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Calculator Exam – 2015

Directions: Write your answers on the answer sheet. **DO NOT detach the answer sheet from your exam.** Answers can be given as integers, fractions or in decimal form. The answers which are given in decimal form should be recorded so that they are **accurate to at least 4 places *after* the decimal**.

DO NOT ROUND YOUR ANSWERS until *after* the 4th decimal place!! For example, suppose a question requests the value of sin(2). Your calculator will tell you that sin(2) is 0.9092974268 (assuming your calculator is in radian mode). Examples of correct responses include 0.9092, 0.90929, 0.90923 and 0.90929116. **The response 0.9093 is not correct.**

It does not matter what appears *after* the 4th decimal place, provided the values up to and including the fourth decimal place are correct.

Good Luck!!

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1. Give the average of the numbers $1, \frac{1}{2}, \frac{1}{3}, \ldots, \frac{1}{100}$.

2. Give the number of positive solutions to $\frac{1}{12}x - 1 = 3\sin(4x-1)$.

3. Give the number of positive integer values that are smaller than 4,913, and are integer multiples of 2, 3, 5, 7 or 11.

4. Give the sum of the reciprocals of the positive integers between 1 and 1,000 that are not integer multiples of 2, 3, 5, 7 or 11.

5. Evaluate the function $f(x) = 2x + \cos(x+1) + 3$ at the average of its x and y intercepts.

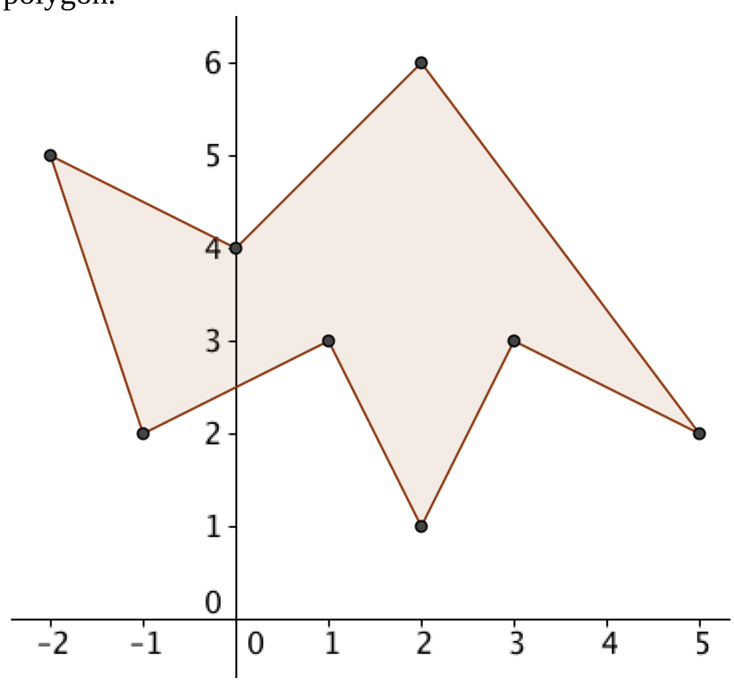
6. Give the fixed point of the function $f(x) = 2x + \cos(x+1) + 3$.

7. Give the x-intercept of the line of best least squares fit for the data (-1,3), (0,-2) and (5,-26).

8. Give the acute angle of intersection (in degrees) of the lines $2x - 3y = 7$ and $5x + 7y = 12$.

9. A right triangle has its legs parallel to the coordinate axes in the xy plane. The hypotenuse is the line segment joining the points of intersection of the graphs of $f(x) = x^2 - 3x + 2$ and $g(x) = x + 4$. Give the area of the triangle.

10. The vertices in of the polygon below have integer coordinates. Find the area of the polygon.



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11. $1,000 is invested at an annual interest rate that compounds quarterly for 12 years, and at the end of that time there is $3,125.43 in the account. Give the interest rate. Note: Answers should be given in the form 13.7226%.

12. $x_0 = 1, x_1 = 4 + \frac{99x_0}{100}, x_2 = 4 + \frac{99x_1}{100}, \ldots$, and the pattern continues. Give the value of x_{127}.

