Questions 1 and 2 pertain to two vectors **A** and **B**. Vector **A** has components  $A_x$  and  $A_y$ , where both components are positive. Vector **B** has magnitude B and makes an angle of  $\theta$  measured counter-clockwise with respect to the positive y axis as shown.

1. The x-component of the vector  $\mathbf{A} - \mathbf{B}$  is given by

(A)  $A_x - B \cos \theta$  (B)  $A_x + B \cos \theta$  (C)  $B \sin \theta + A_x$  (D)  $-B \sin \theta + A_x$ (E) none of the previous answers

2. The positive angle between **A** and **B** is given by

(A)  $\theta - \arctan(A_y/A_x)$  (B)  $\theta + \pi/2 - \arctan(A_y/A_x)$  (C)  $\theta - \arctan(A_x/A_y)$ (D)  $\theta + \pi/2 - \arctan(A_x/A_y)$  (E) none of the previous answers

Questions 3 and 4 pertain to a rock that is thrown from ground level with a speed of  $v_0$  m/s at an angle of  $\theta$  with respect to the horizontal. Let g represent the acceleration due to gravity.

3. The distance the rock has traveled in the horizontal direction when it is at its highest point is given by

| (A) $2 v_0^2 \cos \theta \sin \theta/g$ | (B) $v_0^2 \sin \theta \cos \theta / g$ | (C) $(v_0 \sin \theta)^2/g$ |
|---|---|-----------------------------|
| (D) $(v_0 \cos \theta)^2/g$             | (E) none of the previous answers        |                             |

4. When the rock strikes the ground, the angle the velocity makes with respect to the ground is given by

(A)  $\pi/2$  (B)  $\theta$  (C)  $-\theta$  (D) zero (E) none of the previous answers

Questions 5 and 6 pertain to two blocks (mass M and mass m, with M > m) that are pushed along by a horizontal force **F** as shown.



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5. The magnitude of the normal force of contact between the blocks is given by

 $\begin{array}{ll} (A) (m + M)/MF & (B) (m + M)/mF & (C) MF/(m + M) \\ (D) mF/(m + M) & (E) none of the previous answers \end{array}$ 

6. If the force were applied to the left on m, the magnitude of the normal force of contact between M and m

(A) increases (B) decreases (C) remains the same (D) cannot be determined (E) none of the previous answers

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Questions 7 and 8 pertain to a mass m that slides on a frictionless loop-theloop of radius R as shown. The block is given a sufficient speed  $v_0$  at point A to go completely around the loop without losing contact with it.

7. The contact force that the loop exerts on mass m at the top of the track is given by

(A) mg (B)  $mv_0^2/R$  (C)  $(mv_0^2/R) - 5mg$  (D) $mv_0^2/R - 3mg$ (E) none of the previous answers

8. The vector that could represent the acceleration of the mass when it has traveled  $\frac{1}{4}$  of the circumference is given by

(A)  $\leftarrow$  (B)  $\uparrow$  (C)  $\rightarrow$ (E) none of the previous answers

9. A uniform thin rod can rotate about a pivot fixed at its center as shown. The left end of the rod is tied to a string attached to the floor. A force F is applied at an angle of  $\theta$  and keeps the rod in equilibrium while horizontal. F is applied L/4 from the right end. The mass of the rod is m and its overall length is L. Let g be the acceleration due to gravity.



(D) ↓

The tension in the string is given by

(A)  $(F \cos \theta)/2$  (B)  $(F \cot \theta)/2$  (C)  $(F \sin \theta)/2$  (D)  $(F \tan \theta)/2$ (E) none of the previous answers

10. A mass m is attached to a spring whose natural length is L. The spring is stretched to a length of 1.5 L and released at time t = 0. The time for one complete oscillation is T.

The spring constant k is given by

(A)  $(2\pi/T)^2$ m (B)  $(2\pi m)^2/T$  (C)  $T^2/(2\pi m)$  (D)  $T/(2\pi m)^2$  (E) none of the previous answers



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Questions 11 and 12 pertain to one mole of an ideal gas undergoing the thermodynamic cycle shown. Use  $P_D = P_C = P_o$  and  $P_A = P_B = 3P_o$ ,  $V_C = V_B = 4.0 V_o$  and  $V_A = V_D = V_o$ .



12. The ratio of the temperature at point D to the temperature at point B is given by

(A) (1:3) (B) (1:6) (C) (1:4) (D) (1:12) (E) none of the previous answers

13. A speaker S whose intensity is I emits sound at a frequency f. Point A S\* A represents a car with a listener in it.

In order for the listener to hear the lowest frequency possible

(A) A and S should move away from one another
(B) A should remain stationary and S should move toward A
(C) S should remain stationary and A should move toward S
(D) A and S should move toward each other
(E) A and S should remain stationary

Questions 14 and 15 pertain to standing waves on a string having a linear mass density  $\mu$  as shown. The speed of the waves is v<sub>o</sub>. The string has a length L. The amplitude is greatly exaggerated for clarity.

