

University of Houston Mathematics Contest 2017

Algebra I Exam

Name: _____ School: _____

1. A line passes through the points $(-3, 2)$ and $(6, 5)$. Find the x -intercept of the line.

(A) -10 (B) -9 (C) -8 (D) -7 (E) -6 (F) -5

2. Given the functions $f(x) = x^2 - 2x - 8$ and $g(x) = 9x^2 - 18x$, find the smallest number b such that $f(x) - g(x) \leq b$ for all values of x .

(A) -8 (B) -1 (C) 0 (D) 1 (E) 2 (F) 4

3. Simplify:

$$\frac{\sqrt{2} \cdot \sqrt{6} \cdot \sqrt{12} \cdot \sqrt{20} \cdot \sqrt{30} \cdot \sqrt{42} \cdot \sqrt{56} \cdot \sqrt{72}}{12 \cdot 13 \cdot 14 \cdot 15 \cdot 16}$$

Express your answer as a fraction in lowest terms.

(A) $\frac{2}{13}$ (B) $\frac{3}{13}$ (C) $\frac{9}{13}$ (D) $\frac{1}{26}$ (E) $\frac{3}{26}$ (F) $\frac{9}{26}$

4. Consider the sequence defined recursively by the formula

$$s_{n+1} = f(s_n),$$

where $f(x) = x - 5$. If $s_1 = 2017$, find s_{73} .

(A) 1637 (B) 1642 (C) 1647 (D) 1652 (E) 1657 (F) 1662

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5. Let x be a solution of the equation

$$2x^2 + 24x + 26 = 0.$$

Determine the value of $(x + 6)^2$.

- (A) 18 (B) 23 (C) 28 (D) 33 (E) 38 (F) 43

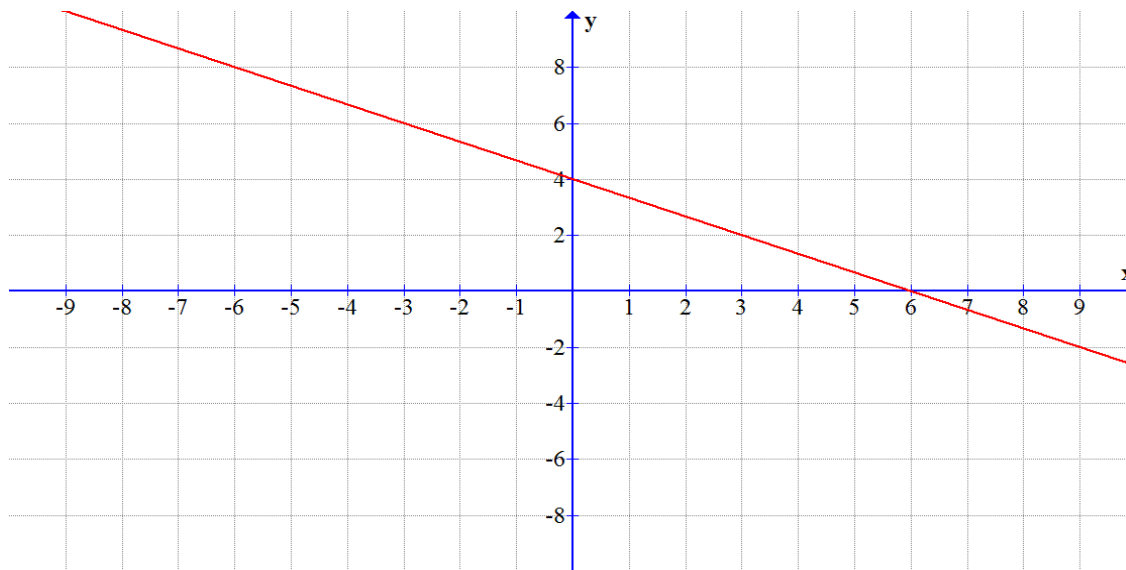
6. Find xy , where x and y satisfy the given pair of equations:

$$6(x + 3) + 8(y - 2) = 18$$

$$9(x - 1) - 2(y + 4) = -14$$

- (A) 1 (B) $\frac{5}{4}$ (C) $\frac{4}{3}$ (D) 2 (E) $\frac{5}{2}$ (F) $\frac{8}{3}$

7. Find the y -intercept of the line which passes through the point $(15, -16)$ and which is parallel to the line whose graph is given below.



- (A) -1 (B) -2 (C) -3 (D) -4 (E) -5 (F) -6

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8. If a , b , c , and d are integers such that

$$\frac{ax + b}{cx + d} = \frac{54x^2 - 150}{18x^2 + 60x + 50},$$

find b/c .

