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University of Houston

High School Math Contest – Spring 2015

Algebra 2 Test

1. A coin is biased so that the probability of obtaining a head is 0.15. Another coin is biased where the probability of obtaining a head is 0.8. If both coins are tossed, find the probability of obtaining at least one head.

A. $\frac{83}{100}$ B. $\frac{17}{20}$ C. $\frac{94}{100}$ D. $\frac{19}{20}$ E. None of these

2. Find all zeros of $f(x) = x^4 - 3x^3 + 6x^2 + 2x - 60$ given that $f(1 + 3i) = 0$.

A.	B.	C.	D.	E.
$x = 1 + 3i$	$x = 1 + 3i$	$x = 1 + 3i$	$x = 1 + 3i$	None of
$x = 1 - 3i$	$x = -1 + 3i$	$x = 3 - i$	$x = -3i$	These
$x = 3$	$x = -3$	$x = -3$	$x = -3$	
$x = -2$	$x = 2$	$x = 2$	$x = 4$	

3. Find the product of the solutions to the following equation:

$$8|x + 2|^2 - 3|x + 2| = 5$$

A. $\frac{-5}{8}$ B. $\frac{693}{64}$ C. -3 D. 3 E. None of these

4. If $\log_b 5 = R$ and $\log_b 4 = S$, find $\log_b 50$.

A. $\frac{R^2 S}{2}$ B. $2R + \frac{S}{2}$ C. $\frac{R}{2} + 2S$ D. RS E. $R^2 + \sqrt{S}$

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5. Find the solution to the following inequality, and write the answer in interval notation.

$$\frac{3}{x-4} \geq \frac{-5}{x+6}$$

A. $(4, \infty)$

B. $\left(-\infty, \frac{1}{4}\right]$

C. $\left(-6, \frac{1}{4}\right] \cup (4, \infty)$

D. $(-\infty, \infty)$

E. No Solution

6. What is the domain of the following function:

$$f(x) = \frac{5x}{\sqrt{8x^2 - 2x - 15}}$$

A. $\left(\frac{-5}{4}, \frac{3}{2}\right)$ B. $\left(-\infty, \frac{3}{2}\right)$ C. $\left(\frac{5}{4}, \infty\right)$ D. $\left(-\infty, \frac{-5}{4}\right) \cup \left(\frac{3}{2}, \infty\right)$ E. $\left[\frac{-5}{4}, \frac{3}{2}\right]$

7. If a, b, and c satisfy the following system of equations, find the product abc.

$$\begin{aligned} a - 2b + 3c &= 9 \\ -a + 3b &= -4 \\ 2a - 5b + 5c &= 17 \end{aligned}$$

A. 2 B. -2 C. 3 D. -3 E. 4

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8. Jackie has $4.85 in coins, comprised entirely of nickels, dimes and quarters. If she has six more nickels than dimes and twice as many quarters as dimes, how many quarters does she have?

A. 7 B. 13 C. 14 D. 18 E. None of these

9. Which of the following expressions is equal to:

$$\frac{(-a)^7 + \left(\frac{1}{a}\right)^{-3}}{-a^2 - \left(\frac{-1}{a}\right)^{-5}}$$

A. $\frac{-(a^2+a)(a^2+1)}{a^2+a+1}$ B. $\frac{(a^3+a^2)(a^2+1)}{a^3+a^2+1}$ C. $\frac{-(a^5+a)(a^3+1)}{a^5+a+1}$ D. $\frac{-(a^3+1)(a^2+1)}{a^4+a^2+1}$ E. None of these

10. At what point(s) do the graphs of the following equations intersect?

$$\frac{(y+3)^2}{4} - \frac{(x-2)^2}{25} = 1$$

$$x = -10(y+3)^2 + 2$$

A. $(-3,2)$ B. $(2,-3)$ C. $(7,-3)$ and $(-3,-3)$ D. $(2,-1)$ and $(2,-5)$ E. No Solution

11. Find x if $A = \begin{bmatrix} 0 & 2 & 1 \\ 3 & -1 & 2 \\ x & x-4 & x-3 \end{bmatrix}$ and det(A) = 14

A. $x = 3$ B. $x = 4$ C. $x = -42$ D. $x = 31$ E. None of these

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12. List all the factors of 2015.How many numbers are on your list?

A. 4 B. 6 C. 8 D. 10 E. 12

13. A colony of bacteria living on a Petri dish under optimal conditions doubles in size every 20 minutes. At noon a certain day, the Petri dish is completely covered with bacteria. At what time was 12.5% of the plate covered by bacteria?

A. 10:40 a.m. B. 11:00 a.m. C. 11:20 a.m. D. 3:00 a.m. E. None of the Above

14. Which of the following numbers is not a perfect square?

A. 18496 B. 20449 C. 24022 D. 26896 E. None of these

15. The graph of the equation $7x^2 - 9y^2 - 72y = 32 - 70x$ is:

A. An ellipse with center $(-4, -5)$.

B. A hyperbola with center $(-5, -4)$.

C. A hyperbola with center $(-4, -5)$.

D. A circle with center $(4,5)$.

E. None of the Above

16. Suppose x varies jointly as 3^{-y} and z^3, and $x = \frac{3}{2}$ when $y = 2$ and $z = 3$. What is the value of x when $y = 3$, and $z = 2$?

