

**University of Houston**  
**High School Math Contest – 2017**  
**Algebra II Test**

1. In a classroom, the ratio of the number of boys to the number of girls is  $\frac{3}{7}$ . 20% of all boys in this class play football. What percent of this class consist of boys who do not play football?

A) 12%  
B) 24%  
C) 25%  
D) 30%  
E) 45%  
F) None of these

2. Find the product of all real roots of the equation:

$$(2x + 5)(x - 2) + (2x - 5)(x + 4) = 0.$$

A) 1  
B) -1  
C)  $\frac{15}{2}$   
D)  $-\frac{15}{2}$   
E) -30  
F) None of these

3. Given:  $x + 4 = y + 6 = z + 8 = x + y + z + 15$ . Find the value of the sum:  $x + y + z$ .

A) 6  
B)  $-\frac{25}{2}$   
C) -12  
D) -9  
E)  $-\frac{27}{2}$   
F) None of the above

4. Let  $f$  and  $g$  be two functions defined on the set of positive real numbers  $\mathbb{R}^+$ . Given:

$$(f \circ g)(x) = f(x) \cdot g(x) \quad \text{and} \quad f(x) = 2x + 3,$$

find the value of  $g(1)$ .

- A) 1
- B) 2
- C) 3
- D) 4
- E) 5
- F) None of these

5. Simplify:  $\frac{3^{2017} \cdot 4^{1010}}{\sqrt{6^{4032}}}$

- A) 96
- B) 24
- C) 48
- D) 72
- E) 144
- F) None of these

6. Operations  $\otimes$  and  $*$  are defined as:  $a \otimes b = \frac{ab}{a+b}$ , and  $a * b = \frac{a}{a-b}$ .

Find the value of:  $(2 \otimes 4) * (6 \otimes 2)$ .

- A)  $3/12$
- B)  $-11/6$
- C) 8
- D) -8
- E)  $-4/3$
- F) None of these

7. Consider the rational function:  $f(x) = \frac{x^2 - 3x - 4}{x^2 - 1}$ .

Let  $k$  be the smallest positive integer such that  $f(k) > 0$ .

Let  $m$  be the largest negative integer such that  $f(m) > 0$ .

Find the sum:  $k + m$ .

- A) 4
- B) 3
- C) 2
- D) 1
- E) -1
- F) None of these

8. Let  $n$  be a positive integer and  $i^2 = -1$ . Simplify:  $\frac{i^{8n-1} - i^{2017}}{i^{4n-1}}$

- A)  $1 + i$
- B)  $1 - i$
- C)  $-i$
- D)  $-2$
- E)  $2$
- F) None of these

9. Let  $f(x) = 2e^{3x}$ ,  $g(x) = 2 \ln(2x)$ , and  $h(x) = |x^2 - 5|$ .

Evaluate:  $(g \circ f \circ h)(-2)$ .

- A) 8
- B)  $6 \ln(4)$
- C)  $6 \ln(16)$
- D)  $6 + \ln(16)$
- E)  $16 + \ln(4)$
- F) None of these