- (A) -5 (B) -3 (C) $-\frac{5}{3}$ (D) $\frac{5}{3}$ (E) 3 (F) 5

9. A department store has a clearance sale on shirts and ties. Kevin buys 4 shirts and 3 ties for 65 dollars, and Jesse buys 7 shirts and 5 ties for 111 dollars. Assuming all shirts sell for the same price, and all ties sell for the same price, how much does Henry pay for his purchase of 3 shirts and 6 ties?

- (A) \$82 (B) \$84 (C) \$86 (D) \$88 (E) \$90 (F) \$92

10. Let $f(x) = \left[\left(\frac{1}{x^2 + 6x - 27} \right)^{5-x} \right]^{\frac{2}{3}}$. Find $f(7)$.

- (A) 4 (B) 16 (C) 36 (D) 64 (E) 144 (F) 256

11. Find the sum of all solutions of the given equation.

$$(2x - 7)(x + 2) = (4x + 3)(2 - x)$$

Express your answer as a fraction in lowest terms.

- (A) -4 (B) $-\frac{7}{4}$ (C) $-\frac{1}{3}$ (D) 1 (E) $\frac{4}{3}$ (F) $\frac{11}{4}$

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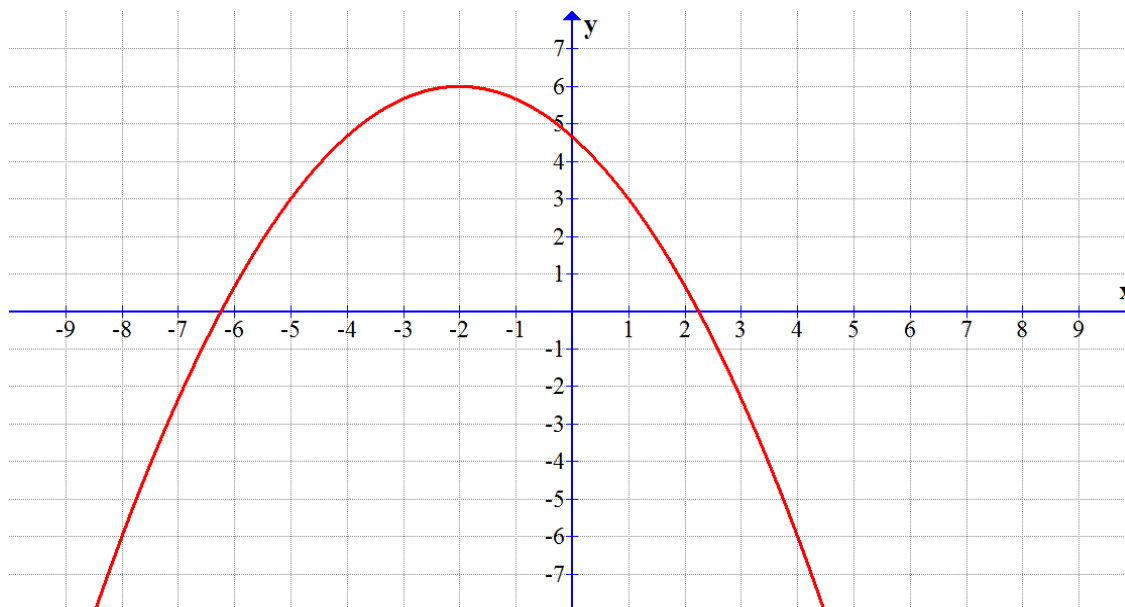
12. Consider a line perpendicular to the x -axis which is given by an equation of the form $Ax + By = C$, where $A + B + C = 12$. If $(3, -4)$ is a point on the line, find the value of C .

(A) 2 (B) 3 (C) 4 (D) 6 (E) 8 (F) 9

13. Simplify: $\left(\frac{\sqrt{864} + \sqrt{216}}{\sqrt{722} - \sqrt{338}}\right)^2$

(A) 12 (B) 18 (C) 27 (D) 32 (E) 48 (F) 108

14. The graph of the quadratic function $f(x) = ax^2 + bx + c$ is shown below. Find $\frac{a+b}{c}$. (Hint: Look for points on the graph of f with integer coordinates. Look carefully!)



(A) $-\frac{3}{22}$ (B) $\frac{5}{22}$ (C) $\frac{3}{14}$ (D) $-\frac{5}{14}$ (E) $-\frac{3}{10}$ (F) $\frac{1}{2}$

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15. If $f(x) = x^2 - 17x - 60$ and $g(x) = x^2 - 6x - 7$, find the largest integer k for which $f(k + 2) < g(k - 5)$.

