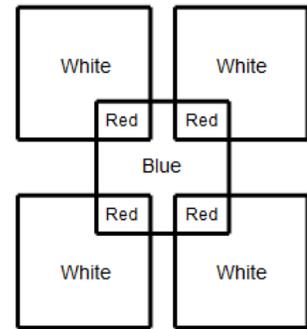


Figure 3

- (A) $\frac{3}{25}$ (B) $\frac{3}{20}$ (C) $\frac{12}{97}$ (D) $\frac{3}{5}$ (E) $\frac{3}{22}$
28. Trapezoid EFGH has bases \overline{EF} and \overline{HG} . The coordinates of E, F, G, and H are shown below in terms of some positive number n .

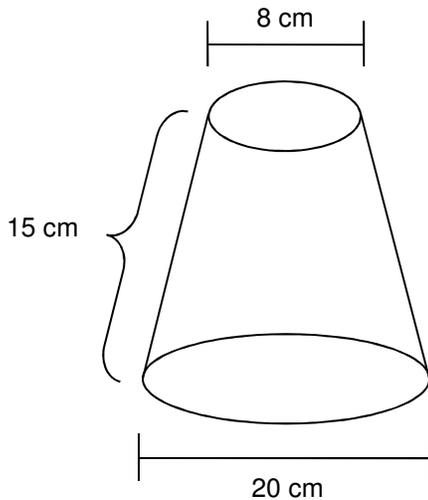
$$E(n+1, 3n+1) \quad F(4n+5, 6n-1) \quad G(5n+4, n+1) \quad H(n-3, -2n+1)$$

Find the equation of the median of trapezoid EFGH.

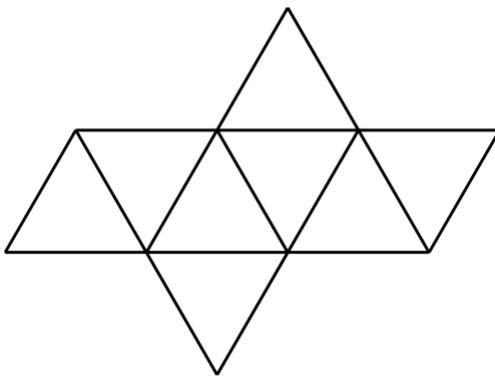
- (A) $y = \frac{5}{2}x - \frac{1}{2}$ (B) $y = \frac{2}{5}x + \frac{8}{5}$ (C) $y = 6x - 39$
 (D) $y = \frac{2}{5}x + \frac{1}{5}$ (E) $y = \frac{5}{2}x - 4$

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29. A spray bottle is in the form of a truncated cone, as shown below. (The attachments for the top sprayer portion are not shown.) The bottle opening has a diameter of 8 cm, the base of the bottle has a diameter of 20 cm, and its slant height is 15 cm. Find the volume of the spray bottle.



- (A) $\frac{500\pi\sqrt{21}}{3}$ cm³ (B) 300π cm³ (C) $468\pi\sqrt{21}$ cm³
(D) 624π cm³ (E) $156\pi\sqrt{21}$ cm³
30. The net for a regular convex polyhedron is shown below. If the volume of the polyhedron is 36 cm³, find the length of its edge in centimeters.



- (A) $6\sqrt{2}$ (B) $2\sqrt{6}$ (C) $2\sqrt{2}$ (D) $3\sqrt{2}$ (E) $2\sqrt{3}$

END OF EXAM ☺