10. The product of 3 consecutive positive even integers is 32 times their sum. Find the sum of their squares.

- A) 116
- B) 200
- C) 308
- D) 440
- E) 596
- F) None of these

11. Let  $x$  and  $y$  be real numbers. Given:  $|x-2|=|x-3|$  and  $|y-3|=|6-y|$ , find the value of  $x+y$ .

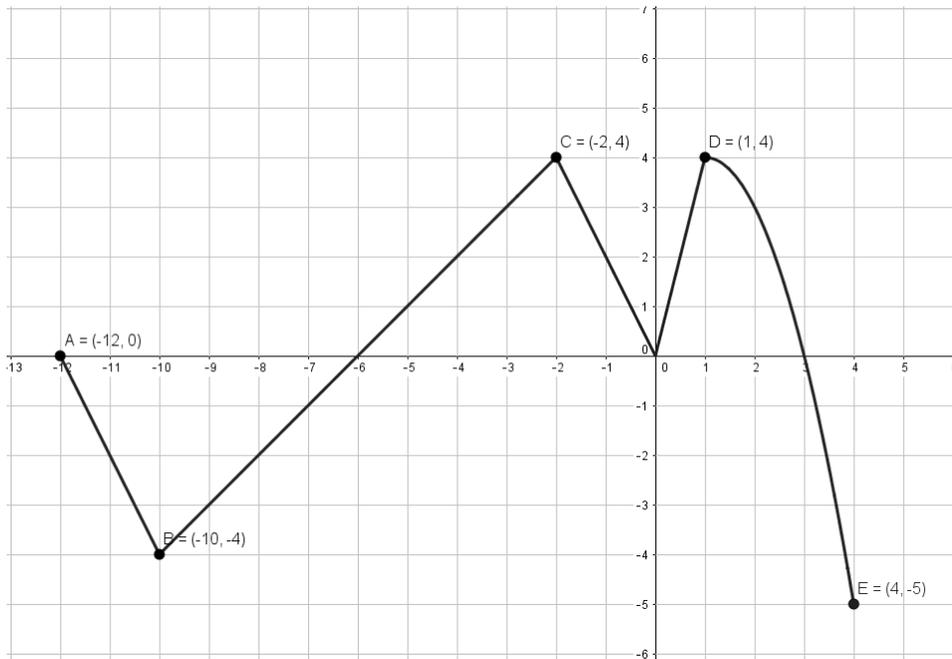
- A) 6
- B) 7
- C) 9
- D) 10
- E) 11
- F) None of these

12. An item is priced as 40% above its cost for the store. During a sale, the price is reduced by 15% of the ticket price. Later, the item was placed at clearance to be sold for half of the reduced price. If the item is sold for this price, what is the loss of the store?

- A) 40.5%
- B) 41.5%
- C) 59.5%
- D) 19%
- E) 58.5%
- F) None of these



16. Consider the function  $f(x)$  whose graph is given below. How many solutions does the equation  $f(f(x)) = 4$  have?



- A) 4  
 B) 5  
 C) 6  
 D) 7  
 E) 8  
 F) None of these.
17. Consider two functions  $f$  and  $g$ ; both have the domain  $\mathbb{Z}$  (the set of integers),

$$f(n) = n + \frac{1}{3} \quad \text{and} \quad g(n) = n + \frac{1}{6}.$$

For which of the following functions would the range (image) consist of integers only?

- I.  $f \circ f \circ f$   
 II.  $f \circ g \circ f$   
 III.  $g \circ f \circ g$
- A) I only  
 B) II only  
 C) I and II only  
 D) I and III only  
 E) I, II and III  
 F) None of these

18. A performance theatre has 10,000 seats. The ticket prices are \$25 for kids and \$35 for adults. Assume that every seat is sold and the return is \$275,000. Let  $x$  be the number of tickets sold to kids, and  $y$  be the number of tickets sold to adults. Find the value of  $\frac{x}{y}$ .

- A) 2
- B)  $\frac{3}{2}$
- C)  $\frac{1}{3}$
- D) 4
- E) 3
- F) None of these

19. Let  $\{a_n\}$  be an arithmetic sequence. The sum of the first 50 terms of this sequence is 200 and the sum of the second 50 terms is 400. Find the value of  $a_{11}$ .

- A) 2.04
- B) 2.12
- C) 3.12
- D) 2.84
- E) 1.82
- F) None of these

20. Solve the system:

$$2x + 2y + 3z = 13$$

$$-3x + 4y - z = 5$$

$$5x - 3y + z = 2$$

What is the value of  $xy + x^2z$ ?

- A) 18
- B) 15
- C) 8
- D) 10
- E) 12
- F) None of these

21. Consider the quadratic equation:  $2x^2 - 42x + c = 0$ . Given that both roots of this equation are prime numbers, find the value of  $c$ .

- A) 38
- B) 34
- C) 76
- D) 78
- E) 82
- F) None of the above

22. Let  $m$  and  $k$  be positive integers such that  $\frac{1}{2} + \frac{1}{3} + \frac{1}{7} + \frac{1}{m} = k$ .

Find the value of:  $\frac{m^2 - k^2}{m + \frac{1}{k}}$ .

- A) 84
- B) 82
- C) 81
- D) 42
- E) 41
- F) None of these

23. May's room is twice as big as John's room and three times as big as David's room.

Assume that all rooms are empty. David's iRobot cleans half as fast as John's iRobot, and one third as fast as May's iRobot. If they all start the robots at the same time, whose iRobot will finish the first?

- A) May's
- B) John's
- C) David's
- D) May and David's robots tie for first place.
- E) John and David's robot tie for first place.
- F) None of these

24. Let  $f(x)$  be a linear function and  $g(x)$  be a quadratic function that has only one  $x$ -intercept. These functions intersect at points  $(0,0)$  and  $(3,1)$ . Find the value of  $f^{-1}(6) + g(9)$ .