(A) 41 (B) 42 (C) 43 (D) 44 (E) 45 (F) 46

16. Adventure Cruises charges 270 dollars per person for parties of up to 10 people. For parties larger than 10 people but less than 30 people, the cost per person decreases by 5 dollars for every additional passenger after the 10th person. For example, a party of 14 people would pay 250 dollars per person. If the total cost for a party of n people is 5,040 dollars, and if p is the price per person, find $p - n$.

(A) 138 (B) 152 (C) 186 (D) 208 (E) 232 (F) 262

17. Consider the function

$$f(x) = \frac{12^{3x+5}}{2^{7x+8} \cdot 6^{3x+1}}.$$

Find $f(0.25)$.

(A) $\frac{3}{8}$ (B) $\frac{4}{9}$ (C) $\frac{27}{8}$ (D) $\frac{9}{2}$ (E) 27 (F) 81

18. A space probe transmits a signal to Earth every hour. The intensity of the signal at time t is given by a function $I(t) = ab^t$ where a and b are positive rational numbers. If the intensity of the signal decreases by 36% every two hours, and the intensity of the signal is 20 units at time $t = 0$, determine ab .

(A) $\frac{8}{5}$ (B) 6 (C) $\frac{36}{5}$ (D) 12 (E) $\frac{64}{5}$ (F) 16

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19. Each day a store owner makes 7 dollars more than she made the previous day. If the the owner made 192 dollars on January 20, 2017, how much money did she make on December 27th, 2016?

(A) \$3 (B) \$10 (C) \$17 (D) \$24 (E) \$31 (F) \$38

20. A cashier at a concession stand begins the day with 10 twenties, 20 tens, 40 fives, and 80 ones. After one hour, the ratio of tens to twenties in the cash register was the same as the ratio of fives to ones. If every customer purchased a 1 dollar drink and paid with a twenty dollar bill, how many customers were served? (Assume the concession stand never ran out of cash, and that each customer was given exact change consisting of the fewest number of bills possible.)

(A) 6 (B) 8 (C) 10 (D) 12 (E) 14 (F) 16

21. Find the difference of the x -intercepts (largest minus smallest) on the graph of the quadratic function which has a vertex at $(2, 9)$ and which passes through the point $(5, -27)$.

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5 (F) 6

22. How many (unordered) pairs of positive integers sum to 180 and have a greatest common divisor of 6?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6 (F) 7

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23. The ratio of women to men at a local college is $3 : 5$. If the ratio of women to men among seniors is $4 : 3$, and if seniors make up one sixth of the student body, what fraction of non-senior students are male?

- (A) $\frac{93}{140}$ (B) $\frac{69}{112}$ (C) $\frac{62}{105}$ (D) $\frac{51}{70}$ (E) $\frac{31}{56}$ (F) $\frac{23}{35}$

24. If the 11th term in the sequence

$$\frac{3645}{112}, \frac{1215}{56}, \frac{405}{28}, \frac{135}{14}, \dots$$

is written in the form a/b where a and b are positive integers with no common factors, find $b - a$.

- (A) 29 (B) 67 (C) 143 (D) 247 (E) 539 (F) 1061

25. Determine the number of even positive integer divisors of 1764.

- (A) 15 (B) 16 (C) 17 (D) 18 (E) 19 (F) 20

26. The number of cars produced in a week by an auto factory is directly proportional to the number of workers at the factory. If 6 additional workers are hired, then the factory will produce 20% more cars. On the other hand, a 30% decrease in the number of workers will result in 6 fewer cars produced per week. If W represents the number of workers currently employed by the factory, and C represents the number of cars currently produced per week, find $W + C$.

- (A) 46 (B) 48 (C) 50 (D) 52 (E) 54 (F) 56

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27. Determine the number of ordered pairs of integers (x, y) that satisfy the inequalities $y - x < 6$, $y + x > -2$, and $x < 2$.

- (A) 25 (B) 28 (C) 30 (D) 36 (E) 45 (F) 49

28. Two runners each complete a 10 kilometer race. If their combined time is 128 minutes, and the difference in their running speeds is 5 kilometers per hour, find the sum of their speeds in kilometers per hour.

- (A) 15 (B) 16 (C) 17 (D) 18 (E) 19 (F) 20

29. Two lines intersect at the point $(2, 4)$. The product of their slopes is -2 , and the quotient of their y -intercepts is 4. Assuming that neither line has slope 1, find the product of their x -intercepts.

- (A) -32 (B) -24 (C) -20 (D) -18 (E) -12 (F) -8

30. Assume x , y , z , and w are positive integers such that the following three equations hold:

$$y^z = w^x, \quad y = 4w, \quad x - z = 1.$$

Find the smallest possible value of $\frac{y^x}{w^z}$.

- (A) 4 (B) 8 (C) 16 (D) 32 (E) 64 (F) 128