A. $\frac{1}{2}$ B. 3 C. 8 D. $\frac{4}{27}$ E. None of these

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17. Find the inverse function of $y = \frac{\sqrt{8x+4}}{2}$.

A. $f^{-1}(x) = \frac{4x^2-6}{3}$;$x \geq 0$

B. $f^{-1}(x) = \frac{x^2-1}{2}$; $x \geq 0$

C. $f^{-1}(x) = \frac{8x^2+4}{2}$; $x \geq 0$

D. $f^{-1}(x) = \frac{6x^2-3}{3}$; $x \geq 0$

E. None of the Above

18. It takes you 9 minutes to wash a car and it takes your friend 12 minutes to wash a car. How long does it take the two of you to wash 7 cars if you work together?

A. 36 minutes B. 45 minutes C. 63 minutes D. 90 minutes E. None of these

19. Given that $f(x) = \frac{x^2}{x+4} - x$, find $f(3p)$.

A. $f(3p) = \frac{3p^2}{3p+4} - 3p$

B. $f(3p) = \frac{9p^2}{3p+4} - p$

C. $f(3p) = \frac{12p}{3p+4}$

D. $f(3p) = \frac{-12p}{3p+4}$

E. None of the Above

20. If $i = \sqrt{-1}$, then find $i^{34} + i^{95} - i^{108} + i^{53}$.

A. 0 B. 1 C. i D. -2 E. 3

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21. The sum of the repeating decimals $4.1\overline{23}$ and $3.\overline{132}$ is

A. 7.3

B. $7.\overline{255}$

C. $7.2553\overline{644}$

D. $7.2\overline{553644}$

E. None of the Above

22. How far apart are the vertices of the parabolas $x^2 - 2x + 25 = 8y$ and $y^2 - 4y + 8 = 2x$?

A. $\sqrt{5}$ B. $\sqrt{3}$ C. $\sqrt{2}$ D. 2 E. None of these

23. What are the vertical asymptotes for the graph $= \frac{2x^2-5x-3}{x^3-2x^2-x+2}$?

A. $x = 2, x = 1$

B. $x = 2, x = 1, x = -1$

C. $x = -1, x = 2$

D. $x = -2, x = -1$

E. None of the above

24. Define $N(A)$ as the number of elements in set A and let A^c be the complement of A.Let A, B, C be sets for which of the following are true: $A \subset B$, $3N(A) = N(B)$, $N(C) = 80$, $4(N(B \cap C)) = N(C)$, $N(A) = 20$ and $N(A \cap C) = 0$.

Find $N(A^c \cap B \cap C^c)$.

A. 10 B. 15 C. 20 D. 25 E. 5

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25. Let $f(x)$ be a function with the property that $f(a+b)=f(a)f(b)$ for all real numbers a and b. If $f(4)=5$, find $f(-8)$.

A. -25 B. -10 C. $\frac{1}{5}$ D. $\frac{-1}{8}$ E. $\frac{1}{25}$

26. Find a quadratic equation, $y=ax^2+bx+c$, whose graph passes through the points $(-1,3)$, $(1,1)$, and $(2,6)$.

A. $y=2x^2-x+7$

B. $y=2x^2-x$

C. $y=3x^2-3x+1$

D. $y=4x^2-2x+2$

E. None of the above

27. Expand using log properties

$$\ln\left(\frac{(x+3)^2}{x\cdot\sqrt[3]{x-2}}\right)$$

A. $\ln(x+2)-\ln(x)+\frac{1}{3}\ln(x-2)$

B. $2\ln(x+3)-\ln(x)-\frac{1}{3}\ln(x-2)$

C. $3\ln(x+3)-\frac{1}{2}\ln(x)+\ln(x-2)$

D. $2\ln(x+3)+\ln(x)+\frac{1}{3}\ln(x-2)$

E. None of the Above

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28. Let $f(x) = cx^{2.11}$. Suppose that $f(a)$ is 9 times as large as $f(b)$. Find the ratio of a to b.

A. $9^{\frac{1}{2.11}}$ B. $9^{2.11}$ C. 2.11^{9} D. $2.11^{\frac{1}{9}}$ E. None of these

29. Two positive numbers exist such that their product, sum, and difference of squares are equal. Find the product of the two numbers.

A. $\frac{3+\sqrt{3}}{2}$ B. $\sqrt{5}$ C. 1 D. $\frac{3+4\sqrt{3}}{4}$ E. $2+\sqrt{5}$

30. Consider the intersection of the graphs of the following equations. If the *x*- and y-values of the point(s) of intersection are listed in a set S, what is the smallest element of S?

$$x^2 - 8x + 2y^2 = 2$$
$$2x^2 - 5y^2 + 4x = 3$$

A. 4

B. -3

C. 3

D. $\frac{71}{81}$

E. $\frac{-71}{81}$