- A) 27
- B) 15
- C) 11
- D) 99
- E) 83
- F) None of these

25. Given that  $x$  and  $y$  are real numbers satisfying:

$$-9 \leq 2x + 1 \leq -3 \quad \text{and} \quad |y - 4| \leq 3, \text{ respectively.}$$

Find the largest possible value of:  $\frac{4x + y}{x}$ .

- A)  $18/5$
- B)  $23/6$
- C)  $19/5$
- D)  $11/3$
- E)  $31/8$
- F) None of these

26. Let  $x$  (where  $x > 0$ ) be the solution for the equation:

$$5^{6+12+24+\dots+6x} = (0.04)^{-108}.$$

Find the value of:  $\sqrt[3]{x^3 - 3x^2 + 3x - 1} + \sqrt{4 - 4x + x^2}$ .

- A) 0
- B) 13
- C) 14
- D) 16
- E) 17
- F) None of these

27. A company manufacturing cylindrical tanks decided to change the dimensions of the tanks without changing the volume. The diameter of each tank is decreased by 25%. All Type I tanks originally had a height of 90 inches. What should the height of the Type I tanks be after company changed the dimensions?

- A) 13 feet and 9 inches.
- B) 13 feet and 6 inches.
- C) 12 feet and 10 inches.
- D) 13 feet and 4 inches.
- E) 12 feet and 8 inches.
- F) None of these

28. Let  $f$  be a function defined as:

$$f(2x) = \begin{cases} 4, & \text{if } x \leq 1, \\ x \cdot f(x), & \text{if } x \text{ is an integer greater than } 1, \\ 0, & \text{otherwise} \end{cases}$$

Find the value of:  $f(128)$ .

- A)  $2^{24}$
- B)  $2^{10}$
- C)  $2^{21}$
- D)  $2^{22}$
- E)  $2^{23}$
- F) None of these

29. Let  $x$  and  $y$  be the solutions of the following equations:

$$\log 100 + \log \sqrt{1+x} + 3 \log \sqrt{1-x} = \log \sqrt{1-x^2}$$

and  $y$  be the solution of the equation:

$$\log_3 4 \cdot \log_4 5 \cdot \log_5 6 \cdot \dots \cdot \log_{y+1}(y+2) = 4$$

Find the value of the expression:  $\frac{10}{9}x(y+1)$ .

- A) 160/9
- B) 88
- C) 800/9
- D) 820
- E) 82
- F) None of these

**The following questions are part of this test, but they are not multiple choice. For the following 3 questions, write your answer on the answer sheet as a number. For example:**

**25, 0, 4.5, -2.7,  $1+2\sqrt{3}$ ,  $4\sqrt{3}$ ,  $1/4$ ,  $12/13$  or  $50/11$**

**are acceptable answers. Radical expressions should be reduced; for example,  $\sqrt{12}$  should be written as  $2\sqrt{3}$ . Show your work on the empty space below each question and write your final answer on the answer sheet.**

30. Let  $R$  be the region **above the**  $x$ -**axis** consisting of all points satisfying the inequalities:

$$y \leq -\frac{2}{3}|x - 2| + 3$$

$$|x - 1| < 4$$

$$|y + 2| \geq 3$$

List all points in this region that have integer coordinates in the form  $(x, y)$ . What is the sum of the  $x$ -coordinates of all points in this list?

31. Let  $\triangle ABC$  be a right triangle with right angle  $C$ . Let  $M$  be the midpoint of side  $AC$  and  $N$  be the midpoint of side  $BC$ . If the length of  $AN$  is 12 and the length of  $BM$  is 16, find the length of  $AB$ .

NAME \_\_\_\_\_

SCHOOL \_\_\_\_\_

32. Sam's school starts at 8:00 am. According to his calculations, if he drives at an average speed of 30 miles per hour, then he will be late by 5 minutes. If he drives at an average speed of 45 miles per hour instead, he will be early by 5 minutes. Due to traffic, for the first  $\frac{1}{5}$ <sup>th</sup> of the distance, he travels with an average speed of 12mph. For the remaining distance, his average speed was 36 mph. How many minutes did it take to reach the school?

*THE END.*

*Make sure your answers are recorded on the answer sheet.*