13. $f(x) = x^3 + x + 2$. Solve $f(x) = 3.71$.

14. $f(x)$ is the quadratic function that passes through the points (1,2), (2,5) and (3,-2). $f(2.13) =$

15. Sandra's annual car insurance premium is decreased by 2% in any year when she does not have an accident, and it increases by 10% every time she has an accident, regardless of whether or not it is her fault. The decreases and increases are realized on the following year's policy renewal. Her initial policy was issued on February 1, 1992, with an annual premium of $1132. The policy renews on February 1 of each year. Between February 1, 1992 and January 31, 2015, she had wrecks on the dates 03/15/1991, 09/21/1996, 07/11/2002, 03/21/2010, 11/17/2010, 06/07/2014, and 01/11/2015. What is the annual premium on February 1, 2015?

16. A curve C is parameterized by $(\cos(2t), 2 - 3\sin(t))$. Give the y coordinate of the point of intersection of the curve C with the graph of $y = x^2 + x + 1$.

17. The mass of US coins is given in the table below. Preston has a pile of US coins consisting of pennies, dimes and quarters, with at least one of each. The mass of the entire stack is smaller than 0.25kg. How many possible totals are there for the pile of coins?

Denomination	**Penny**	**Nickel**	**Dime**	**Quarter**
Weight	2.500 g	5.000 g	2.268 g	5.670 g

18. x is 1.52 less than 3 times the difference of 136.5213 and 123.4217. y is 3.7146 more than one third of the sum of the first number and the 43rd prime number.

$$\frac{2x - y}{\cos(x + 3y)} =$$

19. Give the value of a for which the system below has a solution.

$$\begin{pmatrix} 2x - 3y = 6 - 7a \\ -3x + 5y = 7 + 2a \\ 7x + 11y = 4 - 6a \end{pmatrix}$$

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20. All angles in this problem are measured in radians. Positive angles indicate turns in the counter clockwise direction, and negative angles indicate turns in the clockwise direction. A particle starts at the point (1,0) and moves 3 units in a direction that is 0.62 radians with the positive *x* axis. Then it moves 2 units in a direction that is -0.27 radians from the old direction. Subsequent movements are shown in the table below.

1.7 units	in a direction that is 0.13 radians from the previous direction
2 units	in a direction that is 1 radian from the previous direction
1.7 units	in a direction that is -0.42 radians from the previous direction
1 unit	in a direction that is 0.27 radians from the previous direction

Give the *x* coordinate of the final position of the particle.

21. A ball of radius 0.1 is rolling on a flat surface modeled by the portion of the *xy* plane that lies within the rectangle with vertices (0,0), (3,0), (3,2) and (0,2). The ball begins rolling with speed 1 unit per second, and when it strikes a wall, it reflects in the natural way (angle of reflection is equal to the angle of incidence), and loses a fraction of its speed given by $\frac{1}{10}\sin(\theta)$, where θ is the angle in radians of incidence of the collision. The ball starts at (1,1), rolling in the direction of the point (1.1,1.7). Give the *x* coordinate of the ball 4 seconds later.

22. Choose integers *b* and *c* at random, between -5 and 5. Then create the sequence (b_i, c_i), where $(b_0, c_0) = (b, c)$, and $\begin{pmatrix} b_i = 12 + \frac{1}{2}\sin(b_{i-1} + c_{i-1}) \\ c_i = 7 - \frac{1}{3}\cos(b_{i-1} - c_{i-1}) \end{pmatrix}$. Give the integer part of $1201c_{100}$.

23. A triangle is created with vertices $A = (-1, 2)$, $B = (2, 7)$, and a point C on the parabola $y = x^2$ with x coordinate strictly between -1 and 2. Give largest possible area of triangle ACB.

24. Give the sum of all integers between 1 and 1,000 whose reciprocal has the digit 2 in one of the first 5 positions after the decimal point.

25. Give the total area enclosed by the circles of radius 2 centered at (0,0) and the circle of radius 4 centered at (5,3).