14. The wavelength for the case shown is

(A) L (B) (3/2)L (C) 3L (D) (2/3)L (E) none of the previous answers

15. The tension in the string is given by

(A)  $\mu/v_o^2$  (B)  $v_o^2/\mu$  (C)  $\mu v_o^2$  (D)  $\mu v_o$  (E) none of the previous answers

16. Work is most closely related to

(A) change in total energy (B) change in kinetic energy (C) change in potential energy (D) change in linear momentum (E) change in angular momentum

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17. A skater spins with angular velocity  $\omega_0$  and moment of inertia  $I_0$ . Initially, she has kinetic energy  $K_0$  and angular momentum  $L_0$ . She pulls her arms in to reduce her moment of inertia to  $I_0/2$ . Her new kinetic energy is

(A)  $K_o$  (B)  $K_o/2$  (C)  $4K_o$  (D)  $2K_o$ (E) none of the previous answers

18. Bernoulli's principle is a statement of

(A) hydrostatic equilibrium (B) thermal equilibrium for fluids (C) mechanical equilibrium for fluids (D) energy conservation in moving fluids (E) momentum conservation for moving fluids

19. If a charge -Q is placed inside a hollow isolated conductor having a wall thickness t that is originally neutral and the charge does not touch that conductor at any time,

(A) the inside surface of the conductor will become positively charged.

(B) the outside surface of the conductor will become positively charged.

(C) both the inner and outer surfaces will remain neutral.

(D) both the inner and outer surfaces will become negatively charged.

(E) none of the previous outcomes will occur.

Questions 20 and 21 pertain to the set of charges shown. A = 2Q = B = C - Q = Dx = x = x

20. The most likely place for the magnitude of the electric field to be zero is at point

(A) A (B) B (C) C (D) D (E) none of the previous answers

21. The most likely place for the electric potential to be maximum is at point

(A) A (B) B (C) C (D) D (E) none of the previous answers

22. Two capacitors with capacitances of  $C_1$  and  $C_2$  are connected in series with a battery having an emf of E. The charge that accumulates on one plate of  $C_2$  is

(A)  $E/C_2$  (B)  $EC_2$  (C)  $EC_1C_2/(C_1+C_2)$ (D)  $(C_1 + C_2)/EC_1C_2$  (E) none of the previous answers 23. Light that has a wavelength  $\lambda$  in a vacuum travels in a material whose index of refraction is n. Let c be the speed of light in a vacuum. Its wavelength in the material is

(A)  $n\lambda$ (B)  $\lambda/n$ (C)  $(n-1)\lambda$ (D)  $\lambda$ (E) none of the above

24. A magnetic field is along the +x direction as shown. A charge q enters the region with velocity **v** whose direction is arbitrary. The maximum magnetic force occurs

(A) only when  $\mathbf{v}$  is along the x axis (B) only when **v** is along the y axis (C) only when  $\mathbf{v}$  is along the z axis (D) when  $\mathbf{v}$  is along the x or z axis (E) when **v** is along the y or z axis

25. Linearly polarized light whose intensity is  $I_0$  passes through a polarizer whose transmission axis makes an angle  $\theta$  with respect to the electric field of the light. If the intensity of the wave after passing through the linear polarizer is f  $I_0$ , where f is between zero and 1, the angle  $\theta$  is

(A) arccos (f) (C)  $\arccos(f)^{1/2}$ (D)  $\arccos(1/f)^{1/2}$ (B)  $\arccos(1/f)$ (E) none of the previous answers

26. Diamond has an index of refraction n<sub>d</sub>. If a diamond is immersed in oil having an index of refraction  $n_o$ , where  $n_o < n_d$ , the critical angle for total internal reflection is

(C)  $\arcsin(n_d/n_o)$ (D)  $\arcsin(n_0/n_d)$ (A)  $n_0/n_d$ (B)  $n_d/n_o$ (E) none of the previous answers

27. A long straight wire is in the same plane as a wooden nonconducting loop. The wire carries an decreasing current in the direction shown. The correct statement concerning this situation is

- (A) there will be no induced emf
- (B) there will be a clockwise induced emf, but no induced current
- (C) there will be an induced current that is clockwise around the loop
- (D) there will be a counter-clockwise induced emf, but no induced current

(E) none of the previous statements



$$e \xrightarrow{v} \xrightarrow{g} \xrightarrow{z \cdot e} x$$

28. A slit of width  $w_0$  is illuminated by a laser beam whose width is much greater than  $w_0$ . The width and intensity of the central diffraction peak are  $b_0$  and  $I_0$ , respectively. If the slit width is decreased to  $2w_0/3$ , the width b and intensity I of the central diffraction peak (assuming far-field diffraction) are most likely to be given by

(A)  $I < I_0$ ,  $b < b_0$  (B)  $I < I_0$ ,  $b > b_0$  (C)  $I > I_0$ ,  $b < b_0$  (D)  $I > I_0$ ,  $b > b_0$  (E) none of the previous answers